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MINE PLANT DESIGN

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BY

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SECOND EDITION

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MINE PLANT DESIGN

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PREFACE TO THE SECOND EDITION

This edition of "Mine Plant Design" has been expanded to include much material suggested by engineers in the field and teachers of mining engineering.

Four new chapters have been added. The chapters on ore bins, skips and cages, and safety catches are new to books on plant design.

The chapter on headframes has been rewritten to include wooden headframes and more complete design data for headframes in general.

For the chapter on mine ventilation it was thought desirable to use an example for illustrative purposes instead of the usual textbook method of employing confusing formulas.

The author wishes to express his appreciation to the various mining companies that have supplied him with drawings and have willingly answered questions, and to John Wiley & Sons, Inc.; the *Canadian Mining Journal;* the Canadian Institute of Mining and Metallurgy; *The Journal of the South African Institution of Engineers;* the Transvaal Chamber of Commerce; *The Compressed Air Magazine;* the Ingersoll-Rand Company; the Jeffrey Manufacturing Company; the Ontario Department of Mines; and many others.

W. W. STALEY

MOSCOW, IDAHO
May, 1949

PREFACE TO THE FIRST EDITION

In preparing this book, it has been the author's intention to present a practical discussion of the methods and procedure for selecting mine-plant equipment and simple structures. He realizes that other methods of approach are available and that the field is by no means completely covered.

The selection of proper equipment for the operation of a mine plant is of primary importance. A good potential property may become a liability if care is not used in the choice of equipment. In many cases such equipment will be purchased secondhand or will be transferred from one part of a property to another. Therefore, it becomes necessary, within reasonable limits, to determine the size and nature of the requirements. In other instances manufacturers' representatives are called upon for estimates and for assistance with the designing. In such cases it is also desirable that their recommendations be verified by the engineer or superintendent. Such a procedure often saves time and financial embarrassment.

The man on the job is far better equipped to understand his needs correctly than is a stranger who may, through necessity, have to get his data by correspondence and who may consequently not select the most suitable equipment. Mining properties located in more or less isolated and inaccessible regions with respect to manufacturing centers or well-equipped designing concerns may find themselves at a loss for proper guidance. It then becomes highly desirable for the mine operator to be able to select properly the equipment best suited to his needs. He is not ordinarily required to make a detailed design.

It might be well to outline briefly what is meant by mine plant. We may consider the subject as being divided into two main parts: surface and underground. The surface plant will consist essentially of buildings and equipment contained therein and headframes. We shall not here consider crushing or other ore-dressing apparatus. The buildings, in general, will be of two types: those for housing the office and supervising staffs, and those containing hoists, compressors, power-generating equipment, machine shops, drill-sharpening layout, and warehouses. Many of the larger companies have some of the above operations located underground. The underground layout consists mainly, from our point of view, of pumping, transportation, drilling, and ventilation equipment.

Fundamental mathematical, physical, or theoretical considerations will be utilized where a clearer understanding will be obtained by so doing. The main purpose sought is to offer, with the least confusion, material of practical value and not the physical laws upon which certain phases of mining are necessarily based. With this in mind the contents have been organized to serve as an introduction to the subject for students of mining engineering and also to answer the need of mine operators and engineers.

At first sight it may appear that acknowledgments to source material are rather freely made. The author feels that such continuous reference to original sources may induce the reader to consult them and thus enlarge his knowledge of details. In addition, the final solution of a given problem may depend on a more extensive insight into the subject than it was felt was justified here; if so, the source will be readily available.

The material has been drawn from a great many sources. In fact, it was because of the wide distribution of the desired information that the book was conceived. To the author's knowledge no single publication contains the data embraced herein. Many books and technical articles cover the individual sections more or less completely, as do handbooks, circulars, etc., of the various manufacturers of equipment.

The author wishes to acknowledge his appreciation to Prof. J. E. Buchanan, Department of Civil Engineering; Mr. Barton Cruikshank, Department of Mechanical Engineering; Prof. R. H. Hull, Department of Electrical Engineering; and Mr. L. L. Schuldt, Department of English, all of the University of Idaho, for giving their time freely to criticizing the material in their respective fields. To Mr. A. J. Nicht, Jr., engineer, Hoisting Division, Allis-Chalmers Manufacturing Company, the author is especially indebted for data and criticism on the chapter on mine hoists. He also wishes to express his gratitude to *Engineering and Mining Journal; Engineering News;* John Wiley & Sons, Inc.; Scientific Book Corporation; American Institute of Mining and Metallurgical Engineers; U. S. Bureau of Mines; the late Mr. M. S. Ketchum; Allis-Chalmers Manufacturing Company; Atlas Car and Manufacturing Company; Goodman Manufacturing Company; General Electric Company; Edison Storage Battery Company; Electric Storage Battery Company; Philco Radio and Television Corporation; Gould Pumps, Inc.; John A. Roebling's & Sons; Ingersoll-Rand Company; Jeffrey Manufacturing Company; Mr. C. A. Newton,

Fairbanks, Morse Company; Westinghouse Electric and Manufacturing Company; Fairbanks, Morse Company; The Aluminum Company of America; and International Nickel Company, for permission to use material.

W. W. STALEY

MOSCOW, IDAHO
February, 1936

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CHAPTER I

GRAPHICAL SOLUTION OF SIMPLE BEAMS AND ROOF TRUSSES

Before beginning a discussion of trusses or headframes, a very brief review of a few of the essential fundamentals relating to the graphical solution of stresses will be taken up. Only those principles which are directly related to the material to be presented here will be discussed. A lengthy dissertation on graphic statics is out of the question.

Force may be defined as the action of one body on another which causes or which tends to cause motion. We have, in general, five types of forces that may act on framed structures: dead loads, snow loads, wind loads, concentrated loads, and live loads. The ultimate result of such loads is compression, tension, shear, or bending in the member.

When two or more forces act in such a way as to neutralize themselves, they are said to be balanced, or in equilibrium. Under the action of a single force a body cannot be in equilibrium.

In order that the effect of a force be known, three things must be considered: (1) the point of application, (2) the magnitude, and (3) the direction. All three of these are necessary to define a force completely. In order to make a graphical or analytical solution, not more than two of the forces can lack being completely defined.

The point of application is usually known and it is the point at which the force is applied and its effect exerted. In Fig. 1*a*, the point of application of the forces F_1, F_2, F_3, and F_4 is at O.

The direction of the force is the line along which the point of application tends to move; for instance, the direction of the force F_1 (Fig. 1*a*) could be represented by the angle α which the force makes with the horizontal, and an arrowhead showing the direction.

By magnitude of a force is meant the pounds, tons, or other units that give the value of the force acting. The magnitude may be represented by the length of a line to some convenient scale.

Force Polygon. A force polygon may be closed (*i.e.*, be in equilibrium) or open (not in equilibrium). The stress diagram of a truss or other structure is simply the combination of all the individual force polygons for each joint. Figure 1*b* is the force polygon representing the system of forces shown in Fig. 1*a*. It would be identified as the force polygon *abcd*. By means of a pair of triangles, *ab* is transferred parallel to F_1. (It is immaterial which force we start with). The length of *ab* depends on the scale chosen to represent the magnitude of

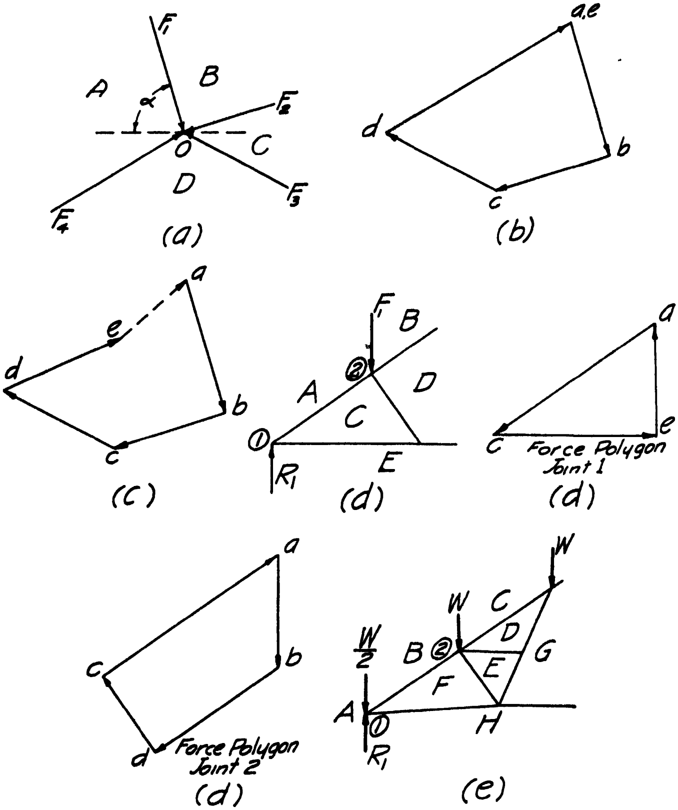

FIG. 1.

the force F_1. For example, if $F_1 = 10$ lb, and the scale selected was 1 in. = 5 lb, then *ab* would be 2 in. in length. From *b*, *bc* is drawn parallel to F_2. The force F_3 is represented by *cd*. If the line *de*, representing F_4, coincides in length (magnitude) and direction with *da*, the system of forces in Fig. 1*a* is in equilibrium (it is a closed polygon). Thus we have the equilibrium polygon. On the other hand, if the point *e* does not coincide with the point *a*, the forces are not in equilibrium (the polygon is open). This condition is shown in Fig. 1*c*. Here the additional force *ea* is necessary to give equilibrium.

In the solution of truss and headframe problems with which we are primarily interested, we have for solution at each joint a system of internal forces (stresses) holding in equilibrium a system of external forces. They form a system similar to that shown in Fig. 1*a*. The external forces, at least so far as we are concerned, are always known. The internal forces, represented by the members of the structure, are usually unknown with respect to their magnitude. It is the stresses in the members that we desire. The magnitude of the internal stresses can be found by the use of individually constructed force polygons, as illustrated by Fig. 1*d*. It is much more convenient, however, to combine the separate polygons into one large stress diagram (see stress diagram, Fig. 8). At joint 1, Fig. 1*d*, there are one known force R_1 and two unknown stresses AC and CE. By laying off *ea* parallel to R_1, and giving it a length representing the magnitude, and drawing *ac* from *a* parallel to AC and *ce* parallel to CE, the force polygon for joint 1 is constructed. The intersection of *ac* and *ce* locates the point *c*. The length of *ac* and *ce* also represents the magnitudes of the stresses AC and CE. At joint 2 we have known F_1, and from joint 1, *ac*. The construction for joint 2 is similar to that for joint 1, and gives the polygon *abdc*. In like manner, it is possible, by these individual polygons, to find values for all the unknown stresses in a system that is statically determinate.

Forces are, in general, of one of two kinds, concurrent or nonconcurrent. They may also be coplanar or noncoplanar. If a system of forces is not coplanar, the forces may be made so by resolving them into the same plane. The component of this resolution will have the same effect as a coplanar force, as would the original force acting as a noncoplanar force. Concurrent forces are those whose lines of action, if produced, would all intersect in the same point. Forces acting at the panel points of a truss or headframe are concurrent (see joint 2, Fig. 1*d*). Nonconcurrent forces do not do this. Some may intersect, but all do not in the same point. In the solution of most beams, trusses, and headframes, the external forces acting are nonconcurrent (see Fig. 1*e*; forces $W/2$ and W are nonconcurrent). The internal forces acting on any one panel point are usually concurrent (Fig. 1*e*: At joint 2, CD, DE, EF, and FB are concurrent). The resultant of any number of forces is the single force that will produce the same result as the combined action of all the forces acting.

It must be kept in mind that if more than two of the forces acting at a point are not completely defined (point of application, magnitude, and direction) a direct graphical solution cannot be made. Figure 1*e* illustrates this important point. The forces *AB* and *BC* are completely defined. The stresses (internal forces) in *BF* and *FH* are readily determined, as previously explained. At joint 2, *FB* and *BC* are known. The stresses in *CD*, *DE*, and *EF* are indeterminable graphically except by special methods to be discussed later (page 20). Their direction and point of application are known but their magnitudes are not. If the member *DE* and *EF* is removed, the problem can be solved. It must be remembered, therefore, that to make a graphical solution, not more than two forces may be unknown. In attacking a problem a point should be selected where only two unknowns are encountered. It may, in some cases, be necessary to jump from one point to another to complete a solution of a given problem.

The stress resulting from the application of a force or system of forces may have one of three senses: zero, tension, or compression. In the case of tension or compression, arrowheads are used to denote the sense of the stress. In this discussion, arrowheads pointing toward each other indicate tension and arrowheads pointing away from each other denote compression. The utilization of this method will be illustrated later. In addition to arrowheads, plus and minus signs are used for compression and tension, respectively.

In the solution of problems by graphical methods we make use of the equilibrium polygon, the force polygon, and the stress polygon. The use of these three figures will be demonstrated by applications to follow. The forces of the polygons must be in a state of equilibrium in order that a complete graphical solution be attained.

ILLUSTRATIONS

A system of nonconcurrent forces is shown in Fig. 2. Their directions and magnitudes are known. The point of application is of no consequence. Expressed in terms of unity the values of F_1, F_2, F_3, F_4, and F_5 are, respectively, 0.75, 1.00, 0.50, 1.25, and 1.50. It is necessary to find the resultant R of these forces and to resolve R into two parallel components V_1 and V_2 passing through the points k and l. It will be noted that the forces, in addition to being distinguished by F_1, F_2, etc., are also located by the letters *BC*, *CD*, *EF*, and *FG*, and that the order of reading advances to the right or clockwise. A clockwise procedure will be used throughout our discussion. The components, V_1 and V_2 are represented by the letters *BA* and *GH*, respectively.

To determine R, V_1, and V_2, we proceed as follows:

The force diagram is drawn, which consists of the forces *bc*, *cd*, *de*, *ef*, and *fg*, parallel to *BC*, *CD*, etc., and which have their magnitudes represented by any suitable scale. Connecting *b* and *g* gives the direction of the resultant force that could be substituted in direction and magnitude for the series of forces. To determine the values of the components and the line of application of *R*, locate the point *O*, called the "pole," at any convenient point. The pole is ordinarily located a distance from the force line equal to about one-half the length of the force line and about in the center of this line. The same results are obtained regardless of its position. The points *b*, *c*, *d*, *e*, *f*, and *g* are connected with *O* by lines called "rays." The rays, in their proper sequence, are transferred to, and made to intersect, the proper force line in the equilibrium polygon. They are now called "strings." The line of action of a force may be prolonged any distance necessary to bring about such an intersection.[1] Starting with the ray *Ob*, we find that it connects the force F_1 with the component V_1, but as V_1 is

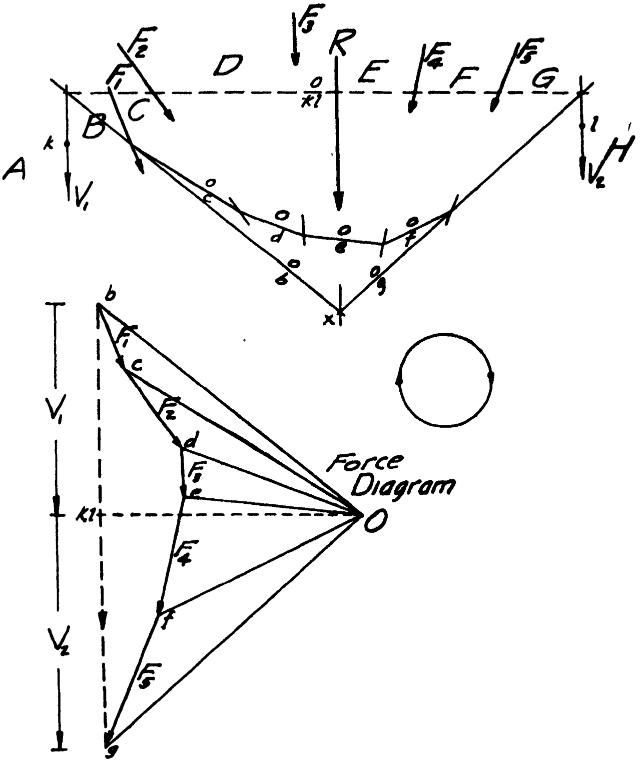

FIG. 2.

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[1] In mechanics this is known as the "principle of transmissibility."

for the present unknown, it cannot be transferred just yet as a part of the equilibrium polygon. The ray *Oc* joins the forces F_1 and F_2. It is transferred to the equilibrium polygon parallel to its direction in the force diagram, intersecting F_1 and F_2. Ray *Od* is transferred in a like manner, intersecting *Oc* on F_2, and giving the string *od*. The remainder of the rays are transferred in a like manner. The strings *ob* and *og* are now drawn, beginning at the intersection of *oc* with F_1 and *of* with F_5, respectively, and intersecting at *x*. If *bg* is transferred to *x*, the intersection of *ob* and *og*, we shall have the line of action of *R*. To resolve *R* into the two components V_1 and V_2, passing through *k* and *l*, *ob* is extended to intersect V_1 drawn through *k*, parallel to *R*, and *og* is extended to intersect V_2 drawn through *l*, parallel to *R*. These points of intersection are connected by the string *okl*, thus closing the equilibrium polygon. Then in the force diagram the ray *Okl* is drawn through *O* parallel to the string *Okl* to intersect *bg*. The value of V_2 is then *gkl* and that of V_1 is *klb*.

It will be noted that arrowheads are placed on F_1, F_2, etc., indicating the direction in which the forces are acting. The arrowheads on the closing line *gb* point in the opposite direction.

Simple Beams. The design of a beam will usually depend on the bending moment and the shear in the beam resulting from the nature and position of the loads supported. It is customary to express bending moment in inch-pounds, and shear in pounds per square inch. In Fig. 3 is shown a simple beam. It is acted upon by the forces F_1, F_2, F_3, and F_4, with the lines of action as shown in the figure. The values of the forces are, in units, 0.53, 0.5, 1.0, and 1.0, respectively. We desire the values of the reactions R_1 and R_2 and the maximum bending moment. The force diagram is drawn as indicated. The forces *ab*, *bc*, *cd*, *de* taken in clockwise order are located. The line *ea* gives the direction of the reactions. If this line is vertical, it means that the sum of the horizontal components is zero. If not vertical, the direction of the closing line will indicate the direction in which the beam tends to move. After connecting *ea*, we next project *ab* on to *ea*, giving *ab′* the vertical component of the force F_1. The remaining forces are likewise projected, the projections in each case being perpendicular to *ea*. The pole *O* is located a distance *H*, known as the "pole distance," from *ea*. The same scale and unit are used in locating *H* as are used in locating the forces F_1, F_2, etc., and are always in pounds or whatever unit of force is used for the loads. The ray *H* must be at right angles to *ea*. Rays are drawn from *O* to *a*, *b′*, *c′*, *d′*, and *e*. The equilibrium polygon is now drawn. The ray *Oa* connects the reaction R_1 with the force F_1.

It is transferred so as to intersect R_1, and the vertical projection of F_1 extended to intersect string *oa*. Rays *Ob′*, *Oc′*, *Od′*, and *Oe* are also transferred. The points at which *oa* and *oe* intersect R_1 and R_2 are connected by the closing string *of* of the equilibrium polygon. The line *of* is transferred to the force diagram passing through *O* and intersecting *ea* at *f*. The distance *ef* then represents the magnitude of R_2 and *fa* the magnitude of R_1. It is to be noted that the forces acting on the beam when taken in a clockwise direction check this. The forces taken in such order are *AB*, *BC*, *CD*, *DE*, *EF* (R_2), and *FA*(R_1). The values of the reactions are found by scaling the distances *ef* and *fa* and multiplying by the factor used. The bending moment at any point in the beam is determined from the equilibrium diagram by multiplying the intercept for that section of the beam by the pole distance. At R_1 the intercept is zero, at F_1 it is 1-1′, at F_2, 2-2′, etc. The maximum

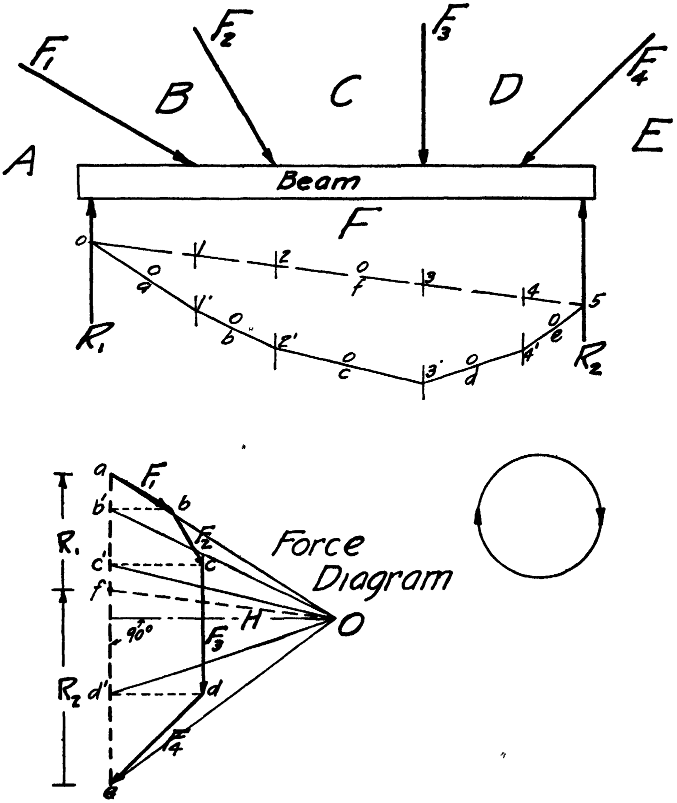

FIG. 3. Bending moment in a simple beam.

intercept is found by inspection to be 3-3′. In determining the value of the intercept, the same scale must be used as was used for laying out the drawing.

$$\text{Maximum bending moment} = M = 3\text{-}3' \times H$$

A cantilever beam is illustrated in Fig. 4. The solution is identical with that of Fig. 3. The forces are laid out in clockwise order.

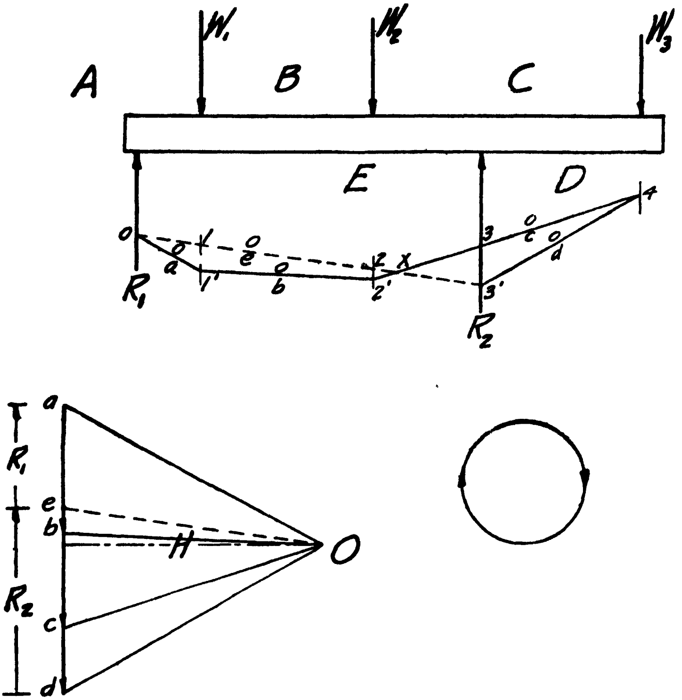

FIG. 4. Bending in a simple cantilever beam.

Attention is called to string *oc*, which is drawn from the extension of W_2 to W_3. From W_3 string *od* is drawn to the reaction R_2. The maximum bending moment is found to be at R_2. The diagrams were drawn to a scale of 1 in. = 1 ft for the beam, and 1 in. = 1 lb for the forces; $H = 2$ lb.

$$\text{Maximum bending moment} = M = 3\text{-}3' \times H = 0.31 \times 2 = 0.62 \text{ ft-lb}$$

Simple Derrick. The stresses in a simple derrick may be determined as shown in Fig. 5. This type of apparatus is quite commonly used in construction work. The load *W* represented by *DA* is laid out to a suitable scale. Examining the diagram, we find that *DA* is held in equilibrium by the two internal forces *AC* and *CD*. We can, therefore, start the solution of the problem at this point. From *d*, *dc* is drawn parallel to *DC*, and from *a*, *ac* is drawn parallel to *AC*. Their intersection locates *c*. It is next found that *CA*, *AB*, and *BC* form an equilibrium triangle. From *a*, *ab* is drawn to intersect *bc*, thus locating *b*. The reaction *BD* and the members *DC* and *CB* are in equilibrium. Connecting *b* and *d* gives the reaction *R*. The nature or sense of the stresses indicated in the stress diagram must be determined now. If the members are considered in clockwise order about

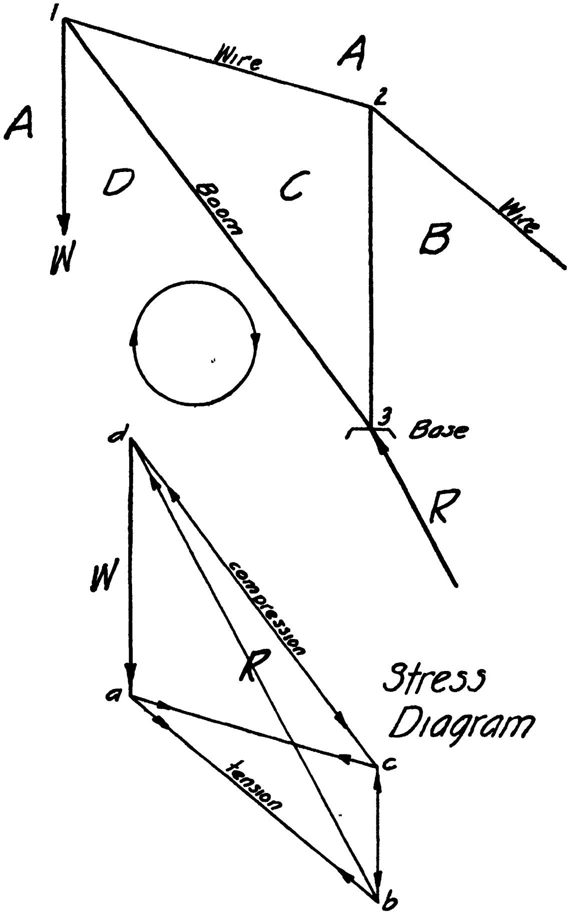

FIG. 5. Stresses in a simple derrick.

a joint, it is found that those construction lines in the stress diagram which are drawn away from the joint under consideration are tension members; and those drawn toward the joint are compression members. It is immaterial at which joint we start or the order in which the joints are taken so long as the members about the joint are taken clockwise. Counterclockwise would do just as well provided that the external forces had been laid out in counterclockwise order. As previously mentioned, the clockwise order will be followed in all solutions given in this text. If, through error, the order is reversed, the sense of the members is reversed, *i.e.*, a tension member becomes a compression member. In most structures such a mistake would be a calamity. Starting at joint 1, we proceed as follows: The line *ac* is drawn *away* from the joint in order to intersect *cd*. (We imagine we start out going from *a* to *c* in the stress diagram; but suddenly find ourselves on *AC* going in the same direction, which will be away from the joint.) Member *AC* is therefore in *tension* and an arrowhead is placed near, but pointing away from, *a*. Passing to *CD*, we see that *cd* is drawn toward the joint and is therefore in *compression*. An arrowhead is placed near, and pointing toward, *c*. The load *DA* is found from *da* to be acting away from the joint, which is as it should be. Continuing the solution at joint 2, *ab* is found to act away from the joint, thus being *tension*. The next member, *bc*, acts toward the joint and is *compression*. The last member, *ca*, is drawn from the joint and is in *tension*, which checks with the result found at joint 1. Passing to joint 3, *cb* is drawn toward the joint and is *compressive*. The reaction *bd* is drawn toward the joint and is therefore acting toward the joint as indicated by the arrowhead. The final member, *dc*, is drawn toward the joint and is in *compression*. From the foregoing analysis we find that the *compression* members are *CB* and *DC* and that the *tension* members are *AC* and *AB*. A circle with arrowheads may be used to indicate the order in which the members are taken around the joint.

By means of suitable column formulas the area for any given material necessary to meet the requirements can be obtained. As an illustration, we can assume that the tension stress in *AB* is equal to 10,000 lb; and the compressive stress in *DC* is equal to 15,000 lb. Assuming a factor of safety of 4 to be used with the wire rope, a rope with an ultimate strength of 40,000 lb is required. Consulting a manufacturer's catalogue, we find that a 9 by 16 plow steel hoisting rope, ¾ in. in diameter, will meet the requirements. For the member *DC*, we shall assume that the length is 25 ft and that the material from which it is made is longleaf yellow pine. The column formula recommended for longleaf yellow pine (see Table 11) is

$$\frac{P}{A} = 1300\left(1 - \frac{l}{60d}\right)$$

where P = total safe load, lb
A = area of cross section, sq in.
l = length of column, in.
d = least transverse dimension, in.

From our assumptions, $l = 25 \times 12 = 300$ in. For a trial, we shall select a 6- by 10-in. timber.

$$\frac{P}{6 \times 10} - 1300 - \frac{1300 \times 300}{60 \times 6} = 1300 - 1083 = 217 \text{ lb per sq in.}$$
$$P = 6 \times 10 \times 217 = 13{,}020 \text{ lb}$$

which is not sufficient.

Trying next an 8- by 8-in. timber,

$$\frac{P}{8 \times 8} - 1300 - \frac{1300 \times 300}{60 \times 8} = 488 \text{ lb per sq in.}$$
$$P = 8 \times 8 \times 488 = 31{,}232 \text{ lb}$$

which is apparently double the required size. For reasons advanced later (see Trusses) it would be used.

A more detailed application of the column formula will be given later.

Simple Trusses. The next example to be considered is the simple truss, with a concentrated load W, shown in Fig. 6. Such a condition of loading might result from a small crane or chain block operating on an I beam attached to the lower chord as shown. The load W would then represent the weight of part of the I beam, the chain block, and the maximum load to be lifted. It is seen from inspection that R_1 equals R_2 because W is equidistant between the reactions. However, a force diagram will be drawn for purposes of illustration. The load line *ef*, representing W, is laid out to a suitable scale. The location of O is chosen and the rays *Oe* and *Of* drawn. In constructing the equilibrium polygon, string *of* must be drawn connecting *EF* with *FA* and string *oe* connecting *AE* with *EF*. This causes the equilibrium polygon to become inverted as compared with examples so far discussed. It does not alter the results. The closing line *oa* is transferred to the force diagram, thus locating *a*. The reactions are *fa* or R_1 and *ae* or R_2. To construct the stress diagram, *eaf* is drawn. We could start the solution at either R_1 or R_2, as each has but two unknown stresses. Selecting the left-hand reaction joint, *ab* is drawn from *a* parallel to *AB* and intersecting *bf* drawn from *f* parallel to *BF*. This locates *b*. The upper left-hand joint now has but two unknown members. Member *ac* is drawn intersecting *cb*, locating *c*. We can continue the solution from either the joint at the right reaction or the middle joint of the lower chord. Choosing the former, *da* is drawn intersecting *ed*. Last *cd* is drawn completing the stress diagram. All members of the stress diagram are, of course, transferred parallel to the corresponding members of the main figure. A check on the drafting is had when member *cd* is transferred. It should close on the point *d* in the stress diagram. The nature of the stresses are now determined. Starting at the left reaction, we find that *ab* is drawn toward the joint and is in compression. The line *bf* is drawn away from the point, thus being in tension. At the upper left-hand joint, *ac* is toward the joint and is compression, while *cb* is away from the joint and is tension, and *ba* is toward the joint and is compression. The stresses in the right half of the truss are identical with the left half.

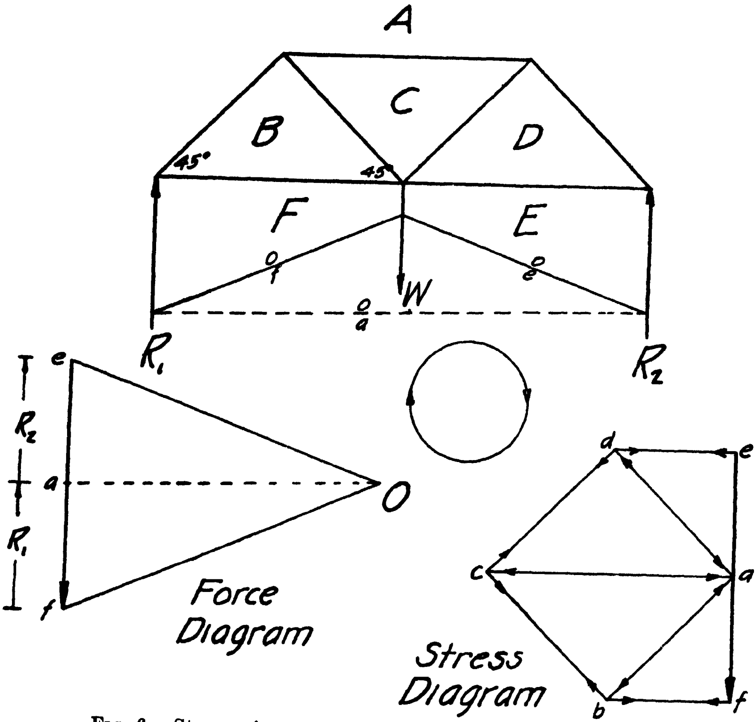

FIG. 6. Stresses in a roof truss carrying a concentrated load.

The solution for dead-load stresses in a simple truss is shown in Fig. 7. The load, while it is evenly distributed, is considered as being concentrated at the panel points as illustrated.

The load line *abcde* ($ab = W_1$; $bc = W_2$; $cd = W_3$; $de = W_4$) is drawn and the pole located. As shown, no rays are drawn to *Oa* and *Oe*. Actually they should be indicated, but as they connect $ab(W_1)$

with *ha*(R_1) and *de*(W_4) with *eh*(R_2), which act along the same line, respectively, but in opposite directions, the rays *Oa* and *Oc* in the equilibrium polygon are but points. The strings *ob*, *oc*, *od*, and the closing line *oh* are drawn and *oh* transferred to the force diagram, thus

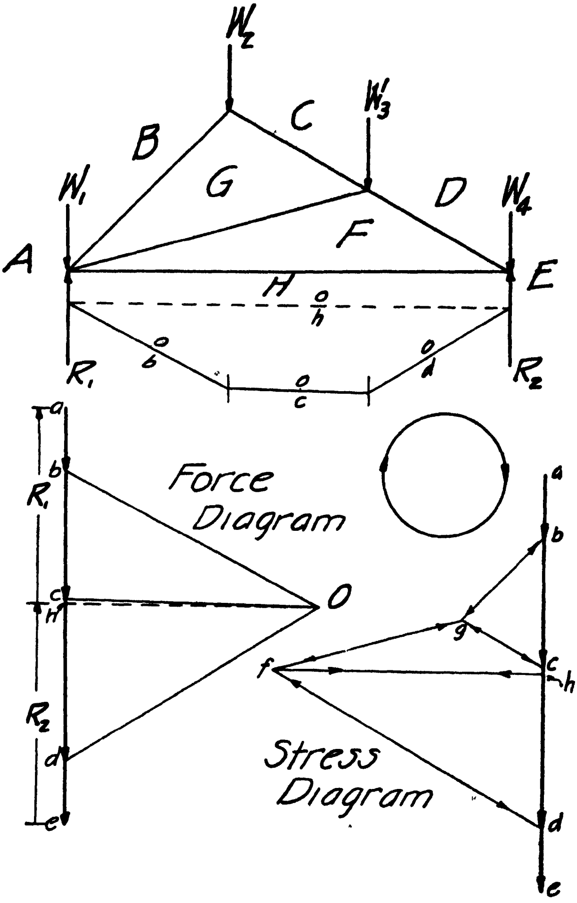

FIG. 7. Dead-load stresses in a simple roof truss.

locating *h* on the load line and determining the values of the reactions. The right reaction R_2 is given by *eh* and the left reaction R_1 by *ha*. To obtain the stresses, draw *abchde*. There are two points at which but two unknown forces are acting, the right reaction and the apex of the truss at W_2. Starting at the panel point under W_2, *cg* is drawn intersecting *gb*, thus locating *b*. We can now solve for the stresses at the left reaction and the right reaction, which completes the stress diagram.

The sense of the members are determined as in previous examples. The only tension member is *fh.*

Figure 8 illustrates the graphical solution for dead loads acting on a simple triangular truss. In this example none of the load is considered concentrated directly above the reactions, this form of solution being sometimes used. The force polygon and equilibrium polygon are used for determining the reactions and the location of ray *On.* Then the stress diagram is started by drawing the load line *abcndef.* The solution of the problem may start at either the left or the right reaction. The individual polygons for the left half (as the truss is symmetrical, the right half is identical with the left half) are *amna; blmab; cklbc; kjnmlk.*

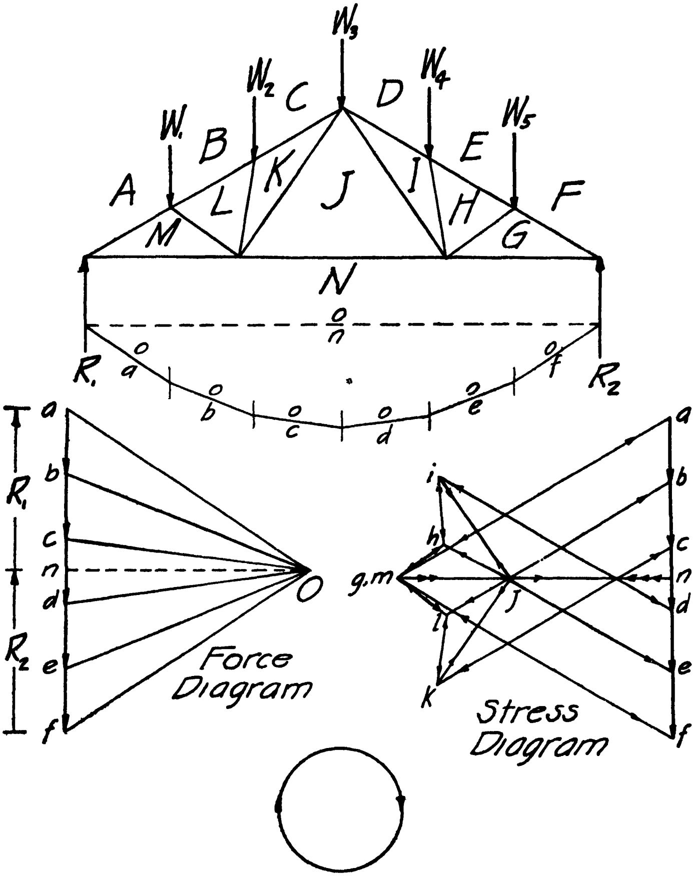

FIG. 8. Dead-load stresses in a triangular truss.

The nature of the stresses is determined as before. We shall, for the purpose of illustration, solve for the *sense* of the stresses at W_2 and the left-hand panel point on the lower chord. Starting at W_2, *ck* is drawn toward the joint and is *compressive; kl* is toward the joint and *lb* is toward the joint, both being *compression.* On the lower chord *jn* is away from the joint and *nm* is away from the joint so that they are *tension; ml* and *lk* are toward the joint and are *compression* members; finally *kj* is away from the joint and is a *tension* member. The rest of the truss is readily determined.

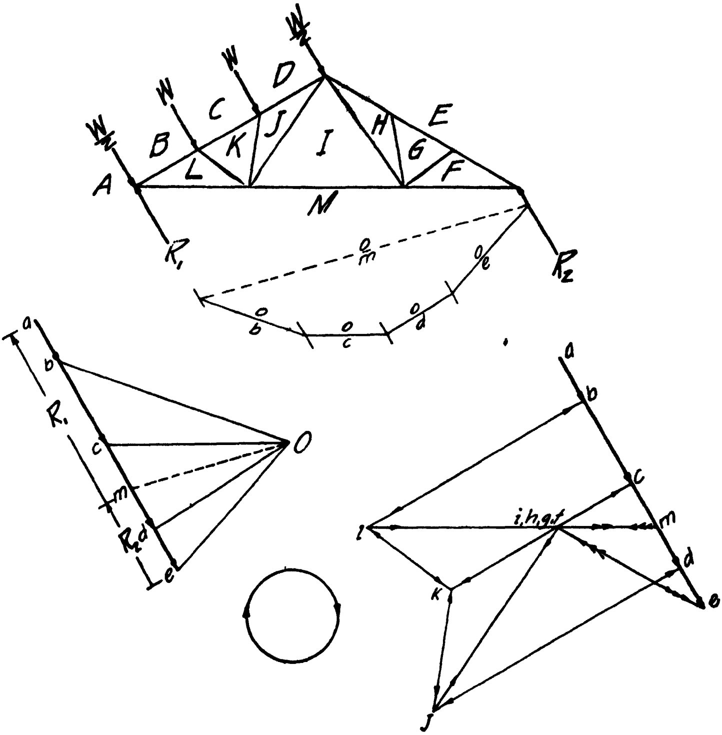

FIG. 9. Wind-load stresses in a triangular truss.

The truss used in the preceding example will now be solved for wind loads. Figure 9 shows the procedure. How the magnitude of the wind load is determined will not be discussed at the present time; it will be investigated later. If the ends of the truss are fixed, the line of the reactions will be parallel to the direction in which the loads W and $W/2$ are acting. The force polygon is used for determining R_1 and R_2. The load line *abcde* is laid out parallel to the line of action for W. In

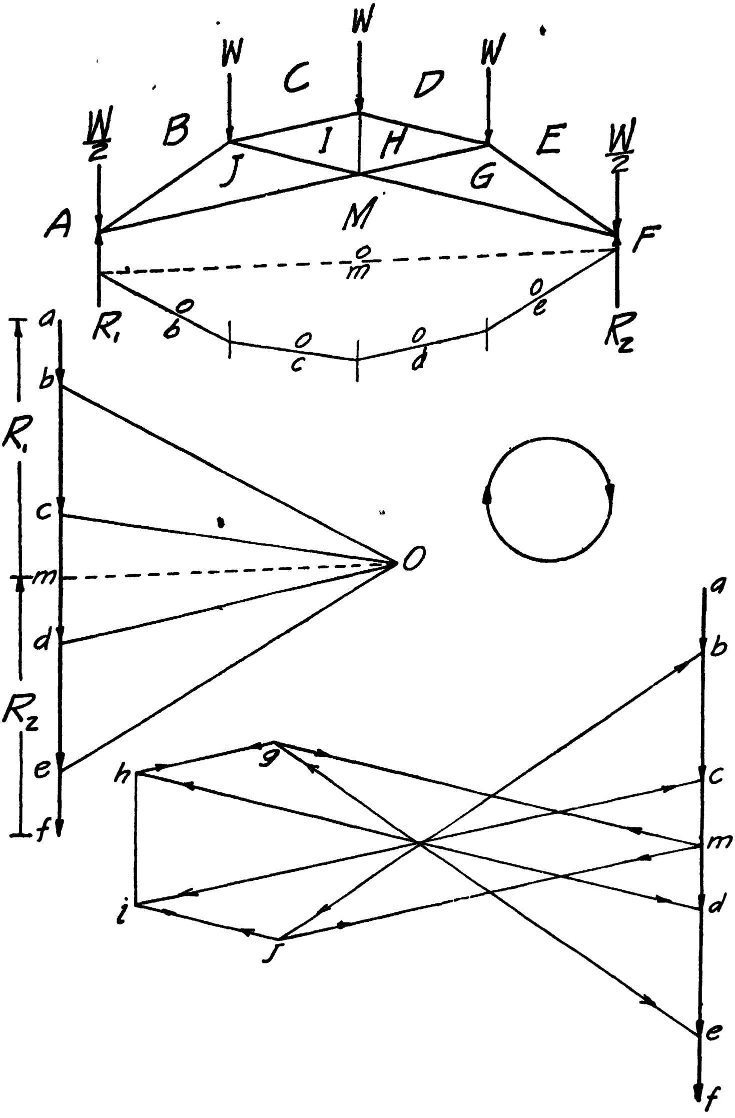

FIG. 10. Dead-load stresses in a simple hipped truss.

the stress diagram, *abcmde* is also parallel to *W*. The solution starts at the left reaction. Members *bl, ck, dj, eh, eg, ef, lk,* and *kj* are *compression;* members *ml, mi, mf,* and *ji* are *tension;* while members *ih, hg,* and *gf* are *zero.*

Figure 10 shows the dead-load solution for a simple hipped truss.

If a horizontal member were placed across the bottom of the truss, the problem would become indeterminate so far as graphical methods are concerned. It might be added that algebraic moment and resolution methods also would prove of no avail. Special methods,[1] beyond the scope of this book, must be applied.

The solution of Fig. 10 offers no particular difficulty, so a detailed discussion need not be made.

The determination of wind-load stresses in a hipped truss are somewhat different than for the simple triangular truss. Figure 11 shows a truss identical with that in Fig. 10, both ends being fixed. The wind load W_1 is normal (perpendicular) to *BH*. Normal to *CG* is W_3. As the slope of *BH* and *CG* differs the line of action of their loads differs. At the middle panel point the wind load W_2 is the resultant of W_1 and W_3. It has a line of action that differs from the other two. The result is that R_1 and R_2 are not likely to be perpendicular to either panel. In the force polygon, *ab*, *bc*, and *cd* are drawn parallel to W_1, W_2, and W_3. They are held in equilibrium by *da;* therefore, *da* is the line of action of the reactions. The equilibrium polygon is constructed and *k* located on *da*. Then *dk* equals R_2 and *ka* equals R_1. In the stress diagram *abcdka* is drawn and the polygon then completed. No further discussion is necessary.

Up to the present, trusses with both ends fixed have been considered. We shall now investigate the action of the wind on a simple triangular truss with one end free. Figure 12 shows the truss with necessary diagrams. The right end is mounted on rollers and is free to move. The reaction here (R_2) can only be vertical. Two solutions are made: one with the wind from the left, the other with the wind from the right.

The construction for determining the reaction by the method here employed (there is another method that we shall not take up) may, depending on the design of the truss, take in considerable territory. A preliminary inspection should be made in order that the intersections *M* and *M'* fall on the paper.

The wind loads are considered as acting normally to the roof. All work is done in clockwise rotation. The solution with the wind from each side is necessary in order that the maximum stresses may be found; for example, we find that the maximum stress in the members

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[1] KETCHUM, M. S., "Design of Steel Mill Buildings" or "The Design of Mine Structures," McGraw-Hill Book Company, Inc., New York.

PARCEL and MANEY, "Statically Indeterminate Stresses," John Wiley & Sons, Inc., New York.

hd and *eg* occur when the wind is from the right, while the wind on the left causes maximum conditions in members *bk* and *cj*. Certain members are zero in one case; others are zero in the other case. If both ends had been fixed, there would be no difference resulting from a change in the direction of the wind. Taking the wind on the left-hand side, we proceed as follows: The reaction at the free end is vertical. The line of action of W is prolonged until it meets the extension of R_2 at M. From M is drawn a line to the left end of the truss. This line gives the direction of R_1. Starting at any convenient point on WM, we lay out ab equal to $W/2$; bc equal to W; and $cdef$ equal to $W/2$.

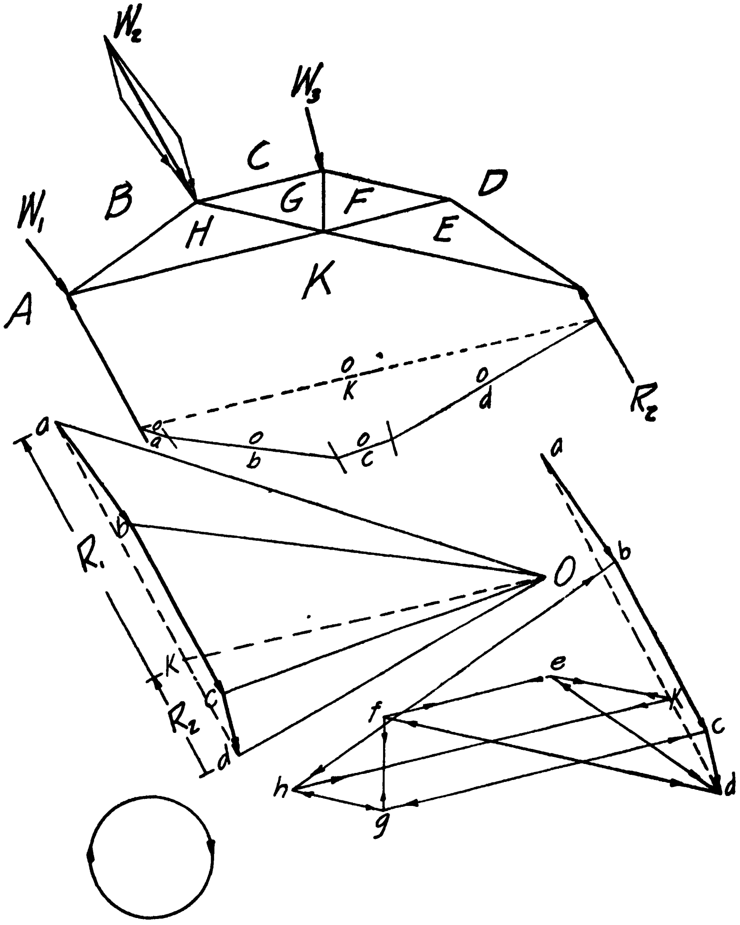

FIG. 11. Wind-load stresses in a simple hipped truss.

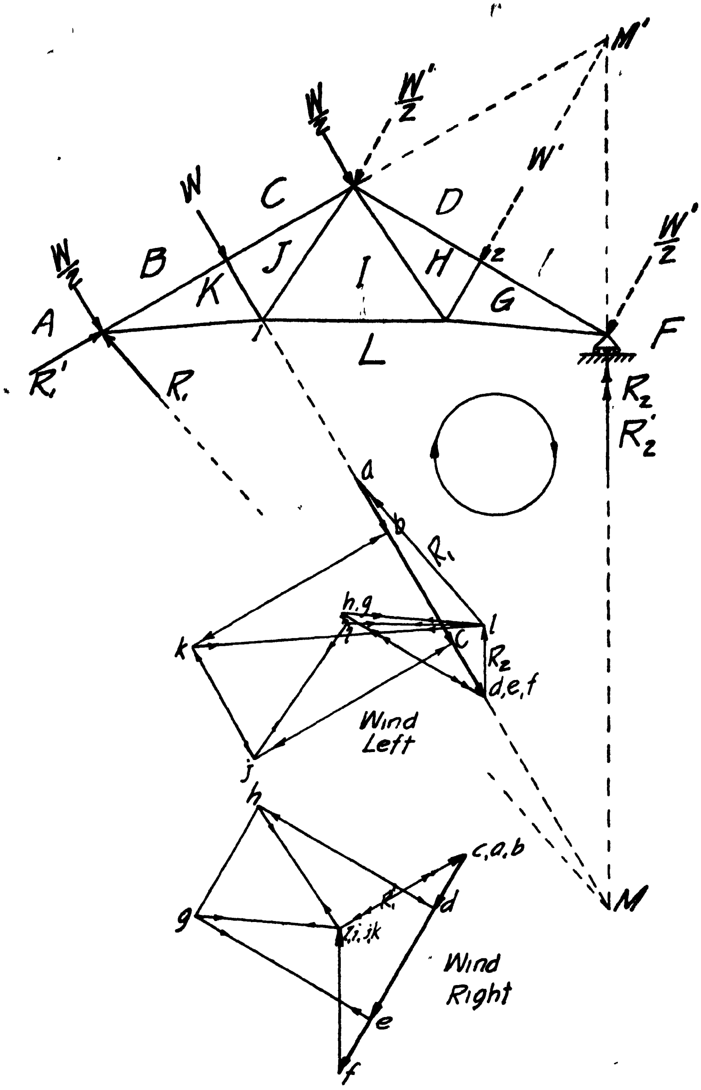

FIG. 12. Wind-load stresses in a triangular truss.

The load line *abcdef* is held in equilibrium by R_2 and R_1. From *f*, R_2 is drawn parallel to MR_2 and from *a*, R_1 is drawn parallel to MR_1. Their intersection locates *l*. The stress diagram may now be constructed. A convenient starting point is the left end. The order of constructing the polygons is *abkla; cjkbc; jilkj; eflge; deghd; hglih;* and last *cdhijc*. This is, of course, not the only sequence. The sense of the stresses are determined as has been done previously. It will be noted that *hg* is zero.

With the wind on the right, very much the same procedure is followed. The loads are represented by $W'/2$ and W'. The line of application of W' is extended to intersect R'_2 at M'. The line of action of R'_1 then falls on a line connecting M' with the left end of the truss. (This line, by chance, falls along the left upper chord.) The loads are now laid out on a line parallel to $W'M'$. These forces are *abcd* equal to $W'/2$; *de* equal to W'; and *ef* equal to $W'/2$. The magnitudes of the resultants are found in a manner similar to that previously used. The stress diagram is then completed. With the wind on the right the stresses in the members *ji*, *il*, *lk*, and *kj* are zero.

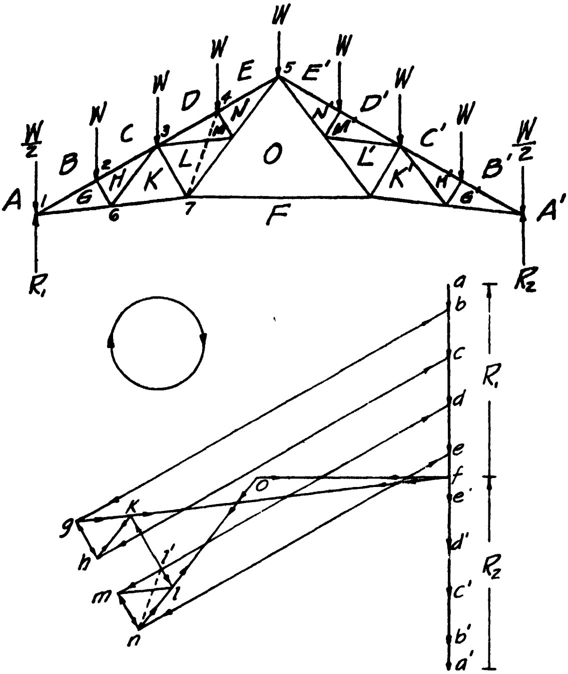

FIG. 13. Method of substitution for more than two unknown stresses at one point.

Figure 13 shows a Fink truss. It is offered mainly for the purpose of illustrating the method of *substitution* available for determining stresses when there are more than two unknown members intersecting at a point. The reactions can be determined by the usual equilibrium polygon. They will be found to be equal. The load line *abcdefe'd'c'b'a'* is constructed. Starting at the left side of the truss, the stresses are easily determined until we reach joint 3, at which there are three unknown stresses: *DM*, *ML*, and *LK*. A simple and acceptable method for arriving at these stresses is as follows. The dotted line 7-4 is drawn and the solution continued as if *NM* and *ML* were absent. Draw *dl′* to intersect *l′k*. Then *en* intersects *nl′*. The remainder of the unknown members can now be determined: *nm* intersects *md*; *ml* intersects *lk*; *lo* intersects *of*, and finally *on* intersects *ne*. The right side of the truss is identical with the left side and need not be determined.

CHAPTER II

REVIEW OF SIMPLE DESIGN PRINCIPLES

Most structures must be designed for one or several of the following stresses:

1. Tension
2. Compression
3. Bending
4. Shear
5. Bearing plates (washers) for tie rods and bolts
6. Base plates for columns

Tension. When a member is so loaded that the tendency is to increase the length, it is in tension. Sufficient net area must be provided to prevent failure. The design of tension members is best approached by first finding the net area necessary. This is determined by dividing the stress in the member by the allowable working stress in tension for the particular material used (see Table 11); *i.e.*,

$$A = \frac{P}{s}$$

where A = area, sq in.
P = stress, lb
s = working stress in tension, lb per sq in.

For wooden members, s will depend on the species [see Table 11, column (4)]; for structural steel, s = 18,000 lb per sq in. (this applies to the various structural shapes, plates, and tie rods). Bolts may also be subjected to shear.

As previously mentioned, the design must be based on the net area; this is necessary in the tension members only. Allowance must be made for daps, notches, and boltholes and also for rivet holes and boltholes in steel. When selecting a tie rod, two options are available: ordinary threaded rods (the net diameter must be used for calculating the area) and upset rods, where the threaded end has been enlarged so that the net diameter equals the diameter of the rod. The upset rods require a larger bolthole than the ordinary rod. A width (for wood) or thickness (for steel) for the tension member is chosen. Closely approximate calculations are made for the notches, daps, boltholes, or rivet holes, and their area is added to the net area. The other dimensions of the member may now be selected. The force represented by the area of final choice should be within 5 per cent of the original force.

Compression. In designing simple structures, two types of compression stresses will usually be encountered: posts where the ratio of length to least transverse dimension is small; and those where a column formula is required. Table 11, column (6), gives values for the former. Column (7) of the same table gives the column formula for various wood species. The tendency in recent years is to use a more complicated column formula. When one considers that the lumberyards provide, as a rule, only standard dimensioned material, making it necessary to select the nearest commercial size, little if anything is gained by using the refined but more complicated formula. The simplified formulas in Table 11 will usually require a slightly larger transverse dimension. This is desirable when we consider the type of lumber often available at mining camps.

In choosing a column formula for steel members, the same discussion holds as already advanced for wood.

Column Formulas for Steel. The allowable unit stress for steel has, in recent years, been increased to 18,000 lb per sq in. There are four column formulas commonly suggested.

American Institute of Steel Construction (AISC) formulas:

1. $$\frac{P}{A} = \frac{18{,}000}{1 + \frac{1}{18{,}000}\left(\frac{L}{r}\right)^2}$$

where $P/A < = 15{,}000$ lb per sq in.

$L/r > 60 < 120$ for main members, or 200 for bracing

2. $$\frac{P}{A} = 17{,}000 - 0.485\left(\frac{L}{r}\right)^2$$

where $L/r < 120$

Formula (2) is based on 20,000 lb per sq in. for steel.

Committee on Steel Research formula

3. $$\frac{P}{A} = 15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2$$

where $L/r < 120$ for riveted ends

American Railway Engineers Association (AREA) (1929) formula:

4. $$\frac{P}{A} = 18{,}000 - 60\frac{L}{r}$$

where $P/A < = 15{,}000$ lb per sq in.

$L/r < 120$ for main members, or 200 for bracing

In the above formulas,

L = length of column, in.

r = least radius of gyration, in.

L/r = the slenderness ratio

Substituting values of L/r ranging from 60 to 120 in the above formulas shows that formula (4) gives within a few per cent the same values as formula (1). When the extreme L/r ratio of 200 for secondary members is used, formula (4) is about 11 per cent higher than formula (1).

If we consider that there is a definite limitation on the commercially available sizes of steel shapes, the straight-line formula, (4), proves satisfactory.

Steel is usually designed, sold, fabricated, erected, and painted on a weight basis. The designer should keep this in mind and try and select L/r ratios, moments of inertia, etc., which provide the right solution to the problem, but at the same time call for the least weight. In some rare instances, dimensional restrictions will supersede weight economy.

Timber is sold by the board foot. Commercial sizes, other than small stuff (less than 4 in.), increase by 2-in. changes and 2-ft. lengths. Then, for the same reason as for steel, a straight-line formula is sufficient. From an economic angle, members should be selected that require the least board feet.

To prevent excessive lateral deflection in beams, the depth should not ordinarily exceed two to four times the breadth. If this ratio is exceeded, there should be lateral bracing between the beams about every 8 ft of length.

EXAMPLE 1

Select a post of Douglas fir to support a load of 20,000 lb. The post is 5 ft long. Consulting Table 11, column (6), Douglas fir has a compressive stress of 900 lb per sq in. for short columns.

$$A = 20{,}000 \div 900 = 22.2 \text{ sq in.}$$

A 4- by 6-in. timber is the nearest size in the rough. If surfaced material is used, the size will be $3\frac{5}{8}$ by $5\frac{1}{2}$ in., which does not give sufficient area. The rough post would just meet requirements of 15 times the least dimension ($15 \times 4 = 60$ in.; the post is 5 ft long). If surfaced lumber is used, a 6- by 6-in. timber ($5\frac{1}{2}$ by $5\frac{1}{2}$ dressed) is necessary.

EXAMPLE 2

A column 15 ft long to support a load of 20,000 lb is needed. The material is Douglas fir. From Table 11, column (7), the formula is

$$\frac{P}{A} = 1200\left(1 - \frac{l}{60d}\right) = 1200 - 20\,\frac{l}{d}.$$

The solution is by cut-and-try methods as both dimensions of the column are unknown.

Trying first a 6- by 6-in. timber,

$$\frac{P}{6 \times 6} = 1200 - 20\,\frac{15 \times 12}{6} = 1200 - 600 = 600 \text{ lb per sq in.}$$
$$P = 6 \times 6 \times 600 = 21{,}600 \text{ lb}$$

A 6- by 6-in. timber is satisfactory if unsurfaced material is used; otherwise the size is 5½ by 5½ in., and the solution becomes

$$\frac{P}{5.5 \times 5.5} = 1200 - 20\,\frac{15 \times 12}{5.5} = 1200 - 654 = 546 \text{ lb per sq in.}$$
$$P = 5.5 \times 5.5 \times 546 = 16{,}500 \text{ lb}$$

A 6- by 8-in. surfaced timber provides

$$P = 5.5 \times 7.5 \times 546 = 22{,}500 \text{ lb}$$

Therefore, either a 6- by 6-in. timber in the rough or a 6- by 8-in. surfaced timber will suffice.

EXAMPLE 3

Select two angles with the long legs back to back to resist a compression load of 20,000 lb in the main member of a truss. The distance between supports (column length) is 15 ft. To solve problems involving steel construction, a steel handbook must be available.[1] The designing engineer should be thoroughly familiar with the contents of such a book. From it may be obtained practically all the design information needed. The back-to-back spacing of angles has apparently been standardized, as the various handbooks give data for only 0, ¼-, ⅜-, ½-, ⅝-, and ¾-in. spacing. The spacing to a close degree depends on the thickness of the gusset plate between the angles. This thickness in turn is related to the diameter of the rivets used. The object is to equalize more or less the rivet shear and the gusset plate and angle bearing on the rivets. For plates up to ½ in., ¾-in. rivets are commonly used. For the present example, the angles will be taken at ⅜ in. back to back.

For main members the l/r ratio must not exceed 120. For a trial, $l/r = 120$; $r = l/120 = 15 \times 12/120 = 1.5$ in.

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[1] "Steel Construction," a manual for architects, engineers, and fabricators of buildings and other steel structures. American Institute of Steel Construction, New York.

In consulting the handbook, the radius of gyration about the *xx* axis (parallel to the short legs), and the *yy* axis (parallel and between the long legs) must both be considered. In the column formulas, *r* is the smaller of these two. Quite often several angle sizes will apparently be satisfactory. Steel construction is usually handled on a tonnage basis. Therefore, the angles with the least weight per foot should be selected. The trial *r* found from $l/r = 120$ is simply to provide a start and to set the *minimum* value of *r*. The value of *r* can be as much *larger* as is necessary. For a start, we can try two 5- by 3½- by ⅝-in. angles with a weight of 33.6 lb per ft; the area of both angles is 8.00 sq in., ⅜ in. back to back. The radii of gyration are *xx* axis = 1.56 in. and *yy* axis = 1.51. These are the minimum data given in the handbook consulted. By using two 6- by 4- by ⅜-in. angles, the weight is 24.6 lb per ft. The smaller *r* is about *yy* and is 1.62 in.; the area is 7.22 sq in.

$$\frac{P}{A} = 18{,}000 - 60\frac{l}{r}.$$

$$\frac{P}{7.22} = 18{,}000 - 60 \times 15 \times \frac{12}{1.62} = 18{,}000 - 6670$$

$$= 11{,}330 \text{ lb per sq in.}$$

$$P = 7.22 \times 11{,}330 = 82{,}000 \text{ lb}$$

This is more than four times the load to be supported. If the l/r ratio is to be adhered to, the selection made must be used. The handbook gives smaller angles with smaller l/r ratios; but they weigh more than those chosen.

Bending. Bending is one of the principle stresses for which structural members must be designed. Such stresses occur in rafters (from the evenly distributed roof covering—sheathing, shingles, corrugated roofing, tile, etc.); purlins, the member which connects the upper chords of the trusses at their panel points (the rafters, resting on the purlins cause a series of evenly spaced concentrated loads) floor beams and joists; bottom of an ore bin; sheave girders of a headframe; line shafting supported by the lower chord of a truss; machinery, such as crushers, screens, etc., supported by the lower chord of a truss or a beam; columns or wall studding from wind pressure; and numerous other examples. Members in bending are usually spoken of as beams. Bending is sometimes combined with direct stresses (tension or compression).

From the cases cited, we see that bending does not necessarily result from vertically acting forces.

The bending stresses often dictate the dimensions of a member instead of the tension or compression stresses.

Before a member can be designed for bending, the maximum bending moment must be found. Fortunately, in practically all instances, the problem can be treated as a simple beam and the formulas for simple beams can be used. Also, usually there are two supports only and the load is a concentrated load, a uniformly distributed load, or both (an exception so far as mine plant is concerned are most ore bins, where the floor beams have a distributed load increasing toward one end).

Bending moments may be obtained graphically or algebraically. Ordinarily the algebraic method is used. The procedure consists of finding the reactions at the ends of the beam and then finding the maximum bending moment. Time is saved by directly consulting tabulated data giving the formulas for maximum bending moments under various conditions of loading. This usually eliminates the step of finding the reactions. In some instances the reactions, as such, are not needed; many times they must be known in order that bearing plates may be provided (see bearing) or that the shear may be determined. Figure 14 gives data for beam problems customarily met in mine plant design.

After determining the *maximum* moment to be resisted, the design of the beam is based on one of two formulas. These two formulas are but convenient expressions of the same basic formula—one is more suitable for wooden members (which invariably have rectangular sections), while the other is more applicable to steel or odd-shaped sections.

For wooden members or other rectangular sections,

$$M = \frac{sbd^2}{6} \qquad \text{in.-lb}$$

For steel or other sections,

$$M = \frac{sI}{c} \qquad \text{in.-lb}$$

where M = moment (usually the maximum, either positive or negative), in.-lb

s = extreme fiber stress [see Table 11 column (1), for wood; 18,000 for steel], lb per sq in.

b = horizontal dimension of member (width), in.

d = depth of member, in.

I = moment of inertia of particular section

= $\frac{1}{12}bd^3$ for rectangular sections

c = distance from neutral axis to extreme outer edge, in. (For most symmetrical sections, *e.g.*, square or rectangular,

c = $d/2$; for most steel shapes c is found from data in the steel handbook. It is the extreme distance to the neutral axis.)

As was true with the column formula, the selection must be made by

w = lb. per ft.; l = span, ft.; P = load, lb.; R = reactions, lb.; W = total distributed load, lb. = wl; V = shear, lb. where M is maximum

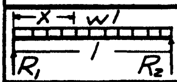
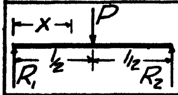
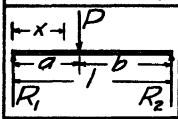
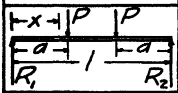
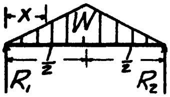
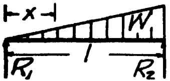
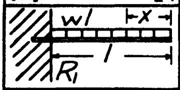
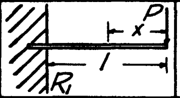
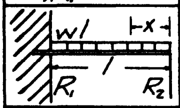
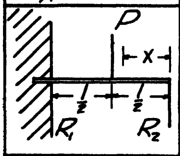
TYPE OF BEAM AND LOADING	REACTIONS	BENDING MOMENTS	MAXIMUM DEFLECTION	MAXIMUM SHEAR
	$R_1 = R_2$ $= \frac{1}{2}wl$	$M = \frac{1}{2}wlx(l-x)$, ft.-lb. $M_{max} = \frac{1}{8}wl^2$, ft.-lb. $= \frac{1}{8}wl^2 \cdot 12$, in.-lb.	(l in inches) $\frac{5wl^4}{384EI}$	At R_1 and R_2 $V = R$
	$R_1 = R_2$ $= \frac{1}{2}P$	$x < \frac{1}{2}l$, $M = \frac{1}{2}Px$ $M_{max} = \frac{1}{4}Pl$ $= \frac{1}{4}Pl \cdot 12$, in.-lb.	$\frac{Pl^3}{48EI}$	$V = R$
	$R_1 = b/l\,P$ $R_2 = a/l\,P$	$x < a$ $M = P\frac{b}{l}x$, $M_{max} = \frac{Pba}{l}$ $= \frac{Pba}{l} \cdot 12$, in.-lb.	$\frac{Pb(6la-3a^2)^{\frac{3}{2}}}{81EIl}$	At max R, $R = V$, when $a < b$
	$R_1 = R_2$ $= P$	$x < a$, $M = Px$ $x > a < (l-a)$ $M = M_{max} = Pa \cdot 12$, in.-lb.	$\frac{Pa(3l^2-4a^2)}{24EI}$	$R = V$
	$R_1 = R_2$ $= \frac{1}{2}W$	$x < \frac{1}{2}$ $M = Wx(\frac{1}{2} - \frac{2x^2}{3l^2})$ $M_{max} = \frac{Wl}{6} \cdot 12$, in.-lb.	$\frac{Wl^3}{60EI}$	$R = V$
	$R_1 = \frac{W}{3}$ $R_2 = \frac{2}{3}W$	$M = \frac{Wx}{3l^2}(l^2-x^2)$ $M_{max} = \frac{2}{9\sqrt{3}}Wl \cdot 12$, in.-lb	$001304\frac{Wl^3}{EI}$	At R_2 $R_2 = V_2$
	$R_1 = wl$	$M = -\frac{1}{2}wx^2$ $M_{max} = -\frac{1}{2}wl^2 \cdot 12$, in.-lb.	$\frac{wl^4}{8EI}$	At R_1 $R_1 = V_1$
	$R_1 = P$	$M = -Px$ $M_{max} = -Pl \cdot 12$, in.-lb.	$\frac{Pl^3}{3EI}$	At R_1 $R = V$
	$R_1 = \frac{5}{8}wl$ $R_2 = \frac{3}{8}wl$	$M = \frac{wx}{8}(3l-4x)$ $M_{max} = -\frac{wl^2}{8} \cdot 12$, in.-lb.	$\frac{wl^4}{185EI}$	At R_1 $R_1 = V_1$
	$R_1 = \frac{11}{16}P$ $R_2 = \frac{5}{16}P$	$x < l \div 2$, $M = -\frac{5}{16}Px$ $x > l \div 2$, $M = \frac{P}{16}(8l-11x)$ $M_{max} = -\frac{3}{16}Pl \cdot 12$, in.-lb.	$\frac{1}{\sqrt{5}}\frac{Pl^3}{48EI}$	At R_1, R_2 $R = V$

FIG. 14. Beam formulas.

cut-and-try methods. In the case of wooden members, b and d are unknown; and with steel I and c are unknown.

EXAMPLE 1

Select a wooden beam to support a load of 10,000 lb at the middle. The length of the beam is 12 ft. The species is tamarack.

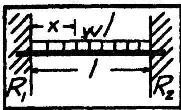
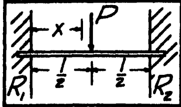
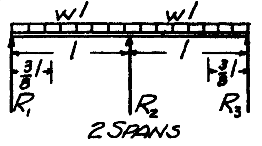
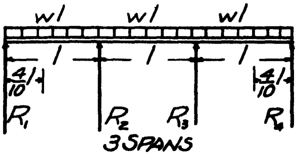
	$R_1=R_2$ $=\frac{1}{2}wl$	$M=\frac{w}{12}(6lx-l^2-6x^2)$ $M_{max}=-\frac{1}{12}wl^2\cdot 12$, in-lb.	$\frac{wl^4}{384EI}$	At R_1, R_2 $R=V$
	$R_1=R_2$ $=\frac{1}{2}P$	$x<l\div 2$, $M=\frac{1}{8}P(4x-l)$ $M_{max}=\pm\frac{1}{8}Pl\cdot 12$, in.-lb.	$\frac{Pl^3}{192EI}$	At center and R_1, R_2 $R=V$
 wl wl l l $\frac{3}{8}l$ $\frac{3}{8}l$ R_1 R_2 R_3 2 SPANS		$R_1=R_3=\frac{3}{8}wl$; $R_2=\frac{10}{8}wl$ M at $\frac{3}{8}l$ = Max. positive $=\frac{9}{128}wl^2\cdot 12$, in-lb. M at R_2 = Max. negative $=\frac{16}{128}wl^2\cdot 12$, in-lb.		At R_2, $R_2=\frac{5}{8}wl$
 wl wl wl l l l $\frac{4}{10}l$ $\frac{4}{10}l$ R_1 R_2 R_3 R_4 3 SPANS		$R_1=R_4=\frac{4}{10}wl$ $R_2=R_3=\frac{11}{10}wl$ M at $\frac{4}{10}l$ = Max positive $=\frac{16}{200}wl^2\cdot 12$, in.-lb. M at R_2, R_3 = Max negative $=\frac{20}{200}wl^2\cdot 12$, in.-lb.		At R_2, R_3 $R_2=\frac{5}{10}wl$ (+) $=\frac{6}{10}wl$ (−) $R_3=\frac{6}{10}wl$ (+) $=\frac{5}{10}wl$ (−)

FIG. 14 (*cont.*). Beam formulas.

From Fig. 14,

$$R_1 = R_2 = \frac{P}{2} = 10{,}000 \div 2 = 5000 \text{ lb}$$

$$M_{\max} = \frac{1}{4} \times Pl^2 \times 12 = \frac{10{,}000}{4} \times 12 \times 12 = 360{,}000 \text{ in.-lb}$$

$$M = \tfrac{1}{6} \times sbd^2$$

For tamarack, from Table 11, s = 900 lb per sq in.

$$d = \sqrt{\frac{6M}{sb}}$$

Assume that b = 14 in.,

$$d = \sqrt{\frac{6 \times 360{,}000}{900 \times 14}} = \sqrt{171} = 13.1 \text{ in.}$$

In the rough a 14 by 14 would do. If surfaced, we should check for a 14 by 16, which proves suitable.

At this point attention should be called to the fact that there are usually a large number of b and d combinations which satisfy the moment formula. It is recommended practice that d never be less than b and preferably greater. Sometimes, however, the space is limited and d controls the design. Where limitations are not imposed, the economic consideration must become a factor. Timber is usually sold on board-foot measure; and pieces with very large dimensions are more costly if not unobtainable. The board-foot measure is based on the nominal size and not the surfaced dimensions. The following data illustrate a number of beams that satisfy the preceding example but have a considerable variance in their board-foot content; the dimensions are based on the nominal size, as timber is sold on this basis. The depth cannot become too large at the expense of the width, or additional cost is involved for lateral bracing.

Dimensions, $b \times d$	Board Feet, per Foot
2×10*	1.67
3×10	2.50
4×8	2.66
6×6*	3.00

* If surfaced lumber is used, these dimensions are too small. A 2×12 and a 6×8 would be required.

EXAMPLE 2

A grizzly is to be built using railroad rails. The unsupported length of a rail is 10 ft, with a maximum uniformly distributed load of 500 lb per ft of rail.

From Fig. 14, the maximum moment is

$$M = \tfrac{1}{8}wl^2 \times 12, \qquad \text{in.-lb}$$
$$= \frac{500 \times 10^2}{8} \times 12 = 75{,}000 \text{ in.-lb}$$

A rail has an odd-shaped section; therefore the most convenient formula is $M = sI/c$.

Consulting a steel handbook, under the ASCE rail section, a choice is made of one of the light sections. Trying a 30-lb rail, I about the xx axis (perpendicular to the web) equals 4.06 in.^{4}; the depth of the rail is $3\tfrac{1}{8}$ in.; from the base of the rail to the neutral axis is $x = 1.52$ in.

$$c = 3.125 - 1.52 = 1.605 \text{ in.}$$
$$M = \frac{sI}{c} = 18{,}000 \times 4.06 \div 1.605 = 45{,}500 \text{ in.-lb}$$

But 75,000 in.-lb are required; therefore the 30-lb rail is too light. Trying a 50-lb rail next,

$$I = 9.94 \text{ in.}^4 \qquad x = 1.88 \text{ in} \qquad depth = 3.875 \text{ in.} \qquad c = 1.995 \text{ in.}$$
$$M = 18{,}000 \times 9.94 \div 1.995 = 89{,}600 \text{ in.-lb}$$

The 50-lb rail is satisfactory. Additional substitutions provide nothing that comes closer to the required moment.

A column headed S, the section modulus, is given in the handbook. $S = I/c$ for each section shown. Therefore

$$M = Ss = 18{,}000S$$

The work is simplified by using this data.

A common type of beam in structures is that similar to a purlin on a truss or a heavy beam supporting the ends of floor joists. The effect here is a series of evenly spaced concentrated loads. The spacing of the rafters on the purlin or the joists on the floor beams determines the position of the concentrated loads. The reactions and maximum moment is found by algebraic methods. An empirical formula is of little value because the spacing varies with the structure.

EXAMPLE 3

The trusses of a building are spaced 15 ft apart. Rafters are spaced 20 in. apart and there is a rafter over each truss. The roof covering and rafter is equivalent to 80 lb at each rafter. Find the size of a suitable purlin if Douglas fir is used.

$$R_1 = R_2 = 10 \times 80 \times \tfrac{1}{2} = 400 \text{ lb}$$

By inspection the maximum moment is seen to occur midway between the fifth and sixth rafters. (If unsymmetrical loading is involved, a moment diagram will have to be drawn or moments calculated under each load until a maximum is finally reached.)

$$\begin{aligned} M &= 400 \times 7\tfrac{1}{2} - 80 \times 7\tfrac{1}{2} - 80 \times 5\tfrac{5}{6} - 80 \times 4\tfrac{1}{6} - 80 \times 2\tfrac{3}{6} - 80 \times \tfrac{5}{6} \\ &= 1333 \text{ ft-lb} \\ &= 1333 \times 12 = 15{,}996, \text{ or } 16{,}000 \text{ in.-lb} \\ M &= \tfrac{1}{6} sbd^2 \end{aligned}$$

Trying $b = 4$ in.,

$$d = \sqrt{\frac{6 \times 16{,}000}{1200 \times 3\tfrac{5}{8}}} \times \sqrt{22.1} = 4.7 \text{ in.}$$

A 4 by 6 is satisfactory.

Shear. *Wooden Members.* There are three types of shearing stresses for which wood members must be designed.

1. Parallel to the grain
2. Longitudinal in beams
3. Perpendicular to the grain

Table 11 gives values for each of these.

Parallel to the grain. This type of shearing is important when designing connections where the force is at an angle to the grain or parallel to the grain. Bolts must be spaced so that the segment of wood between them does not shear out. At a later point its application will be illustrated (design of a wooden truss).

Longitudinal shear in beams. This is failure that results in the beam parting along the grain because of bending. If the deflection from bending is too great, the fibers part and slide over each other. It very seldom occurs when the bending stress for the wood is properly selected. The design is checked by $bd = \frac{3}{2}(V/s)$ (for V, see Fig. 14; s = value for longitudinal shear, Table 11).

Perpendicular to the grain. Shear perpendicular to the grain must be considered when using planks or beams for footings to posts, resting beams on masonry walls, butting compression members against the grain, etc. It must be investigated in practically all timber design.

Stresses on surfaces inclined to the grain. A number of expressions have been developed for determining the compression strength of a surface inclined to the load. The one recommended by the U.S. Forest Products Laboratory will be given.[1] It is convenient to express the formula in two forms. Figure 15*a* illustrates the members of the formula.

$$N = \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

or

$$N = \frac{PQ}{Q \sin^2 y + P \cos^2 y}$$

where N = allowable unit stress on inclined surface, lb per sq in.
P = unit stress in compression parallel to grain, lb per sq in.
Q = unit stress in compression perpendicular to grain, lb per sq in.
θ = angle between direction of load and direction of grain
y = angle between bearing surface (1-2, Fig. 15*a*) and direction of grain

It is well to remember that y and θ are complimentary.

EXAMPLE

A stress of 24,000 lb in the upper chord of a truss acts at an angle of 30 deg to the lower chord. If the material is Douglas fir, what is the allowable unit stress on the inclined face and what must be the area of the face?

From Table 11,

$$P = 1200 \text{ lb per sq in.} \quad \text{and} \quad Q = 310 \text{ lb per sq in.}$$

$$N = \frac{1200 \times 310}{1200 \sin^2 30^\circ + 310 \cos^2 30^\circ} = 697 \text{ lb per sq in.}$$

$$\text{Area required} = \text{load} \div N = 24{,}000 \div 697 = 34.5 \text{ sq in.}$$

Further examples of the use of the formula will occur in the design of joints.

Occasionally the shear in beams at the point of greatest shear must be checked. It will be remembered that, as a rule, the shear is a maximum where the moment is zero; and the shear is zero where the moment is a maximum. This is true except in the case of restrained beams; but even so the shear is a maximum at the supports (see Fig. 14). When the dimensions of a beam just satisfy the moment requirements it is well to check the beam for shear; for example, the problem on page 29 may be used. From Fig. 14 it is found that $R = V$. The reaction was found to be 5000 lb. From Table 11, column (4), the unit shear stress is 100 lb per sq in.

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[1] "Wood Handbook," p. 56, Forest Products Laboratory, U.S. Department of Agriculture, Washington, D.C.

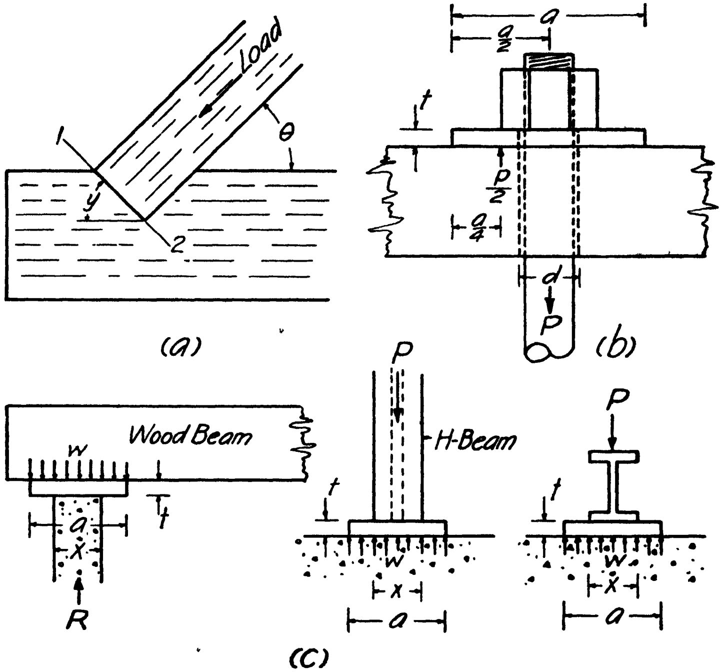

Fig. 15.

$$bd = \frac{3}{2} \times \frac{V}{s} = \frac{3}{2} \times \frac{5000}{100} = 75 \text{ sq in.}$$

The member selected on the basis of bending has $13\frac{1}{2} \times 15\frac{1}{2} =$ 209.25 sq in. if surfaced, and $14 \times 14 = 196$ sq in. if used in the rough. Either of these sizes will prove satisfactory to meet the requirements for shear.

Shear. *Steel.* For all practical considerations, it is assumed that steel has the same shear value for all directions. The numerical value used depends on the steel (structural shapes, rivets, plates), the part of the connection under consideration, the relative dimensions, and other items. There are too many considerations to try to cover them all in a work of this kind. The following data[1] should prove sufficient for most designs. If more information is desired, a steel handbook should be consulted.

Hand-driven rivets and unfinished bolts

Shear = 10,000 lb per sq in. = s_s
Bearing = 16,000 lb per sq in. (piece in single shear) = s_b
Bearing, enclosed = 20,000 lb per sq in. (piece in double shear) = s_d

Power-driven rivets and turned bolts in reamed holes

Shear = 13,500 lb per sq in. = s_s
Bearing = 24,000 lb per sq in. (piece in single shear) = s_b
Bearing, enclosed = 30,000 lb per sq in. (piece in double shear) = s_d

EXAMPLE

The compressive stress in a member is 47,000 lb. The member is made up of two angles back to back. Gusset plate and angles are each ⅜ in. thick. The rivets are ¾ in. How many rivets are needed? The rivets are to be driven in the field.

Rivets are in double shear (two angles). Allowable stress per rivet is

$$\begin{aligned}\text{Double shear} &= 2\frac{\pi d^2}{4} \times 10{,}000\\ &= 2 \times 3.14 \times \frac{(3/4)^2}{4} \times 10{,}000 = 8900 \text{ lb}\end{aligned}$$

For bearing on a ⅜-in. gusset plate,

$$\text{Stress} = 3/4 \times 3/8 \times 20{,}000 = 5620 \text{ lb}$$

For bearing on angles,

$$\text{Stress} = 2 \times 3/4 \times 3/8 \times 16{,}000 = 9000 \text{ lb}$$

The bearing on the gusset plate is the controlling factor; therefore

$$47{,}000 \div 5620 = 8.4, \text{ or 9 rivets required}$$

---

[1] "Carnegie Pocket Companion," 24th ed., pp. 326–327, Carnegie Steel Company, Pittsburgh, Pa.

The formulas used above may be written as follows:

Single shear (rivets) $= \frac{\pi d^2}{4} s_s$ lb

Double shear (rivets) $= \frac{2\pi d^2}{4} s_s$ lb

Bearing (gusset plate) $= tds_b$ lb

Bearing (on angles) $= 2tds_d$ lb

where s = values as already given
d = diameter of rivet, in.
t = thickness of gusset plate or angles, in.

Rivet holes are $\frac{1}{8}$ in. larger than the diameter of the rivet.

Spacing of rivets. The recommended minimum spacing of rivets is as follows:

Rivet diam., in.	Minimum center to center, in.	Maximum pitch, in.	End distance, in.	Edge distance, in.
d	$3d$	$16t$, or 6 in.	$2d$	$2d$

These are the recommended minimum distances; actually, they may be $\frac{1}{8}$ to $\frac{1}{4}$ in. less in practice. Distance t is the thickness of the piece.

Bearing Plates and Washers. Bearing plates and washers are necessary for distributing the load over the surface of the bearing member. Most commercial sizes of washers have insufficient area; therefore, if crushing of the wood fibers is to be avoided with development of the full strength of the bolt, the areas should be checked. The thickness of the washer or plate must be determined to prevent its bending.

Of the many formulas suggested for finding the thickness, the following seems to be the most logical (see Fig. 15*b*).

$$t = \sqrt{\frac{0.75Pa}{(a - d)s}}$$

where t = thickness, in.
a = long side of washer, in.
d = diameter of hole, in.
s = allowable unit bending stress of washer material, lb per sq in.
= 18,000 lb per sq in. for steel

P = stress in bolt, lb
b = width of washer, in.
A = gross area of washer, sq in.
A_n = net area of washer, sq in.
f_1 = allowable stress in wood, perpendicular to grain, lb per sq in.; or N if plate is at an angle to the grain

Area of washer to withstand P,

$$A = (b \times a) \quad \text{sq in. (gross area)}$$

$$A_n = (b \times a) - \frac{\pi d^2}{4} \quad \text{sq in. (net area)}$$

$$= \frac{P}{f_1}$$

$$A = \frac{P}{f_1} = \frac{\pi d^2}{4}$$

The supporting member should project at least ½ in. beyond the edge of the plate or washer. The hole in the washer is usually ¼ in. larger than the diameter of the rod.

EXAMPLE

A 6- by 10-in. timber supports a vertical tie rod carrying a load of 8800 lb. The rod is 1 in. in diameter. The timber is surfaced tamarack. Design a bearing washer.

$$\text{Width of plate} = b = 5.5 - 2 \times \tfrac{1}{2} = 4.5 \text{ in.}$$

From Table 11,

$$f_1 = 220 \text{ lb per sq in.}$$

$$A = \frac{P}{f_1} = \frac{\pi d^2}{4} = \frac{8800}{220} = \pi \frac{(1 + \frac{1}{4})^2}{4} = 41.2 \text{ sq in.}$$

$$a = \frac{A}{b} = 41.2 \div 4.5 = 9.2, \text{ or } 9\tfrac{1}{4} \text{ in.}$$

The washer is 4½ by 9¼ in.

$$t = \sqrt{\frac{0.75Pa}{(a - d)s}} = \sqrt{\frac{0.75 \times 8800 \times 9.25}{(9.25 - 1.25) \times 18{,}000}} = 0.65 \text{ in., or } \tfrac{11}{16} \text{ in.}$$

The washer would be 4½ by 9¼ by ¾ in.

Bearing plates are sometimes required to distribute the reaction at the end of a truss or other member (see Fig. 15c).

To find the thickness of the plate,[1]

$$t = \sqrt{\frac{3wa(a - x)}{4f}} \quad \text{in.}$$

---

[1] "Carnegie Pocket Companion," p. 352, or "Steel Construction," p. 136.

where w = allowable unit pressure on masonry or wood, whichever is smaller, lb per sq in.
a = length of bearing plate, in.
x = thickness of wall or width of flange if beam is structural member (I or H beams) or shoe angles on a truss, in.
f = unit bending stress of bearing plate material, lb per sq in.
= 18,000 lb per sq in. for steel
A = area of plate, sq in.
R = reaction or load, lb
b = width of plate, in.

EXAMPLE 1

An 8- by 10-in. beam rests on an 8-in. concrete wall. The beam is of tamarack. The reaction at the wall is 22,000 lb. Select a bearing plate. Bearing for concrete is 400 lb per sq in.

The wooden beam has the lower resistance to shear or crushing, so that the bearing plate must be selected to distribute the load over that member and not the concrete.

$$b = 7.5 - 2 \times \tfrac{1}{2} = 6.5 \text{ in.}$$

$$A = \frac{P}{w} = 22{,}000 \div 220 = 100 \text{ sq in.}$$

$$a = 100 \div 6.5 = 15.4, \text{ or } 15.5 \text{ in.}$$

$$t = \sqrt{\frac{3 \times 220 \times 15.5(15.5 - 8)}{4 \times 18{,}000}} = 1.04, \text{ or } 1\tfrac{1}{8} \text{ in.}$$

The plate is 1⅛ by 6½ by 15½ in.

EXAMPLE 2

A 15- by 6-in. at 60.8-lb American standard beam is to rest on a 6-in. concrete wall. The reaction at the wall is 30,000 lb. What size bearing plate must be used? Concrete has a bearing stress of 400 lb per sq in.

$$b = 6 - 2 \times \tfrac{1}{2} = 5 \text{ in. bearing width of wall}$$
$$= \text{width of plate}$$

$$A = \frac{P}{w} = 30{,}000 \div 400 = 75 \text{ sq in.}$$

$$a = \tfrac{75}{5} = 15 \text{ in.}$$

$$t = \sqrt{\frac{3 \times 400 \times 15(15 - 6)}{4 \times 18{,}000}} = 1.5 \text{ in.}$$

The plate is 1½ by 5 by 15 in.

Bolts.[1] The following discussion applies to bolted joints similar to those shown in Fig. 16. Tables 1 to 3 give the necessary data.

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[1] "Wood Handbook," p. 130, Forest Products Laboratory, U.S. Department of Agriculture, Washington, D.C.

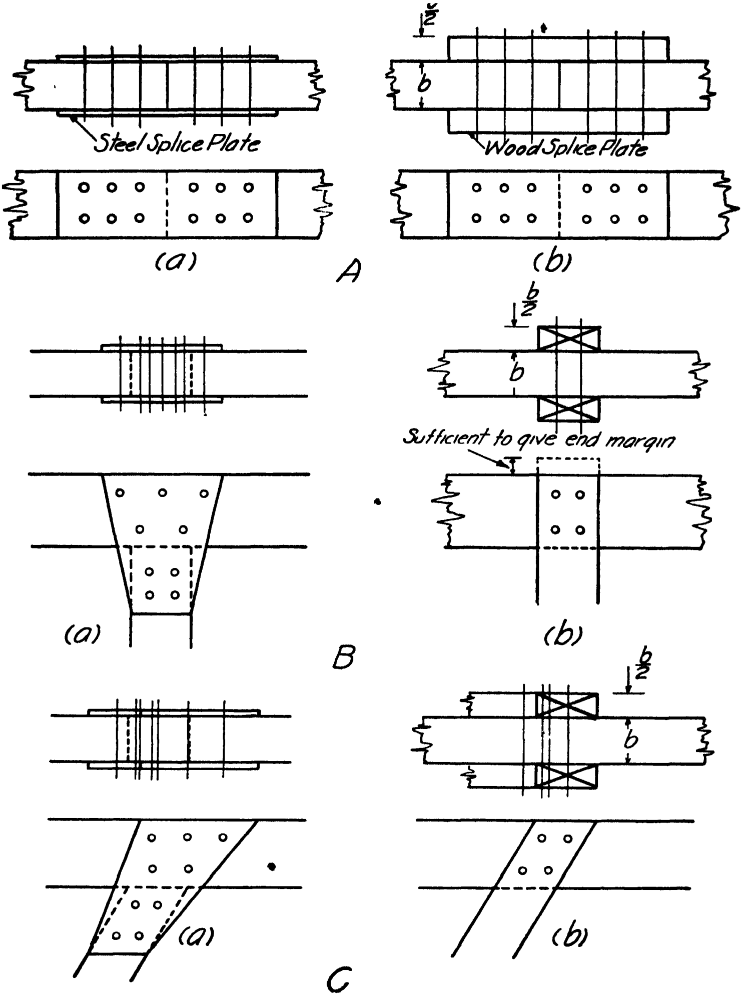


Fig. 16. Bolted joints.

EXAMPLE 1

A butt tension joint similar to that in Fig. 16*A* is to be designed. The wood is Douglas fir, Rocky Mountain region. The member is in tension with a load of 30,000 lb. Common bolts are to be used.

From Table 11, Douglas fir will stand 1200 lb per sq in. in tension.

$$\text{Area of member} = 30{,}000 \div 1200 = 25 \text{ sq in.}$$

A 6- by 6-in. timber has 5.5 × 5.5 = 30.3 sq in.

TABLE 1. BASIC STRESSES FOR CALCULATING SAFE LOADS FOR BOLTED JOINTS

Group	Species of wood	Basic stress, S	
		Parallel to grain, lb per sq in.	Perpendicular to grain, lb per sq in.
	Softwoods (conifers):		
1	Cedar, northern and southern; fir, balsam and commercial white; hemlock, eastern; pine, lodgepole, ponderosa, sugar, northern white, and western white; spruce, Englemann, red, Sitka, and white......	800	150
2	Cedar, Alaska, incense, Port Orford, and western red; Douglas fir (Rocky Mountain region); hemlock, western; pine, Norway........	1000	200
3	Cedar, eastern red; cypress, southern; Douglas fir (coast region); larch, western; pine, southern yellow; redwood; tamarack.............	1300	275
	Hardwoods (broad-leaved species):		
1	Ash, black; aspen and large tooth aspen; basswood; birch, paper; butternut; chestnut; cottonwood, black and eastern; poplar, yellow	925	175
2	Alder, red; elm, American and slippery; gum, black, red, and tupelo; hackberry; magnolia; cucumber and evergreen; maple, big-leaf and red and silver (soft); sugarberry; sycamore.............	1200	250
3	Ash, commercial white and Oregon; beech; birch, sweet and yellow; cherry, black; elm, rock; hickory, true and pecan; honey locust; locust, black; maple (hard), black and sugar; oak, commercial and white; walnut, black ...	1500	400

Consulting Table 1, the basic stress is found under group 2, and is 1000 lb per sq in.

Trying ¾-in. bolts,

$$\frac{L}{D} = \frac{5.5}{0.75} = 7.3$$

The stress percentage from Table 2 is 62.4 (by interpolation).

$$1000 \times 62.4\% = 624 \text{ lb per sq in.}$$
$$\text{Per bolt} = 624 \times 5.5 \times \tfrac{3}{4} = 2580 \text{ lb}$$

TABLE 2. PERCENTAGE OF BASIC STRESS PARALLEL WITH THE GRAIN FOR CALCULATING SAFE BEARING STRESSES UNDER BOLTS

(The product of the basic stress parallel with the grain selected from Table 1 and the percentage for the particular *L/D* ratio and species group, taken from this table, is the safe working stress at that ratio for joints with metal splice plates. When wood splice plates one-half the thickness of the mean timber are used, 80 per cent of this product is the safe working stress.)

Length of bolt in main member divided by its diameter (*L/D*)	Percentage basic stress for					
	Common bolts*			High-strength bolts†		
	Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
1–2 inclusive	100.0	100.0	100.0	100.0	100.0	100.0
2.5	100.0	100.0	99.7	100.0	100.0	100.0
3	100.0	100.0	99.0	100.0	100.0	100.0
3.5	100.0	99.3	96.7	100.0	100.0	99.7
4	99.5	97.4	92.5	100.0	100.0	99.0
5	95.4	88.3	80.0	100.0	99.8	96.0
5.5	91.4	82.2	73.0	100.0	98.2	93.0
6	85.6	75.8	67.2	100.0	95.4	89.5
6.5	79.0	70.0	62.0	98.5	92.2	85.2
7	73.4	65.0	57.6	95.8	88.8	81.0
7.5	68.5	60.6	53.7	92.7	85.0	76.8
8	64.2	56.9	50.4	89.3	81.2	73.0
8.5	60.4	53.5	47.4	85.9	77.7	69.6
9	57.1	50.6	44.8	82.5	74.2	66.4
9.5	54.1	47.9	42.4	79.0	71.0	63.2
10	51.4	45.5	40.3	75.8	68.0	60.2
10.5	48.9	43.3	38.4	72.5	64.8	57.4
11	46.7	41.4	36.6	69.7	61.9	54.8
11.5	44.7	39.6	35.0	66.8	59.2	52.4
12	42.8	37.9	33.6	64.0	56.7	50.2
12.5	41.1	36.4	32.2	61.4	54.4	48.2
13	39.5	35.0	31.0	59.1	52.4	46.3

* Bolts having a yield point of approximately 45,000 lb per sq in.

† Bolts having a yield point of approximately 125,000 lb per sq in.

Two rows of six bolts each on each side of the joint are satisfactory.

A total of 24 bolts is necessary.

If 1-in. bolts are used, eight are necessary for each side. Two rows of four on each side are sufficient. Sixteen 1-in. bolts would probably be a better choice than twenty-four ¾-in. bolts.

TABLE 3. PERCENTAGE OF BASIC STRESS PERPENDICULAR TO THE GRAIN USED IN CALCULATING SAFE BEARING STRESSES UNDER BOLTS

(The safe working stress for a given value of L/D is the product of three factors: (1) the basic stress perpendicular to the grain taken from Table 1, (2) the percentage from this table, and (3) the factor for bolt diameter, also from this table.)*

Length of bolt in main member divided by its diameter, (L/D)	Percentage for common bolts†				Percentage for high-strength bolts,‡ all groups
	Group 1, conifers; group 1, hardwoods	Group 2, conifers	Group 2, hardwoods; group 3, conifers	Group 3, hardwoods	
1–5 inclusive	100.0	100.0	100.0	100.0	100.0
5.5	100.0	100.0	100.0	99.0	100.0
6	100.0	100.0	100.0	96.3	100.0
6.5	100.0	100.0	99.5	92.3	100.0
7	100.0	100.0	97.3	86.9	100.0
7.5	100.0	99.1	93.3	81.2	100.0
8	100.0	96.1	88.1	75.0	100.0
8.5	98.1	91.7	82.1	69.9	99.8
9	94.6	86.3	76.7	64.6	97.7
9.5	90.0	80.9	71.9	60.0	94.2
10	85.0	76.2	61.2	55.4	90.0
10.5	80.1	71.6	62.9	51.6	85.7
11	76.1	67.6	59.3	48.4	81.5
11.5	72.1	64.1	55.6	45.4	77.4
12	68.6	61.0	52.0	42.5	73.6
12.5	65.3	58.0	49.0	40.0	70.2
13	62.2	55.3	45.9	37.5	66.9

Diameter of bolt, in.	¼	⅜	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2½	3 and over
Diameter factor	2.50	1.95	1.68	1.52	1.41	1.33	1.27	1.19	1.14	1.10	1.07	1.03	1

---

* No reduction need be made when wood splice plates are used except that the safe load perpendicular to the grain should never exceed the safe load parallel with the grain for any given size and quality of bolt and timber.

† Bolts having a yield point of approximately 45,000 lb per sq in.

‡ Bolts having a yield point of approximately 125,000 lb per sq in.

On the basis of 1-in. bolts the splice plates are found as follows:

Each plate takes one-half the load $= 30{,}000 \times \frac{1}{2} = 15{,}000$ lb
Net area $= 15{,}000 \div 18{,}000 = 0.833$ sq in.

Two rows of boltholes must be allowed for. The bolts are 1 in. in diameter. Therefore, an equivalent width of plate equal to $5.5 - 2 \times 1 = 3.5$ in. would be used for finding the thickness.

$$0.833 \div 3.5 = 0.236 \text{ in., or } \tfrac{1}{4}\text{-in. plate}$$

EXAMPLE 2

If wood splice plates are used instead of steel, find the number of bolts in Example 1.

Each splice plate must have a thickness equal to not less than one-half the main member. Then, $5.5 \times \frac{1}{2} = 2.75$ in. would be the thickness of the splice plates.

Trying 1-in.-diameter bolts,

$$\frac{L}{D} = \frac{5.5}{1} = 5.5$$

The stress percentage, from Table 2, is 82.2.

$1000 \times 82.2\% = 822$ lb per sq in.
$822 \times 80\% = 658$ lb per sq in. load because of wooden splice plates

Load per bolt $= 658 \times 5.5 \times 1 = 3620$ lb
Number of bolts $= 30{,}000 \div 3620 = 8.3$

Two rows of four bolts on each side would probably do.

EXAMPLE 3

A connection similar to Fig. 16 *Bb* is to be designed. The horizontal member is a 6- by 14-in. timber. The load on the vertical member is 6000 lb, and the vertical ties are 4- by 10-in. Douglas fir, Rocky Mountain region.

Trying 1-in. bolts,

The basic stress, from Table 1, is 200 lb per sq in.

$$\frac{L}{D} = \frac{5.5}{1} = 5.5$$

The stress percentage from Table 3 is 100.0; the diameter of bolt factor from the same table is 1.27.

Safe stress $= 200 \times 100\% \times 1.27 = 254$ lb per sq in.
Load per bolt $= 254 \times 5.5 \times 1 = 1400$ lb
Number of bolts $= 6000 \div 1400 = 4.3$, or 6 bolts

Two rows of three each are used. The vertical member must project 5 in. above the 6 by 14 to allow for end margin.

For the type of connection shown in Fig. 16 *Fc*, the stresses must be used in the formula

$$N \quad \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

Additional design data:

Center-to-center spacing between bolts acting parallel to the grain is at least $4d$.

It is recommended that an even number of bolts always be used to avoid staggering of bolts.

In tension,

Distance from end of timber to center of first bolt

$= 7D$ for coniferous woods

$= 5D$ for hardwoods

Under compression,

Distance from end of timber to center of first bolt

$= 4D$ for all woods

Distance from edge of timber to center of bolt acting parallel with

grain $= 1\frac{1}{2}D$ for $L/D < 6$

$= 1\frac{1}{2} +$ for $L/D > 6$

All of the above are minimum requirements.

Center-to-center spacing across the grain:

For loads acting perpendicular to the grain through metal splice plates there need be only sufficient distance to permit tightening the nuts.

For loads acting perpendicular to the grain, the margin between the edge toward which the bolt pressure is acting and the center of the bolt nearest this edge is equal to $4D$.

The margin at the other edge is of relatively no importance.

Laminated Columns. It is believed by some designers that built-up or laminated columns are as strong as columns made from a single piece. Test data does not support this belief.[1] The strength of laminated columns, whether spiked, bolted, constructed with timber connectors, or combinations of these, should be based on the following data as compared with solid columns. A built-up column requires greater cross-sectional dimensions.

L/D Ratio	Per Cent of Solid Column Strength
6	82
10	77
14	71
18	65
22	74
26	82

The ends of the various planks should butt tightly together. Bolts ¾ in. in diameter are usually sufficient. Their edge margin is 2 in. to the center of the bolt; the end margin is 3 in.; and the spacing longitudinally is not greater than 30 in. In addition the various planks are thoroughly spiked together.

TABLE 4. WEIGHT PER CUBIC FOOT OF VARIOUS WOODS*

Species	Weight per cubic foot, lb	
	Green	Air dry (12 per cent moisture content)
Cedar, western red	27	23
Cypress, southern	51	32
Douglas fir:		
Coast region	38	34
Inland Empire	36	31
Rocky Mountain	35	30
Hemlock:		
Eastern	50	28
Western	41	29
Oak, white†	63	47
Pine:		
Northern white	36	25
Norway	42	34
Ponderosa	45	28
Western white	35	27
Pine, southern yellow:		
Longleaf	55	41
Shortleaf	52	36
Redwood	50	28
Spruce:		
Eastern‡	34	28
Englemann	39	23
Sitka	33	28
Tamarack	47	37

* "Wood Handbook," p. 46, Forest Products Laboratory, U.S. Department of Agriculture, Washington, D.C.

† Average of bur, chestnut, post, swamp chestnut, swamp white, and white oak.

‡ Average of black, red, and white spruce.

Data useful in designing timber structures are given in Tables 4 to 7.

Before starting the design of a truss, the material from which it is to be made (and, in the case of timber, the type of construction) must

TABLE 5. SIZES OF AMERICAN STANDARD YARD LUMBER AND TIMBER, Inches

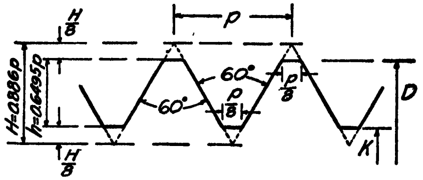
Thickness		Nominal and rough widths							
Nominal and rough	Dressed	2	3	4	5	6	7	8	Wider than 8
		Width of dressed sizes							
1	25/32	1⅝	2⅝	3⅝	4⅝	5⅝	6⅝	7½	½ in. off
1¼	1 1/16								
1½	1 5/16								
2	1⅝								
2½	2⅛								
3	2⅝								
4	3⅝	↓	↓	↓	↓	↓	↓	↓	↓
5	4½				4½	5½	6½	7½	½ in. off
6	5½								
7	6½								
8	7½								
Thicker than 8	½ off				↓	↓	↓	↓	↓

Lengths increase by 2-ft intervals and are usually available up to 40 ft.
The dressed 1 in. thickness is usually taken as ¾ in. for calculations.

TABLE 6. TROLLEY HOISTS, ARMY STYLE

Capacity, tons	Regular lift, ft	Size of I beam
¼	8	5 in. at 10 lb
½	8	5 in. at 10 lb
1	8	6 in. at 12½ lb
1½	8	7 in. at 15.3 lb
2	9	8 in. at 18.4 lb
3	10	9 in. at 21.8 lb
4	10	10 in. at 25.4 lb
5	12	12 in. at 31.8 lb
6	12	12 in. at 31.8 lb
8	12	15 in. at 42.9 lb
10	12	15 in. at 42.9 lb

TABLE 7. SCREW THREADS*



Diameter		Area		Number of threads per in.	Diameter		Area		Number of threads per in.
Total D, in.	Net K, in.	Total D, in.^{2}	Net K, in.^{2}		Total D, in.	Net K, in.	Total D, in.^{2}	Net K, in.^{2}	
¼	0.185	0.049	0 027	20	2¾	2.425	5.940	4 619	4
⅜	0.294	0.110	0.068	16	3	2.675	7.069	5.621	4
½	0.400	0.196	0.126	13	3¼	2.925	8.296	6.720	4
⅝	0.507	0.307	0.202	11	3½	3.175	9.621	7.918	4
¾	0.620	0.442	0.302	10	3¾	3 425	11.045	9.214	4
⅞	0.731	0.601	0.419	9	4	3.675	12.566	10.608	4
1	0.838	0.785	0.551	8	4¼	3.798	14.186	11.330	2⅞
1⅛	0.939	0.994	0.693	7	4½	4.028	15.904	12.741	2¾
1¼	1.064	1.227	0.890	7	4¾	4.255	17.721	14.221	2⅝
1⅜	1.158	1.485	1.054	6	5	4.480	19.635	15.766	2½
1½	1.283	1.767	1.294	6	5¼	4.730	21.648	17.574	2½
1¾	1.490	2.405	1.744	5	5½	4.953	23.758	19.268	2⅜
2	1.711	3.142	2 300	4½	5¾	5.203	25.967	21.262	2⅜
2¼	1.961	3.976	3.021	4½	6	5.423	28.274	23.095	2¼
2½	2 175	4.909	3.716	4					

* American Institute of Steel Construction, 5th ed., p. 164.

first be considered. A critical investigation of the various kinds of trusses suggests, for most purposes, the following types:

Howe Truss. This type of truss is used for the following: conventional timber construction; solid timber members for compression; and steel tie rods for tension members. The use of the Howe type of truss simplifies the design of joints and connections.

Fink and Pratt Trusses. These trusses, particularly the Fink, lend themselves to steel construction, and, if of timber, to the use of timber connectors. The compression members are relatively short, while the tension members are long. Usually the Fink type is selected when timber connectors are to be used, especially when the span is less than 60 to 80 ft and the roof slopes are steeper than 25 deg. For greater spans and flatter slopes, the Belgian truss is more economical.

CHAPTER III

ROOF TRUSSES, COLUMNS, AND FOOTINGS

We shall consider the design of a simple triangular roof truss with both ends fixed. Before doing so, it is necessary to outline certain fundamental considerations.

For purposes of designing a truss, three and often five kinds of loads will influence the selection of members. These loads are

1. Dead load
2. Snow load
3. Wind load
4. Concentrated loads
5. Live loads

The first four are all that we need consider for the usual mine structure.

Dead Loads. The dead load is made up of the weight of the truss and the covering material. The weight of roof covering is easily and fairly accurately found; that of the truss is but an approximation. In some instances, after figuring the size and weight of the various members, we shall find it necessary to recalculate the entire problem on the basis of the new weight because of a radical difference between the estimated and determined weight. For purposes of estimation, the following formulas may be used. There are other formulas available.[1]

Weight of Steel Trusses[2]

$$W = 0.75 \times a \times L(1 + 0.1 \times L)$$

Weight of Wooden Trusses[3]

$$W = 0.5 \times a \times L(1 + 0.15 \times L)$$

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[1] Hool and Kinne, "Steel and Timber Structures," 1st ed., p. 147, McGraw-Hill Book Company, Inc., New York.

[2] Ketchum, M. S., "Design of Steel Mill Buildings," 2d ed., p. 5, McGraw-Hill Book Company, Inc., New York.

[3] Peele, R. (ed.), "Mining Engineers' Handbook," 3d ed., Sec. 43, p. 26, John Wiley & Sons, Inc., New York.

where W = weight of truss, lb
a = distance center to center of trusses, ft
L = span, ft

Weight of Roof Covering. The approximate weight per square foot of some of the common roofing materials is

Material	Weight per Square Foot
Corrugated iron	No. 26, 1 lb, to No. 16, 3.3 lb
Felt and asphalt	2 lb
Felt and gravel	8–10 lb
Slate 3⁄16 to ¼ in. thick	7–9 lb
Tin	1–1½ lb
Skylight glass 3⁄16 to ½ in. including frames	4–10 lb
White pine, hemlock, and spruce sheathing, 1 in. thick	3 lb
Yellow pine sheathing, 1 in. thick	4 lb
Tiles, flat	15–20 lb
Tiles, corrugated	8–10 lb
Plastered ceiling	10 lb

Snow Loads. The snow load is always used as pounds per square foot of horizontal projection. The amount that is apt to be on the roof at any one time depends more or less on the latitude. Various localities specify it as follows:[1] Chicago and New York, 20 lb; Cincinnati and St. Louis, 10 lb; New England, 30 lb. If the slope of the roof is steeper than 60 deg, the snow load is not considered.

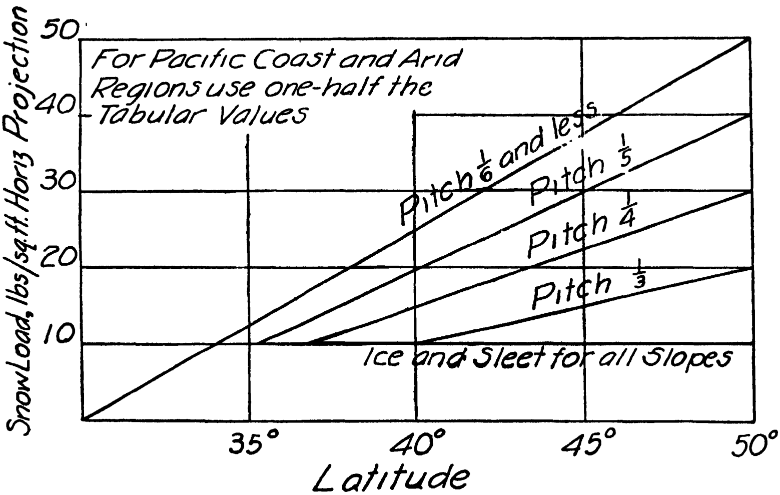

FIG. 17. Diagram for determining snow loads in various localities. (*Ketchum*, "*Design of Steel Mill Buildings*," *McGraw-Hill Book Company, Inc.*, *New York*, 1932.)

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[1] KETCHUM, *op. cit.*, p. 11.

Figure 17 shows the snow load on roofs for various latitudes.[1]

Wind Loads. The wind is considered as acting horizontally and the component normal to the roof surface taken as the force acting on the roof. If the velocity of the wind is known, its equivalent pressure normal to a flat surface can be calculated[2] from

$$P = 0.004V^2$$

where P = pressure, lb per sq ft
V = velocity, mph

From this the following velocities give the corresponding pressures:

Velocity, Mph	Pressure, Lb per Sq Ft
10	0.4
20	1.6
30	3.6
40	6.4
50	10.0
60	14.4
70	19.6
80	25.6
90	32.4
100	40.0

For determining the pressure normal to the roof, various formulas have been suggested. Of these Duchemin's is most commonly used.

Duchemin's Formula

$$P_n = P_h \frac{2 \sin \theta}{1 + \sin^2 \theta}$$

where P_n = normal pressure, lb per sq ft on surface
P_h = horizontal pressure on vertical surface
θ = angle of inclination to the horizontal

Table 8 gives pressures derived by the use of Duchemin's formula.

TABLE 8. WIND LOAD IN POUNDS PER SQUARE FOOT (DUCHEMIN'S FORMULA)

Inclination	Normal Pressure $P_h = 30$ lb
5°	5.1
10°	10.1
15°	14.6
21°48′ (1/5 pitch)	19.8
26°34′ (1/4 pitch)	22.4
30°	24.0
33°41′ (1/3 pitch)	25.5
45° (1/2 pitch)	28.3
60°	29.7
90°	30.0

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[1] *Ibid.*

[2] *Eng. News*, Dec. 13, 1890.

Tables 9 and 10 give further data on snow and wind loads.[1]

TABLE 9. SNOW LOADS FOR ROOF TRUSSES
Pounds per Square Foot of Roof Surface

Location	Pitch of roof				
	½	⅓	¼	⅕	Flat
	* †	* †	* †		
Southern states and Pacific slope	0– 0	0– 5	0– 5	5	5
Central states	0– 5	7–10	15–20	22	30
Rocky Mountain states	0–10	10–15	20–25	27	35
New England states	0–10	10–15	20–25	35	40
Northwest states	0–12	12–18	25–30	37	45

* For slate, tile, or metal roofs.
† For shingle roofs.

TABLE 10. COMBINED WIND AND SNOW LOADS
Pounds per Square Foot of Roof Surface

Location	Pitch of roof					
	60°	45°	⅓	¼	⅕	Flat
Northwest states	30	30	25	30	37	45
New England states	30	30	25	25	35	40
Rocky Mountain states	30	30	25	25	27	35
Central states	30	30	25	25	22	30
Southern and Pacific Coast states	30	30	25	25	22	20

Concentrated Loads. This type of load requires no particular discussion. They result ordinarily from stationary objects and may be acting on the upper chord or be attached to the lower chord, as an I beam for carrying a chain block, or platform for supporting equipment. They are most readily determined by using a diagram drawn separately from the other loads (see Fig. 21).

Live Loads. Live loads around mining works would result from traveling cranes, moving cars, hoisting, etc. They will not, with the exception of the live loads on the headframe, be considered. The reader is referred to standard textbooks treating that subject[2].

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[1] HOOL and KINNE, *op. cit.*, pp. 149–151.

[2] CATHCART and CHAFFEE, "Elements of Graphic Statics," D. Van Nostrand Company, Inc., New York.

MARBURG, E., "Framed Structures and Girders," Vol. 1, Part 1, McGraw-Hill Book Company, Inc., New York.

TABLE 11. ALLOWABLE WORKING STRESSES IN VARIOUS TIMBERS

Pounds per Square Inch

Kind of timber	Bending	Shearing stresses			Compressive stresses			Modulus of elasticity
	Working stress in extreme fiber (1)	Parallel to grain (2)	Longitudinal shear in beams (3)	Perpendicular to grain (4)	Parallel to grain (5)	Columns under 15 dia. (6)	Formulas for over 15 dia. (7)	(8)
Douglas fir	1200	170	110	310	1200	900	$1200\left(1 - \frac{l}{60d}\right)$	1,510,000
Longleaf pine	1300	180	120	260	1300	980	$1300\left(1 - \frac{l}{60d}\right)$	1,610,000
Shortleaf pine	1000	170	130	170	1100	830	$1100\left(1 - \frac{l}{60d}\right)$	1,480,000
White pine	900	100	70	150	1000	750	$1000\left(1 - \frac{l}{60d}\right)$	1,130,000
Spruce	1000	150	70	180	1100	830	$1100\left(1 - \frac{l}{60d}\right)$	1,310,000
Norway pine	800	130	100	150	800	600	$800\left(1 - \frac{l}{60d}\right)$	1,190,000
Tamarack	900	170	100	220	1000	750	$1000\left(1 - \frac{l}{60d}\right)$	1,220,000
Western hemlock	1100	160	100	220	1200	900	$1200\left(1 - \frac{l}{60d}\right)$	1,480,000
Redwood	900	80	...	150	900	680	$900\left(1 - \frac{l}{60d}\right)$	800,000
Bald cypress	900	120	...	170	1100	830	$1100\left(1 - \frac{l}{60d}\right)$	1,150,000
Red cedar	800	...	...	230	900	680	$900\left(1 - \frac{l}{60d}\right)$	860,000
White oak	1100	210	110	450	1300	980	$1300\left(1 - \frac{l}{60d}\right)$	1,150,000

l = length in inches; d = least side in inches.

Stresses are for railroad bridges and trestles. For enclosed and protected buildings free from impact, increase as follows: columns (1), (5), (6), and (7), 50 per cent; column (4), 25 per cent if structural-grade timber is used; for common-grade timber use values in table. The bending-stress values may be used for tension.

Allowable Stresses. In Table 11 are given[1] the working stresses for timber commonly used in construction work.

The bearing resistance of common soils and rocks is given in Table 12.[2] For a very complete and extensive table of the bearing resistance of soils, see "Foundations, Abutments and Footings" by Hool and Kinne.

TABLE 12. SAFE BEARING RESISTANCE OF SOILS

Material	Safe bearing, tons per square foot	
	Minimum	Maximum
Rock, hardest, in thick layers in bed	200	
Rock equal to best ashlar masonry	25	30
Rock equal to best brick	15	20
Rock equal to poor brick	5	10
Clay in thick beds, always dry	4	6
Clay in thick beds, moderately dry	2	4
Clay, soft	1	2
Gravel and coarse sand, well cemented	8	10
Sand, compact and well cemented	4	6
Sand, clean and dry	2	4
Quicksand, alluvial soils, etc	0 5	1

DESIGN OF A WOODEN TRUSS

The truss shown in Fig. 18 will be designed with the following data: The span is 60 ft; the pitch is $\frac{1}{3}$; the distance between trusses is 15 ft; the distance between rafters is 18 in.; the wind load is 30 lb per sq ft horizontal pressure; the snow load is 20 lb per sq ft horizontal projection; the roof covering is 1-in. white-pine sheathing and No. 18 corrugated 26-in. sheeting at 2.32 lb per sq ft; the trusses will be fixed at the ends and supported by 8-in. concrete walls. The Howe type of truss will be used, and will be made of Douglas fir.

The pitch of a truss is equal to the rise or height at the peak divided by the span

$$\text{Pitch} = \frac{H}{L} \quad \text{or} \quad H = PL$$

$$H = \frac{1}{3} \times 60 = 20 \text{ ft}$$

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[1] JACOBY and DAVIS, "Timber Design and Construction," 2d ed., p. 320, John Wiley & Sons, Inc., New York.

[2] BAKER, I. O., "Treatise on Masonry Construction," 9th ed., p. 194, John Wiley & Sons, Inc., New York.

The truss may be divided into six panels, of 10 ft each, on the lower chord.

$$^{60}\!/_{6} = 10 \text{ ft for each lower chord panel}$$

From Table 8, a $\frac{1}{3}$ pitch equals 33°41′.

$$\text{Length of upper chord panel} = 10 \div \cos 33°41' = 12 \text{ ft (approximately)}$$

(Calculations may be made with a slide rule for determining the size of the rafters and purlins and, in fact, for most of the truss calculations.)

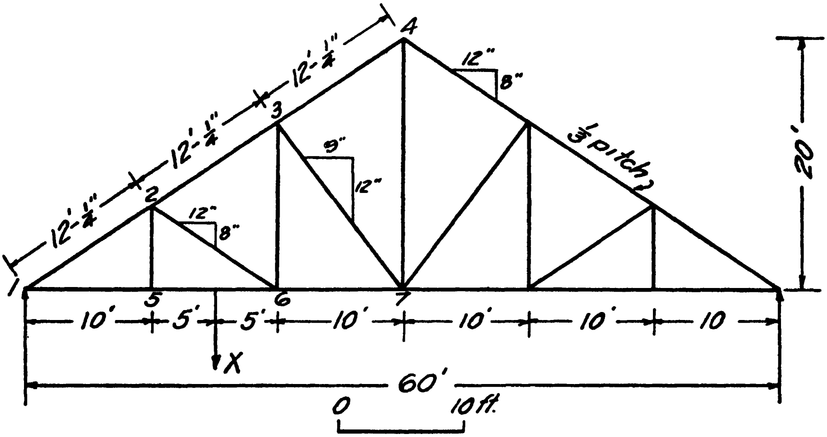

FIG. 18. Layout for a Howe truss.

Each truss supports the load one-half the distance to its neighbor; therefore, each truss supports (with the exception of the end truss) a load equal to the distance between trusses.

The rafters are spaced 18 in. apart. This places a rafter over the center of each truss; the purlins will have to support nine rafters acting as concentrated loads.

Rafters. Each rafter is 12 ft long between supports (purlins). It has a roof load 18 in. by 12 ft acting as a uniformly distributed load. White pine weighs 25 lb per cu ft. The weight of a square foot, remembering that 1-in. sheathing has a surfaced thickness of $\frac{3}{4}$ in., is

$$^{25}\!/_{12} \times \tfrac{3}{4} = 1.6 \text{ lb per sq ft}$$
$$\text{Weight of covering} = 1.6 + 2.3 = 3.9 \text{ lb per sq ft}$$

Assuming the rafters at 2 lb per sq ft,

$$\text{Weight of rafter and covering} = 3.9 + (2 \times {}^{18}\!/_{12}) = 6.9 \text{ lb, per ft of rafter}$$

Snow Load. Snow load is 20 lb per sq ft of horizontal projection. The horizontal projection of the chord is 10 ft, and the upper chord is 12 ft; therefore

$$^{10}\!/_{12} \times 20 = 16.7 \text{ lb per sq ft per ft of rafter}$$
$$16.7 \times ({}^{18}\!/_{12}) = 25 \text{ lb per ft of rafter}$$

Load on Rafters. From Table 8, the wind load normal to the roof is 25.5 lb per sq ft for a $\frac{1}{3}$ pitch. The covering and the snow load must be resolved normal (perpendicular) to the roof. Hence, the total load is

$$25.5 + (6.9 + 25) \times \cos 33°41' = 52.0 \text{ lb per ft of rafter}$$

The full snow load is used instead of one-half the snow load as recommended in textbooks on design when combining snow and wind loads. Anyone familiar with the Western mining country will appreciate the fact that a heavy snow load with a strong wind is not at all uncommon.

From Fig. 14,

$$\begin{aligned} M &= \tfrac{1}{8}wl^2 \times 12 \quad \text{in.-lb} \\ &= \tfrac{1}{8} \times 52 \times 12^2 \times 12 = 11{,}200 \text{ in.-lb} \end{aligned}$$

From $M = \frac{1}{6}sbd^2$,

$$d = \sqrt{\frac{6M}{sb}} \text{ in.}$$

Consulting Table 11, column (1), Douglas fir stands

$$1200 + (50\% \times 1200) = 1800 \text{ lb per sq in.}$$

Assuming $b = 2$ in. (actually $1\frac{5}{8}$ in.),

$$d = \sqrt{\frac{6 \times 11{,}200}{1800 \times 1.625}} = 4.8 \text{ in.}$$

The nearest commercial size is 6 in. ($5\frac{5}{8}$ in. dressed); therefore, a 2- by 6-in. timber is used for the rafters.

Each rafter supports an area equal to 12 ft by $\frac{18}{12}$ = 18 sq ft. One rafter weighs

$$\begin{aligned} 1\tfrac{5}{8} \times 5\tfrac{5}{8} \times 12 \times \tfrac{1}{144} &= 0.76 \text{ cu ft} \\ 0.76 \times 32 \text{ lb} &= 24.3 \text{ lb} \\ 24.3 \div 18 &= 1.35 \text{ lb per sq ft of roof} \end{aligned}$$

The assumed weight of rafter (2 lb) was too much; it is on the safe side and would not materially affect the calculations for rafter size.

Purlins. The load on a purlin extends halfway to each adjacent purlin. It has been found that the rafters carry 52 lb per ft of normally projected load. From center of purlin to center purlin there are 11 rafters. One-half of the load on each of the two end rafters is assumed as acting on the purlins. The effective load therefore comes from 10 rafters. The load on each rafter is 52 lb × 12 ft = 624 lb.

$$\text{Total load} = 10 \text{ rafters} \times 624 \text{ lb} = 6240 \text{ lb}$$

The reaction at each purlin is $\frac{1}{2} \times 6240 = 3120$ lb.

One of the rafters will fall halfway between purlins. The others, being equally spaced from the central one, indicates that the maximum bending moment is under the middle rafter. Taking moments about the center rafter,

$$\begin{aligned} M &= 3120 \times 7\tfrac{1}{2} - 624 \times 6 - 624 \times 4\tfrac{1}{2} - 624 \times 3 - 624 \times 1\tfrac{1}{2} \\ &= 23{,}400 - 9360 = 14{,}040 \text{ ft-lb} \\ &= 14{,}040 \times 12 = 168{,}500 \text{ in.-lb} \end{aligned}$$

Assuming that $b = 8$ in.,

$$d = \sqrt{\frac{6 \times 168{,}500}{1800 \times 7.5}} = 8.6 \text{ in.}$$

The purlin could be an 8- by 10-in. piece. A 6- by 12-in. piece is just as satisfactory and has considerably less timber. It will be used.

Adjustment for weight of purlin. A 6- by 12-in. purlin weighs

$$5\tfrac{1}{2} \times 11\tfrac{1}{2} \times \tfrac{1}{144} \times 15 \times 32 = 211 \text{ lb}$$
$$211 \div 15 = 14.1 \text{ lb per ft}$$
$$M = \tfrac{1}{8} \times 14.1 \times 15^2 \times 12 = 4750 \text{ in.-lb}$$
$$\text{Total } M = 168{,}500 + 4750 = 173{,}250 \text{ in.-lb}$$
$$d = \sqrt{\frac{6 \times 173{,}250}{1800 \times 5.5}} = 10.3 \text{ in.}$$

The 6- by 12-in. purlin is satisfactory.

The purlin weight is equivalent to

$$211 \div (12 \times 15) = 1.17 \text{ lb per sq ft of roof supported}$$

The weight of truss is

$$\begin{aligned} W &= 0.5 \times a \times L(1 + 0.15 \times L) \\ &= 0.5 \times 15 \times 60(1 + 0.15 \times 60) = 4500 \text{ lb} \end{aligned}$$

Dead Load. The roof area supported by each truss is

$$2(15 \times 30 \div \cos 33°41') = 1083 \text{ sq ft}$$

The dead-load weight is

No. 18 corrugated sheeting	2.32 lb per sq ft
Rafters	1.35 lb per sq ft
Purlins	1.17 lb per sq ft
Sheathing	1.6 lb per sq ft
Total	6.44 lb per sq ft, or 6.4

Total weight supported by truss = 1083 × 6.4 = 6940 lb

Total dead load = 4500 + 6940 = 11,440 lb

There are six panels of equal size. The weight on each panel is

$$11{,}440 \div 6 = 1910, \text{ or } 1900 \text{ lb}$$

The load at panel point AB and GH = 1900 × ½ = 950 lb, or 900 lb

and

The load at point $BC = CD = DE = EF = FG = 1800$ lb

Figure 19 shows the determination of the stresses resulting from the dead load.

Snow Load. The snow load is 20 lb per sq ft of horizontal projection. The actual weight of snow is

$$20 \times 15 \times 60 = 18{,}000 \text{ lb}$$

It is not necessary to draw a separate snow-load diagram; snow loads are treated exactly as are dead loads. All we need to do is determine the ratio of snow load to dead load and multiply the stresses in the members caused by the dead load by this factor. The values thus obtained are added to the stresses produced by the other conditions. This factor equals

$$\frac{18{,}000}{11{,}440} = 1.57$$

Wind Load. The horizontal wind pressure will be taken at 30 lb per sq ft. Consulting Table 8, we find that this is equal to 25.5 lb per sq ft normal to the surface when acting on a roof having a slope of 33°41′. The wind can act on but one side at a time, although some investigators have found that suction is created on the leeward side.[1] The total area is 1083 sq ft. The pressure on half of the roof is

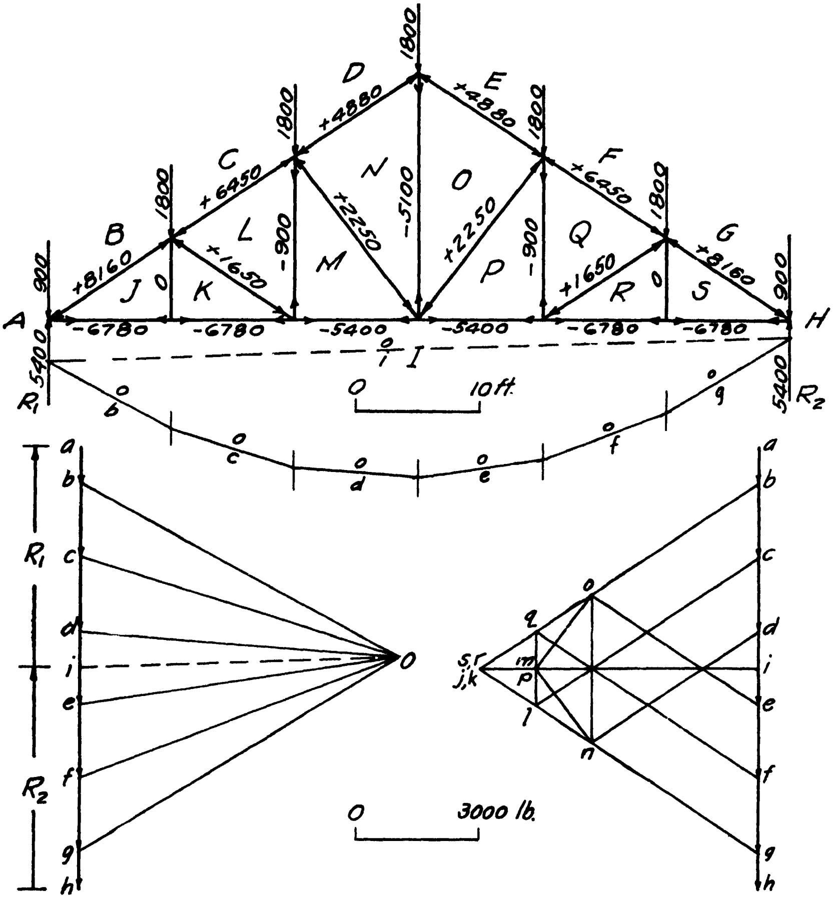

FIG. 19. Dead-load stresses for Howe truss.

$$1083/2 \times 25.5 = 13{,}810 \text{ lb}$$

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[1] *Proc. Inst. Civil Eng.*, **165**, 78; **171**, 175.

As there are only three panels, the load per panel is 4603, or 4600 lb. Therefore, at each panel point there will be

$$A = D = 4600/2 = 2300 \text{ lb}$$
$$B = C = 4600 \text{ lb}$$

As the truss has fixed ends, only one wind-load solution is necessary. If one end had been free, two solutions would have been required in order to find the maximum stresses. Figure 20 shows the wind-load diagrams.

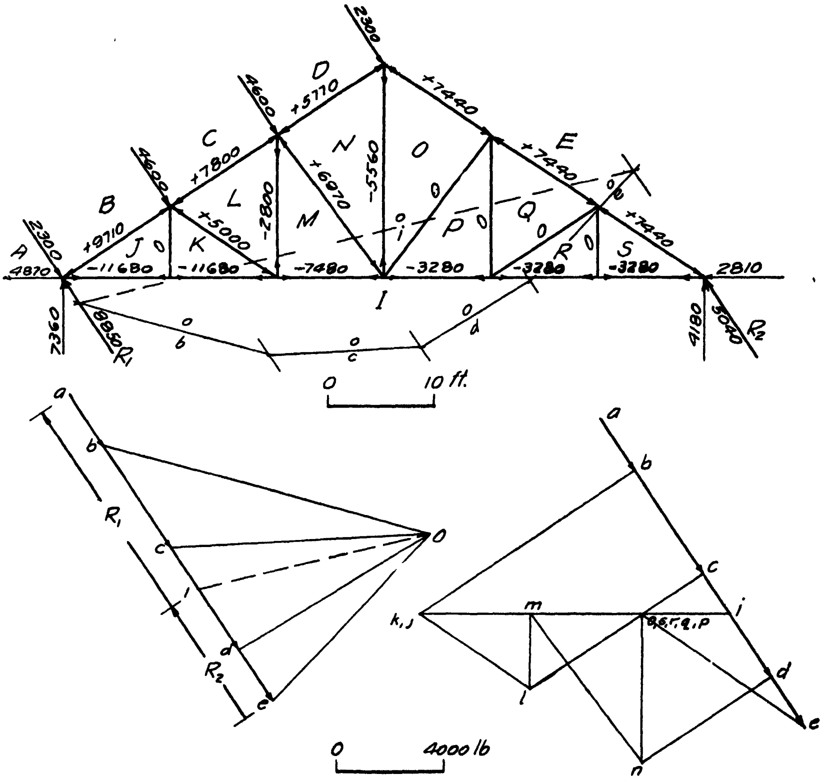

Fig. 20. Wind-load stresses for Howe truss.

Concentrated Load. The concentrated load X shown in Fig. 18 has a value of 10,000 lb. Its effect at the panel points may be found by treating the chord as a simple beam. As the load is in the center of the chord member, the reaction at each end is 5000 lb. The stresses caused by this load are found by using a separate figure (Fig. 21).

The solution for concentrated loads is similar to that illustrated in Fig. 6.

The data given in Fig. 21 are added to the stresses caused by the dead load, snow load, and wind load in order to obtain the total stresses.

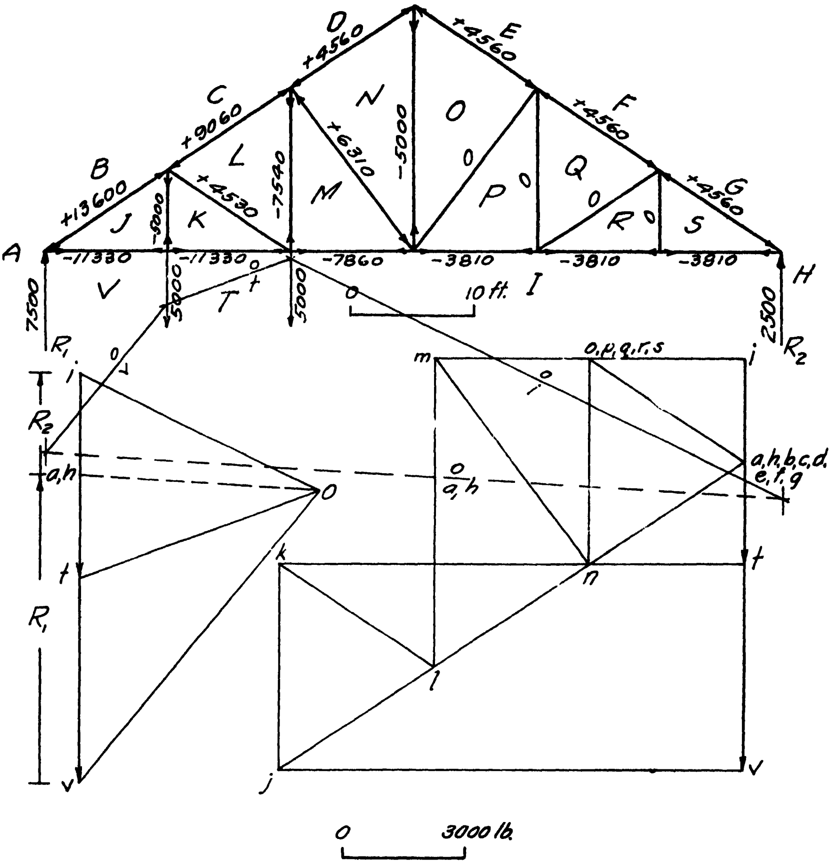

FIG. 21. Concentrated-load stresses for Howe truss.

It is believed that for structures in mining camps, because of their topographic location, the full value of the loads determined should be used. It no doubt is permissible, in the case of similar structures erected in more favorable localities, to combine but part of the snow load and wind load to obtain the total stresses.

In localities where high winds following a heavy snowfall are not likely to occur, it is customary to use the following combinations, the maximum stress resulting from these combinations being used:

Dead load + snow load + concentrated load + ½ wind load

or

Dead load + concentrated load + wind load + ½ snow load

Table 13 gives the stresses caused by the various loads considered.

From Table 13, we now have data on which to base calculations for selecting the size of the material to go into the various members.

By definition from strength of materials, compression members act as columns and must be treated by a column formula. Tension members are based on the tensile strength of the material used. The formulas commonly used for wooden columns are those specified in Table 11. We shall use the one applying to Douglas fir. If other kinds of wood are used, the appropriate formula must be selected. As the truss is under cover and protected, the values listed in Table 11 will be increased as suggested in the footnote. The column formula simply gets us started. In many cases the size of the member must be increased to provide bearing area for framing and connections.

The formula, after increasing 50 per cent, is

$$\frac{P}{A} = 1800\left(1 - \frac{l}{60d}\right) = 1800 - 30\frac{l}{d}$$

where P = total safe load, lb
A = area of section, sq in.
l = length of column, in.
d = least transverse dimension, in.

For tension members, except the lower chord, steel tie rods are used. Their size is based on a working tensile strength for steel of 18,000 lb per sq in.

From Table 13, we find that the greatest compressive stress occurs in the upper chord member *BJ* and is +46,030 lb. There would be no advantage in making the members of this chord of different-sized materials. Better framing of joints and connections is obtained if the entire chord is one size. Therefore, the upper chord will be made of material to take the maximum load.

Selection of Members. *Members BJ, CL, DN, EO, FQ, and GS.* As a trial, we shall select a 6- by 8-in. timber. Substituting in the formula,

$$\frac{P}{6 \times 8} = 1800 - 30 \times \frac{12 \times 12}{6} = 1800 - 720 = 1080 \text{ lb per sq in.}$$

$$P = 6 \times 8 \times 1080 = 51{,}840 \text{ lb}$$

TABLE 13. SUMMATION OF STRESSES IN TRUSS

Member	Stresses due to				Maximum stress	Length of member
	Dead load	Snow load	Wind load	Concentrated load		
BJ	+8820	+13,900	+ 9,710	+13,600	+46,030	12′¼″
CL	+6960	+10,950	+ 7,800	+ 9,060	+34,770	12′¼″
DN	+5270	+ 8,100	+ 5,770	+ 4,560	+23,900	12′¼″
JI	−7330	−11,500	−11,680	−11,330	−41,840	10′
JK	0	0	0	− 5,000	− 5,000	6′8½″
KI	−7330	−11,500	−11,680	−11,330	−41,840	10′
KL	+1780	+ 2,800	+ 5,000	+ 4,530	+14,110	12′¼″
LM	− 970	− 1,520	− 2,800	− 7,540	−12,830	13′4¼″
MI	−5830	− 9,200	− 7,480	− 7,860	−30,370	10′
MN	+2430	+ 3,830	+ 6,970	+ 6,310	+19,540	16′8¼″
NO	−5510	− 8,700	− 5,560	− 5,000	−24,770	20′
EO	+5270	+ 8,300	+ 7,440	+ 4,560	+25,570	12′¼″
FQ	+6960	+10,950	+ 7,440	+ 4,560	+29,910	12′¼″
GS	+8820	+13,900	+ 7,440	+ 4,560	+34,720	12′¼″
SI	−7330	−11,500	− 3,280	− 3,810	−25,920	10′
RS	0	0	0	0	0	6′8½″
RI	−7330	−11,500	− 3,280	− 3,810	−25,920	10′
RQ	+1780	+ 2,800	0	0	+ 4,580	12′¼″
QP	− 970	− 1,520	0	0	− 2,490	13′4¼″
PI	−5830	− 9,200	− 3,280	− 3,810	−22,120	10′
OP	+2430	+ 3,830	0	0	+ 6,260	16′8¼″
R_1	5825	9,160	7,360	7,500	29,845	20′
R_2	5825	9,160	4,180	2,500	21,665	20′

The actual size of a 6- by 8-in. piece is 5½ by 7½ in. If these dimensions are used the member will stand only 41,800 lb and is too small. An 8- by 8-in. piece is satisfactory but has more material in it than a 6- by 10-in. piece.

Trying a 6- by 10-in. piece (actually 5½ by 9½), $P = 53{,}200$ lb. Therefore, the upper chord will tentatively be made of this size material with the 10-in. side vertical.

Members KL and QR. The member is 12 ft long and must withstand a load of +14,110 lb. One dimension must be 6 in. to detail with the upper chord. On trying a 4- by 6-in. piece, it is found to be too small. A 6- by 6-in. piece must be used.

Members MN and OP. The maximum stress is +19,540 lb and the length is 16 ft 8 in. A 6- by 6-in. piece is large enough.

Lower Chord, Members JI, KI, MI, PI, RI, and SI. The maximum stress is in *JI* and *KI* and is −41, 840 lb. Because of the bending moment resulting from the concentrated load on *KI*, the member must be checked for bending. The selection of the lower chord is, at this time, indefinite because we do not have the detail on the chord connection, nor do we know yet how much of the section must be removed because of the joints and tie rods. For these reasons it is better to pass the selection up until more details are obtained. The horizontal dimension will tentatively be 6 in. to detail with the upper chord and the struts selected.

Member JK and RS. The load in *JK* is −5000 lb. The tie rod must have a net area, after threading, of 5000 ÷ 18,000 = 0.278 sq in. Consulting Table 7, we find that a ¾-in. rod with a net area of 0.302 sq in. is nearest. It will be used.

Member IM and PQ. In *IM* there is a stress of −12,810 lb. The area required is 12,830 ÷ 18,000 = 0.712 sq in. A 1¼-in. rod is necessary.

Member NO. The stress is −24,770 lb. The net area is

$$24{,}570 \div 18{,}000 = 1.365 \text{ sq in.}$$

A 1⅝ in. piece is needed.

Lower Chord Connection. The joint shown in Fig. 22*a* will be investigated for connecting the two chords. It is first necessary to determine the area required for the face at *a* − *b*. This is an example of a force acting at an angle to the grain.

$$N = \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

From Table 11, $P = 1200 + 50\% \times 1200 = 1800$ lb per sq in. and

$$Q = 310 + 25\% \times 310 = 388 \text{ lb per sq in.}$$

$$N = \frac{1800 \times 388}{1800 \sin^2 33°41' + 388 \cos^2 33°41'} = 846 \text{ lb per sq in.}$$

Area needed = 46,030 ÷ 846 = 54.3 sq in.

A 6- by 10-in. piece has an area of 5½ × 9½ = 52.25 sq in., and is too small. An 8- by 8-in. piece has an area of 7½ × 7½ = 56.25 sq in. The width of the cut is 7½ in.; the length *a-b* is 54.0 ÷ 7.5 = 7.2 in.

The depth of the cut below the surface of the lower chord is

$$7.2 \times \cos 33°41' = 6 \text{ in.}$$

To resist the tension of 41,840 lb in the lower chord, a net area of

$$41{,}840 \div 1800 = 23.2 \text{ sq in.}$$

is needed. The tie rod at *LM* is 1¼ in. in diameter. The hole through the lower chord would be ¼ in. larger, thus becoming 1½ in. in diameter. The depth of the net section is 23.2 ÷ 7.5 = 3.1 in. For the tie-rod hole *LM*, 3.1 × 1½ = 4.6 sq in. must be added, and this is equivalent to 4.6 ÷ 7.5 = 0.6 in. of additional depth added to the lower chord. The angle washers on the lower end of the bolts will be assumed to set into the timber ¾ in. The total depth of the lower chord becomes

$$6.0 + 3.1 + 0.6 + 0.75 = 10.45 \text{ in.}$$

The nearest size is 12 in., which has an actual depth of 11.5 in. An 8- by 12-in. piece would appear to be suitable for the lower chord. The suitability of this timber must be investigated for the bending caused by the 10,000-lb concentrated load.

$$M = \tfrac{1}{4}Pl \times 12 = \tfrac{1}{4} \times 10{,}000 \times 10 \times 12 = 300{,}000 \text{ in.-lb}$$

$$d = \sqrt{\frac{6 \times 300{,}000}{1800 \times 7.5}} = 11.5 \text{ in.}$$

The 8- by 12-in. timber just satisfies the requirements.

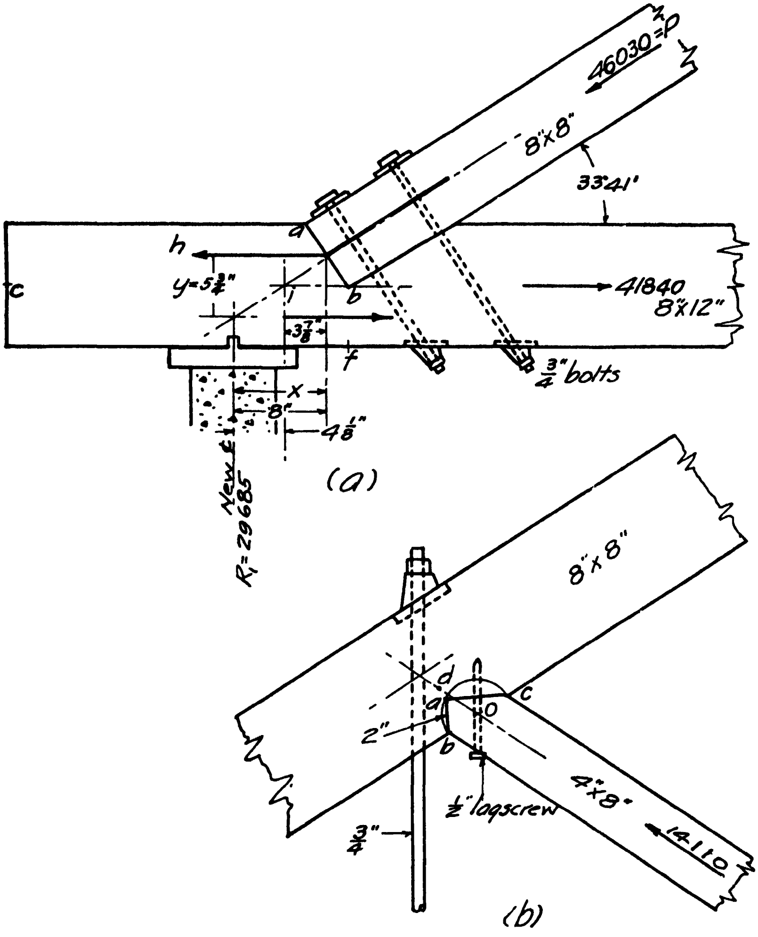

FIG. 22. Joints for Howe truss.

Because of the material removed for making the joint, a new center line for the wall supporting the truss must be found. Shifting the center line will compensate for the eccentricity resulting from removing material for the connection. The force P is assumed to act at the center of the face a-b. There is a couple formed by the stress in the lower chord having a moment arm y. The arm y is equal to one-half the depth of the lower chord, and in this case is $\frac{1}{2} \times 11.5 = 5.75$ in. This couple is resisted by another consisting of the reaction R_1 whose arm x equals the distance from the new center line through the reaction to the point where P intersects the face a-b.

A large-scale drawing of the joint should be made so that the intersection of P with a-b can be located accurately.

With the exception of x, the other members of the couples are known.

$$y = \tfrac{1}{2} \times 11.5 = 5.75 \text{ in.}$$
$$41{,}840 \times 5.75 = 241{,}000 \text{ in.-lb}$$
$$29{,}845x = 241{,}000 \text{ in.-lb}$$
$$x = 8.06, \text{ or } 8 \text{ in.}$$

From Fig. 22a the intersection of the center lines of the chords is measured and is found to be 3⅞ in. from the intersection of P with the center of a-b. Then,

$$8 - 3\tfrac{7}{8} = 4\tfrac{1}{8} \text{ in.}$$

The new wall center line is to the left of the intersection of the center line of the members.

The distance from b to the end of the lower chord must be such that the stress in the lower chord will be resisted by the horizontal shear (shear parallel to the grain). From Table 11, column (2), the allowable shear parallel to the grain is 170 lb per sq in.

$$41{,}840 \div 170 = 246 \text{ sq in.}$$
$$246 \div 7.5 = 32.8 \text{ in.}$$

The distance b-c is, then, 2 ft 9 in.

The preceding calculations for the heel connection have been carried out to illustrate the method of making this very common type of connection. Under most conditions such a connection would prove satisfactory. As the present example draws to a conclusion, it is found that, because of the length of b-c, excessively large purlins would be required to place the rafters above the projection of the lower chord. It is always better to use as few cuts as possible; in the present case, because of the stresses resulting from the concentrated load, the stresses at the heel connection are larger than are ordinarily found, and a double notch joint must be used.

The connection is recalculated as shown in Fig. 23a.

As there are two cuts, each will take one-half the load, and will require $54.0 \times \frac{1}{2} = 27.0$ sq in. of bearing area.

$$\text{Minimum length of face} = 27.0 \div 7.5 = 3.6 \text{ in., or } 3\tfrac{3}{4} \text{ in}$$

The timber is actually 7½ in., and the upper member will be framed as shown in Fig. 23a.

$$\text{Face } a\text{-}b = c\text{-}d = 3\tfrac{3}{4} \text{ in.}$$

Point d is 3¾ in. below the surface of the timber.

To resist the tension of 41,840 lb in the lower chord, a net area of

$$41{,}840 \div 1800 = 23.2 \text{ sq in.}$$

is needed. *The hole for the tie rod LM is 1½ in. in diameter. The depth of the* net section is 23.2 ÷ 7.5 = 3.1 in. To replace the area removed by the hole in the net section, an additional area of 3.1 × 1½ = 4.6 sq in. must be added. This is equivalent to 4.6 ÷ 7.5 = 0.6 in. The angle washers on the lower end of the heel-connection bolts will be assumed to be set ¾ in. into the timber. The total depth of the lower chord becomes

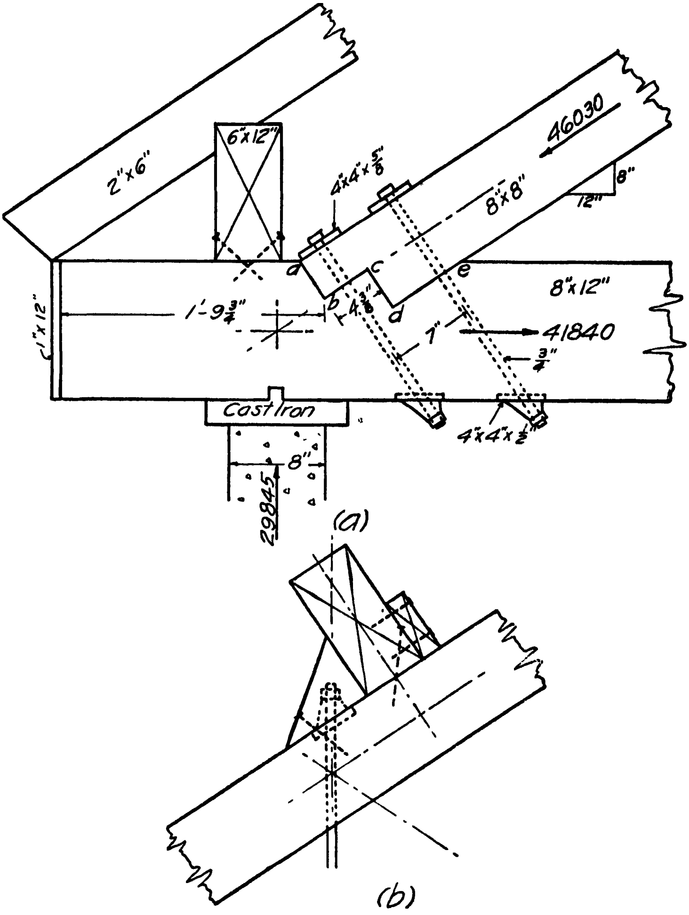

Fig. 23. Joints for Howe truss.

$$3.75 + 3.1 + 0.6 + 0.75 = 8.2 \text{ in.}$$

It would appear that an 8- by 10-in. timber should be used; but, as found from previous calculations, an 8 by 12 in. is needed because of the bending stresses.

The points *b* and *d* are at different depths. Therefore the horizontal area needed to resist the 41,840-lb tension in the lower chord is resisted by two layers.

$$\tfrac{1}{2} \times 41{,}840 = 20{,}920 \text{ lb}$$

Area required = 20,920 ÷ 170 = 123 sq in.
Distance of b to end of chord = 123 ÷ 7.5 = 16.4, or 16½ in.

This is the minimum distance.

Because of the joint detail, it became necessary to change the size of the upper chord to an 8 by 8 in. . The members KL, QR, MN, and OP must be recalculated, using 8 in. for one of the dimensions.

Members KL and QR. Trying a 4 by 8 in.,

$$\frac{P}{3.5 \times 7.5} = 1800 - 30 \times \frac{12 \times 12}{7.5} = 1800 - 576 = 1224 \text{ lb per sq in.}$$
$$P = 3.5 \times 7.5 \times 1224 = 32{,}200 \text{ lb}$$

This is far more than needed. It is not customary to use less than 4-in. material.

Members MN and OP. For these members a 4 by 8 in. is large enough, although later calculations for the joint may require a larger size.

Bolts. To tie the two chords together ¾-in. bolts will be used. Theoretically there is no stress in the bolts, as sufficient area to resist horizontal shear has been provided. There is no certain method for determining the size of the bolts. For small to medium trusses a ¾-in. bolt is usually assumed. Washers are designed to resist the strength of the bolt.

A ¾-in. bolt has a net area of 0.302 sq in. The working strength is 0.302 × 18,000 = 5440 lb. For bearing on the top chord, the area of the washer is

$$5440 \div 388 = 14 \text{ sq in.}$$

To this must be added the area of the hole, which will be taken at ⅞ in. and which equals 0.6 sq in.

Total area = 14 + 0.6 = 14.6 sq in.
Length and width = $\sqrt{14.6}$ = 3.8, or 4 in.

The washer is 4 by 4 in.

The thickness if found from

$$t = \sqrt{\frac{0.75Pa}{(a - d)f}} = \sqrt{\frac{0.75 \times 5440 \times 4}{(4 - 0.875) \times 18{,}000}} = 0.54 \text{ in.}$$

The washers are ⅝ by 4 by 4 in.

Angle washers

Stress perpendicular to the chord = 5440 sin 56°19′ = 4520 lb
Stress parallel to the chord = 5440 cos 56°19′ = 3030 lb
Area for bearing = 4520 ÷ 388 = 11.6 sq in.

The area of a ⅞-in. circular hole is equal to 0.601 sq in. The bolt makes an angle of 56°19′ with the chord. The area of the elliptical hole in the washer is

$$0.601 \div \sin 56°19' = 0.72 \text{ sq in.}$$
$$\text{Total area} = 11.6 + 0.7 = 12.3 \text{ sq in.}$$

A 4- by 4-in. washer will do.

Depth of indentation in lower chord. From Table 11, column (5), the compression parallel to the grain is 1800 lb per sq in.

Area for compression = 3030 ÷ 1800 = 1.68 sq in.
Depth of indentation = 1.68 ÷ 4 = 0.42 in.

The washer will be set into the chord ½ in.

The distance between washers to resist horizontal shear (parallel to the grain) is

$$3030 \div 170 = 17.8 \text{ sq in.}$$
$$\text{Distance} = 17.8 \div 4 = 4.45, \text{ or } 4\tfrac{1}{2} \text{ in.}$$

The bolts will be spaced 7 in. apart.

Intermediate joint 2. The connection at joint 2, Fig. 18, is designed in the following manner: Fig. 22*b* is a large-scale drawing of the connection. Face *a-b* is assumed at 1½ in. Using *o* as a center with radius *a-b*, a semicircle is drawn. Then with *b* as a center and *a-b* equal to 1½ in. the point *a* is established. Angle *bac* is therefore 90 deg. If the scale of the drawing is large enough, the various angles may be measured with a protractor. They are

Angle *bdo* = 56°
Angle *acb* = 21°35′

From

$$N = \frac{PQ}{P \cos^2 y + Q \sin^2 y}$$

we can find the allowable unit pressure on *a-b* and *a-c*.

Consulting Table 11, $P = 1800$ and $Q = 388$. With $y = 56°$,

$$N = 838 \text{ lb per sq in.}$$

The stress in the strut is +14,110 lb. The component perpendicular to *a-b* is 14,110 × sin 56° = 11,700 lb.

$$\text{Area at } a\text{-}b = 11{,}700 \div 838 = 13.9 \text{ sq in.}$$
$$a\text{-}b = 13.9 \div 7.5 = 1.85 \text{ in.}$$

The 1½ in. assumed is too small. Recalculating and using *a-b* = 2 in.,

Angle *bdo* = 53°30′
Angle *acb* = 29°30′

$$N = 788 \text{ lb per sq in.}$$

Stress perpendicular to *a-b* = 14,100 × sin 53°30′ = 11,300 lb

The length of *a-b* is found to be 1.98 in., so that the 2 in. assumed is sufficient.

Cut a-c. With $y = 29°30'$, $N = 480$ lb per sq in.

Stress perpendicular to *a-c* = 14,110 × cos 53°30′ = 8400 lb
Area at *a-c* = 8400 ÷ 480 = 17.5 sq in.

$$a\text{-}c = 17.5 \div 7.5 = 2.33 \text{ in.}$$

From the drawing, *a-c* scales 2¾ in. The arrangement is satisfactory.

A ¾-in. tie rod was selected at *joint* 2. It has the same strength as the bolts selected for the chord connection. The angle washer required on the upper chord is identical with that on the lower chord end of the bolts. Therefore this washer will have the same dimensions as already calculated.

The washer on the lower end of the tie rod at *joint* 5 will be the same as that on the ¾-in. bolts and is ⅝ by 4 by 4 in.

Joint 3. Figure 24*a* shows a detail drawing of *joint* 3. The strut makes almost a 90-deg angle with the chord. A dowell will be sufficient to hold the members together. The stress in the strut is +19,540 lb. The bearing area on the upper chord is

$$19{,}540 \div 388 = 50.2 \text{ sq in.}$$

$$\text{Area of strut} = 3.5 \times 7.5 = 26.3 \text{ sq in.}$$

This is not sufficient; the size of the strut must be increased. The timber that will satisfy the bearing area requirements is an 8- by 8-in. timber.

$$\text{Area} = 7.5 \times 7.5 = 56.3 \text{ sq in.}$$

The axis of the strut makes an angle of 87 deg with the upper chord, so that the stress makes an angle of 3 deg with the face of the chord. The component of the stress parallel to the chord is

$$19{,}540 \times \sin 3° = 1100 \text{ lb}$$

To resist this component, a dowel will be used to connect the strut to the chord. The dowel will be subjected to a shear stress of 1100 lb.

The area in square inches of a round dowel may be found from the formula

$$bd = \frac{V}{s}$$

where b = width, in.
d = depth, in.
A = area, sq in.
V = shear stress, lb
s = allowable unit stress, lb per sq in.

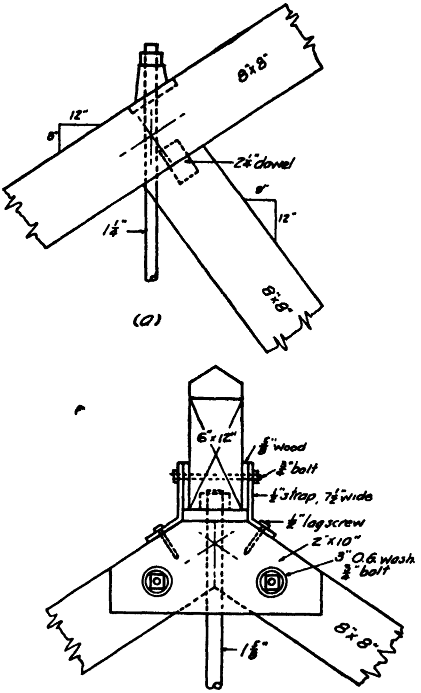

FIG. 24. Howe-truss joints.

$$A = 1100 \div 388 = 2.84 \text{ sq in.}$$

For a round dowel this is equivalent to a diameter of 2 in. The depth of embedment of the dowel is

$$\frac{1100}{\frac{2}{3} \times 2 \times 388} = 1.96, \text{ or } 2 \text{ in.}$$

The length of the dowel is 2 + 2 = 4 in. It seems customary to assume the bearing of a round piece as two thirds of its diameter.

Angle washer at joint 3. The stress in the tie rod is −12,830 lb. The component of this stress perpendicular to the chord is equal to 12,830 × cos 33°41′ = 10,680 lb.

$$\text{Stress parallel to the top chord} = 12{,}830 \times \sin 33°41' \\ = 7120 \text{ lb}$$

Net area of bevel washer = 10,680 ÷ 388 = 27.5 sq in.

This area for an elliptical hole equals 1.5 ÷ cos 33°41′ = 1.8 sq in.

The area of the hole ($1\frac{1}{4} + \frac{1}{8} = 1\frac{3}{8}$ in.) equals 1.5 sq in.

Gross area of washer = 27.5 + 1.8 = 29.3 sq in.

The washer will be $5\frac{1}{2}$ by $5\frac{1}{2}$ in.

Embedment

Area = 7120 ÷ 1800 = 3.95 sq in.

Depth = 3.95 ÷ 5.5 = 0.72, or $\frac{3}{4}$ in.

The washer is $\frac{3}{4}$ by $5\frac{1}{2}$ by $5\frac{1}{2}$ in.

Peak joint. The connection at the peak will be as shown in Fig. 24*b*. The minimum area of contact between the two upper chords should be found. These two members make an angle of 33°41′ with horizontal. Therefore, the angle of the vertical surface of contact is 90° − 33°41′ = 56°19′. Allowable unit pressure on the surface is (y = 56°19′)

$$N = 845 \text{ lb per sq in.}$$

The maximum stress at the peak is in *EO* and is +25,570 lb. The horizontal component of this is

$$25{,}570 \times \cos 33°41' = 21{,}200 \text{ lb}$$

$$\text{Net area of contact} = 21{,}200 \div 845 = 25.1 \text{ sq in.}$$

The tie rod is $1\frac{5}{8}$ in. in diameter; therefore the hole will be $1\frac{5}{8} + \frac{1}{4} = 1\frac{7}{8}$ in. This leaves 7.5 − 1.88 = 5.62 in. net width of the upper chord.

$$\text{Length of contact face} = 25.1 \div 5.62 = 4.47 \text{ in., or } 4\tfrac{1}{2} \text{ in.}$$

Washer for tie rod *NO*: With y = 33°41′,

N = 512 lb per sq in.

Net area of washer = 24,770 ÷ 512 = 48.0 sq in.

Gross area = 48.0 + 2.4 = 50.4 sq in.

Length = 50.4 ÷ 7.5 = 6.72, or $6\frac{3}{4}$ in.

After removing material from the peak to take a washer $6\frac{3}{4}$ by $7\frac{1}{2}$ in., there remains $6\frac{3}{4}$ in. of depth at the contact of the two chords. Only $4\frac{1}{2}$ in. were needed for bearing.

The thickness of washer is

$$t = \sqrt{\frac{0.75 \times 24{,}570 \times 7.5}{(7.5 - 1.88) \times 18{,}000}} = 1.16, \text{ or } 1\tfrac{1}{8} \text{ in.}$$

The washer is $1\frac{1}{8}$ by $6\frac{3}{4}$ by $7\frac{1}{2}$ in.

Joint 6. The detail of joint 6 is shown in Fig. 25*a*. The stress in the strut is +14,110 lb, and in the tie rod it is −12,830 lb; the size of the strut is 4 by 8 in. and of the tie rod is 1¼ in. The face of the short cut makes an angle of about 56°19′ with the axis of the lower chord.

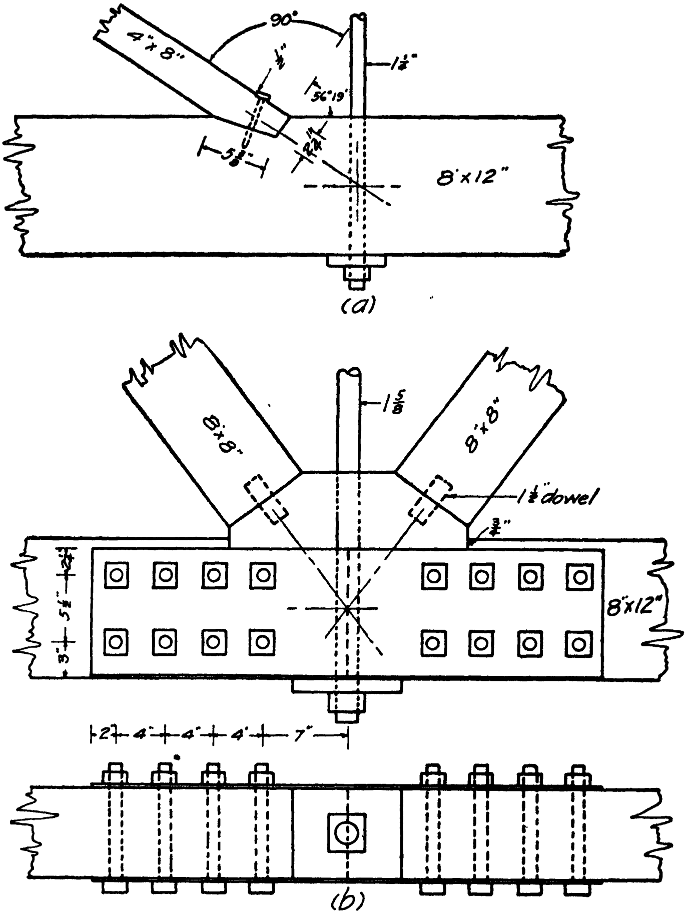

Fig. 25. Howe-truss joints.

N = 845 lb per sq in.
Area = 14,110 ÷ 845 = 16.7 sq in.
Length of face = 16.7 ÷ 7.5 = 2.23, or 2¼ in.

The long cut is 5⅜ in. as scaled from the drawing.

Washer for tie rod

Net area = 12,830 ÷ 388 = 33.1 sq in.
Gross area = 33.1 + 1.5 = 34.6 sq in.
Length of washer = 34.6 ÷ 7.5 = 4.6, or 4⅝ in.
Thickness t = 0.83, or ⅞ in.

The washer is ⅞ by 4⅝ by 7½ in.

Joint 7. Joint 7 must be designed for both the struts and the splice connection. Detail of the joint is shown in Fig. 25*b*. The stresses are as follows: the strut is +19,540 lb; the tie rod is −24,770 lb, and lower chord panel *IM* is −30,370 lb. The wind load stress in the strut is +6970 lb and the concentrated load is 6310 lb. Dimensions of the members are as follows: the strut is 8 by 8 in.; the lower chord is 8 by 12 in.; and the tie rod is 1⅝ in.

The face of the strut makes an angle of about 37 deg with the axis of the tie rod, which is the angle between the face of the block and the horizontal axis. Area needed by the face of the block supporting the strut is

N = 540 lb per sq in.
Area = 19,540 ÷ 540 = 36.2 sq in.
Area available = 7.5 × 7.5 = 56.3 sq in.

The bearing block must be set into the lower chord a sufficient depth to resist the horizontal component of the wind load and concentrated load stresses in the strut.

$$6970 + 6310 = 13{,}280 \text{ lb}$$

Horizontal component = 13,280 × cos 53° = 7960 lb
Area = 7960 ÷ 1800 = 4.42 sq in.
Depth = 4.42 ÷ 7.5 = 0.58, or ¾ in.

To resist horizontal shear from these loads, the block should be 3.28 in. long. The design provides more than this.

For the bearing washer for the tie rod,

Net area = 24,570 ÷ 388 = 63.3 sq in.
Gross area = 63.3 + 2.4 = 65.7 sq in.
Length of plate = 65.7 ÷ 7.5 = 8.75, or 9 in.
Thickness t = 1.13, or 1⅛ in.

The washer is 1⅛ by 7½ by 9 in.

Splice plate and bolts for center connection. The stress is −30,370 lb. A connection similar to Fig. 16*Aa* will be used. Trying 1-in. bolts,

$$\frac{L}{D} = 7.5 \div 1 = 7.5$$

$$1000 \times 60.6\% = 606 \text{ lb per sq in.}$$

Load per bolt = 606 × 7.5 × 1 = 4550 lb
Number of bolts = 30,370 ÷ 4550 = 6.7, or 8 bolts

Two rows of four bolts per row on each side of the joint will be used, a total of 16 bolts. If ⅞-in. bolts were used, a total of 20 would be required.

Thickness of splice plate. Each plate takes one-half of the load.

$$30{,}370 \times \tfrac{1}{2} = 15{,}185 \text{ lb}$$
$$\text{Net area} = 15{,}185 \div 18{,}000 = 0.843 \text{ sq in.}$$

Allowing for two rows of 1-in. bolts, the equivalent width of the plate is $10.5 - 2 \times 1 = 8.5$ in.

$$t = 0.843 \div 8.5 = 0.090 \text{ in.}$$

A $\tfrac{1}{4}$-in. plate will be used.

Purlin connection to upper chord. Several methods may be considered for locating the position of the purlin at the panel point. That shown in Fig. 23*b* seems to be used as much as any.

Connection of truss to wall. The total vertical component of the reaction on the 8-in. concrete wall is 29,845 lb. The horizontal component of the wind load is 4870 lb. Building walls are customarily made of 1: 3.5: 5 (medium mixture) concrete. The working strength of this concrete may be taken at 400 lb per sq in. The bearing pressure on the lower chord will be the controlling factor as the wood will stand a safe working load of only 388 lb per sq in. perpendicular to the grain.

Referring to page 36 for bearing-plate calculations,

$$t = \sqrt{\frac{3wa(a - x)}{4f}} \quad \text{in.}$$

where $f = 4000$ lb per sq in. for cast iron.
$b = 7.5 - 2 \times \tfrac{1}{2} = 6.5$ in.
$A = P/w = 29{,}845 \div 388 = 76.9$ sq in.
$a = 76.9 \div 6.5 = 11.82$, or $11\tfrac{3}{4}$ in.
$x = 8 - 2 \times \tfrac{1}{2} = 7$ in.

$$t = \sqrt{\frac{3 \times 388 \times 11.75(11.75 - 7)}{4 \times 4000}} = 2.02, \text{ or } 2 \text{ in.}$$

To distribute the load on the concrete wall,

Area $= 29{,}685 \div 400 = 74.2$ sq in.
Width of plate $= 8 - 2 \times \tfrac{1}{2} = 7$ in.
Length of plate $= 74.2 \div 7 = 10.6$, or $10\tfrac{1}{2}$ in.

This is less than the $11\tfrac{3}{4}$ in. for timber bearing, so that the thickness of **2 in.** is satisfactory. The bearing plate will be 2 by $10\tfrac{1}{2}$ by $11\tfrac{3}{4}$ in.

Because of the horizontal component of the wind load a lug must be cast on the timber side of the plate. Its dimensions are as follows:

$$\text{Height (indentation in timber)} = 4870 \div (1800 \times 7.5) = 0.36 = \tfrac{1}{2} \text{ in.}$$

Thickness of lug

$$\text{Bending moment on face of lug} = \tfrac{1}{2}Pl = \tfrac{1}{2} \times 4870 \times \tfrac{1}{2} = 1217 \text{ in.-lb}$$

$$t = \sqrt{\frac{6M}{sb}} = \sqrt{\frac{6 \times 1217}{4000 \times 7.5}} = 0.49, \text{ or } \tfrac{1}{2} \text{ in.}$$

To ensure good contact with the timber, the lug will be 1 by 1 in. Figure 26 shows a general drawing of the truss.

JOINTS AND CONNECTIONS

Wood. In designing joints and connections the detailing should not be done blindly.[1] The center lines of all members coming together at the joint should intersect in a common point (be concurrent). If they do not, eccentricity will be developed and stresses set up that are difficult to determine or correct. An advisable way to start is to make a large-scale drawing of the joint. Distances and angles can be scaled from this drawing, thus enabling the designer to have something with which to start the detail. The bearing area of the various members to withstand the load imposed on them must be carefully calculated. The area of washers must be determined in order that injury to the

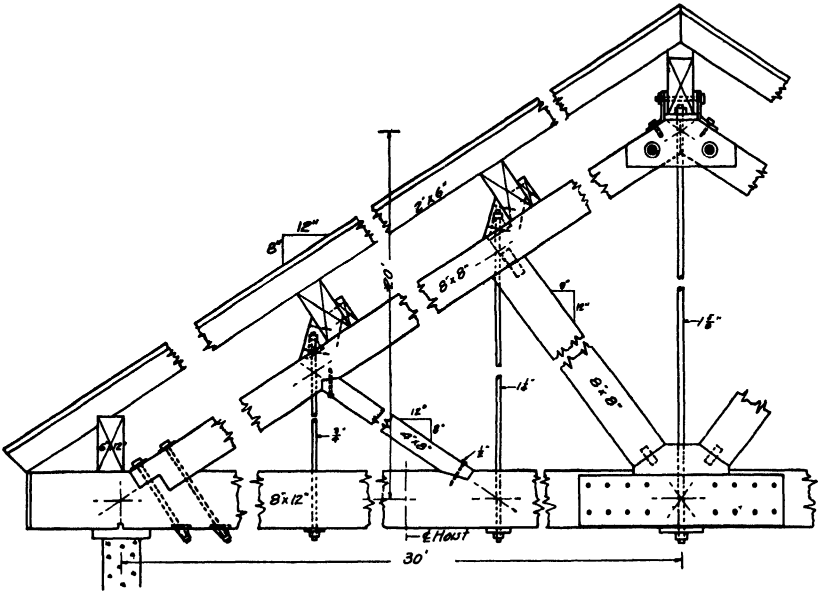

FIG. 26. Howe truss.

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[1] McCULLOUGH, E., "Principal Structural Design," 3d ed., Scientific Book Corporation, New York.

HOOL and KINNE, *op. cit.*

HOWE, M. A., "Design of Simple Roof-trusses in Wood and Steel," John Wiley & Sons, Inc., New York.

JACOBY and DAVIS, *op. cit.*

YOUNG, C. R., "Elementary Structural Problems in Steel and Timber," John Wiley & Sons, Inc., New York.

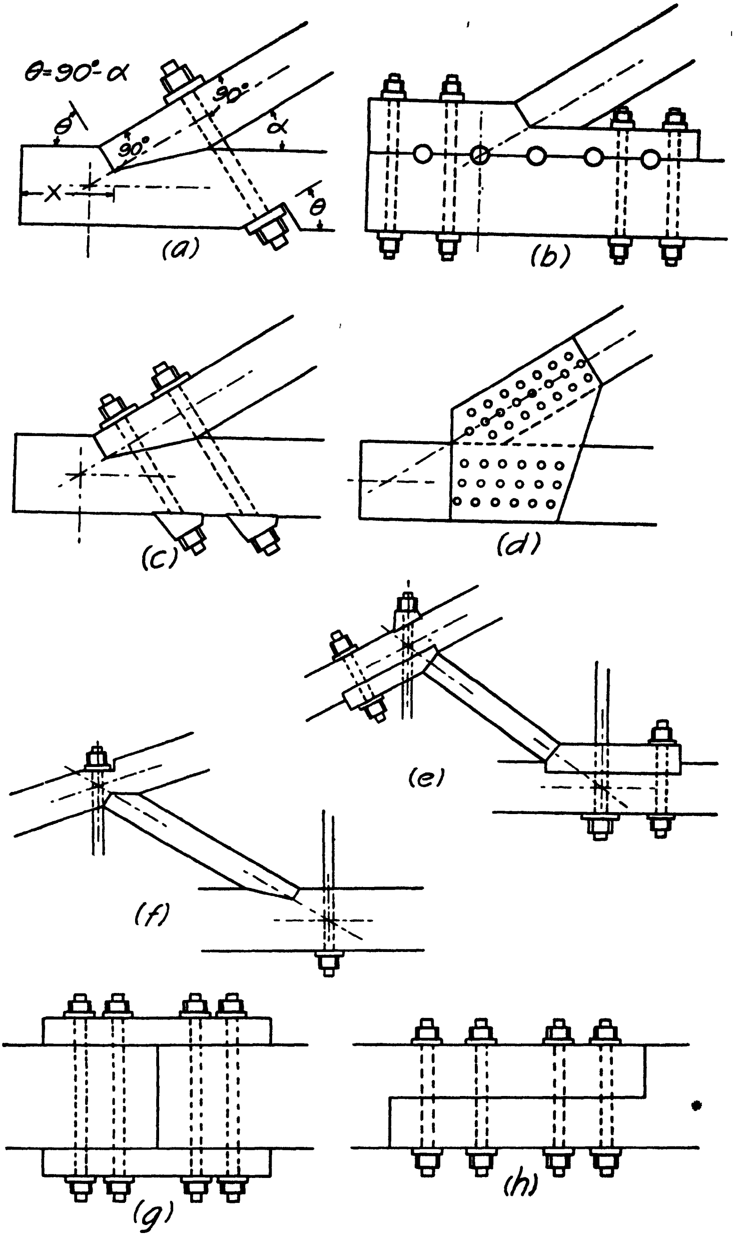


Fig. 27. Joints and connections for wooden structures.

wood fibers be prevented. The size and number of bolts must be figured and additional area added to the section to replace that removed by the boltholes. The timber should be accurately framed so that the members will all have an even contact and not be separated

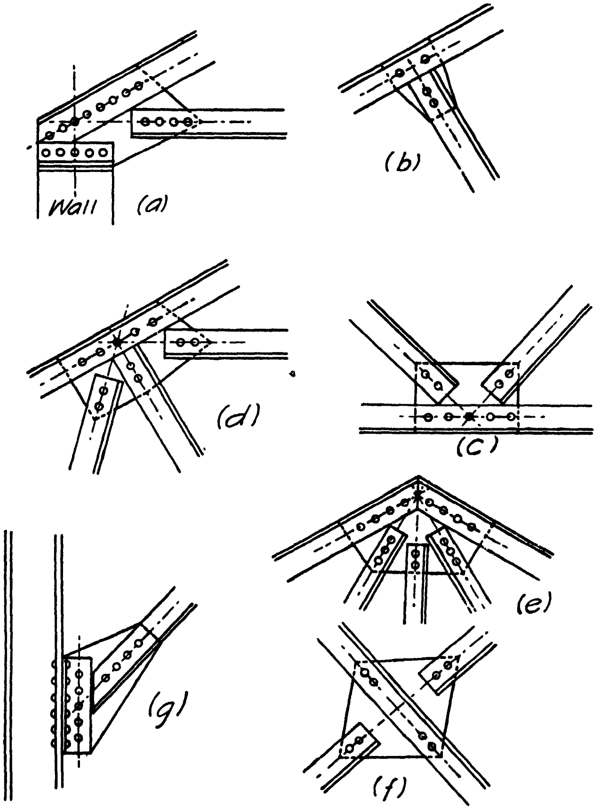


FIG. 28. Joints and connections for steel structures.

by high spots. Joints, where separation may occur through shrinkage of the wood, must be avoided.

In Fig. 27 are shown various types of timber joints. Sketches *a*, *b*, *c*, and *d* illustrate approved methods of connecting the upper and lower chords at their junction. The connection shown in *d* is made with iron plates fastened to the timbers with lag screws. This is considered the most efficient joint but is usually the most costly. The lag screws should be so spaced that their ends do not come opposite each other. The number and size of the lag screws must be carefully calculated.

Figures 27*e* and *f* show methods of making intermediate joints. That at *e* is considered the better connection, although *f* is quite commonly used. Attention is called to the intersection of all center lines. Figures 27*g* and *h* are connections for compression members that must be spliced. The one shown by *g* should be used wherever possible as it has the least number of contact points.

Steel. The same comments made under wooden trusses regarding the intersection of center lines and calculation of component parts of the joint apply with equal force to steel. The number of rivets required for the connection of each member to the gusset plate must be determined. If such calculations indicate that an extra fraction of a rivet is needed, an additional rivet is added, *i.e.*, if 4.2 rivets are required, 5 should be used. In order to ensure rigidity, not less than two rivets should be used for any member, even though a fraction of one rivet was indicated.

Figures 28*a* to *g* show various types of connections for steel. Figure 28*a* is commonly used for connections of the upper and lower chords when the truss is supported on masonry walls. Two angles are fastened to the bottom of the gusset plate and one or two plates are riveted to the horizontal legs to provide bearing surface on the wall. The dimensions of these plates and angles are determined from the reactions of the truss. Figures 28*b* to *f* are connections for the various intermediate joints. Figure 28*f* illustrates a method quite often necessary for connecting bracing in headframe construction. Figure 28*g* shows an approved method for connecting a knee brace to a column.

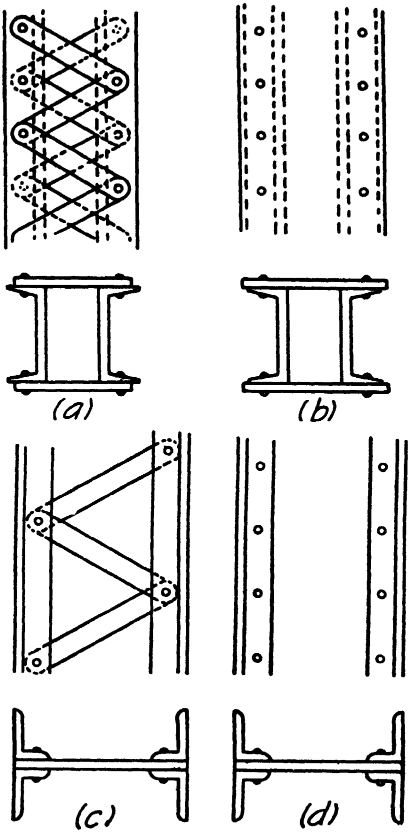


Fig. 29. Commonly used steel columns.

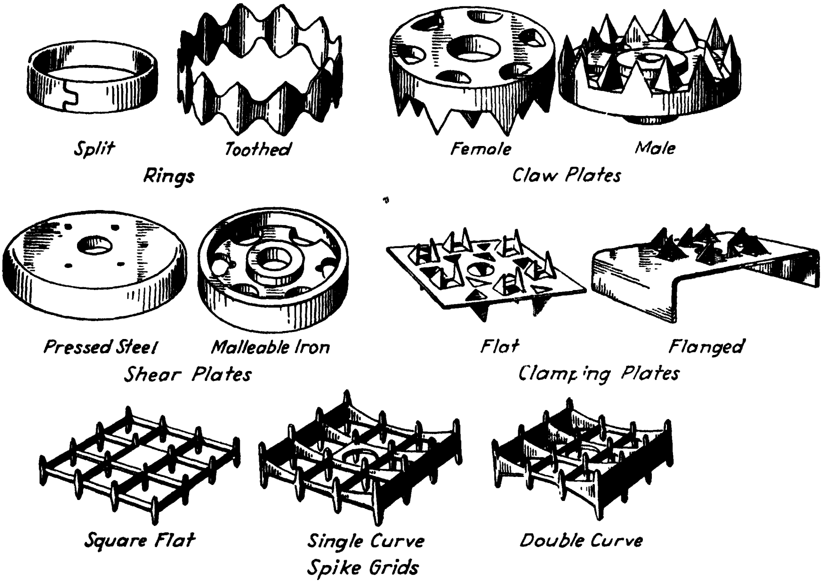
Four commonly used built-up columns are shown in Fig. 29. Figure 29*a* is made of channels connected back to back with lattice work.

The backs of the channels are quite often turned out. Figures 29*c* and *d* show angles laced together with lattice work and plates.

Rolled shapes often used for columns without further alteration are H beams, I beams, wide flange beams, and single angles at corners of the structure.

USE OF TIMBER CONNECTORS

The design of timber structures is not complete without calling attention to the use of timber connectors. The discussion presented applies to the Teco[1] type of connector, which is the one in common use in the United States. Extensive tests have been made by the Forest Products Laboratory; so that there is little doubt of the applicability of connectors. The outstanding advantage, as compared with the conventional type of joint, is that the connector joint very closely permits development of the full strength of the timber. We have found, in the case of the usual method of making connections, that the joining of the timbers at a joint often requires a substantial increase in the size of the lumber in order that sufficient bearing area or net section be provided. This very seldom happens when using timber connectors.



Split Toothed
Rings

Female Male
Claw Plates

Pressed Steel Malleable Iron
Shear Plates

Flat Flanged
Clamping Plates

Square Flat Single Curve Double Curve
Spike Grids

FIG. 30. Teco timber connectors.

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[1] Timber Engineering Company, Washington, D.C. Complete design and erection data may be obtained on application.

Usually there may be expected a 25 to 50 per cent saving of material if connector joints are used. Another distinct advantage is when massive solid timbers are replaced by planks.

From an economic standpoint the use of timber connectors should be considered when designing trusses, ore bins, headframes, platforms, and practically all structures involving connections and joints.

Figure 30 shows the various types of Teco connectors.

Design of a Timber-connected Truss. As previously pointed out, the Fink truss is a very suitable type when the use of timber connectors are contemplated. Using the same data as for the Howe type of truss, the design shown in Fig. 31 was made. A ¼ pitch was used. Figures 32 to 34 show the determination of the various stresses. (This same truss is later used to illustrate the design of a steel truss.)

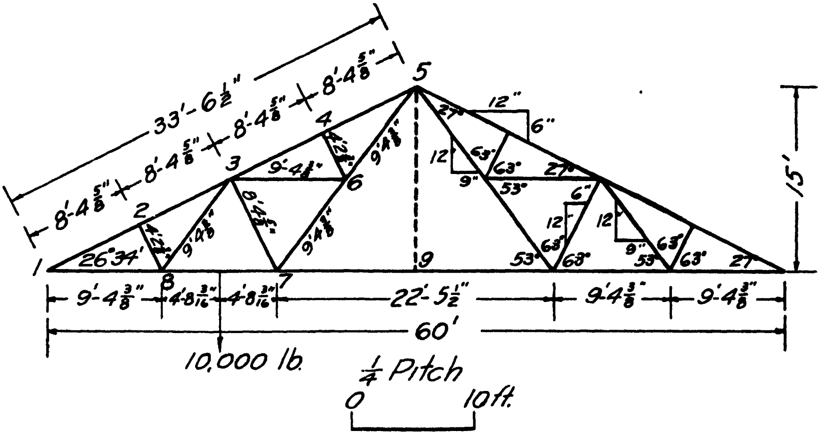

FIG. 31. Layout for timber-connector truss.

Because of the occasional change in connector design data, the tables and charts for such design are not reproduced. This information may be obtained from the Timber Engineering Company. The information thus obtained should be carefully studied before starting a design. Figure 35 shows the completed Fink truss.[1]

Design of a Steel Truss. To illustrate the selection of members and the design of a steel truss, the data obtained for the timber-connector Fink truss will be used.

Because of expansion and contraction, the truss will be free to move at its right support. It then becomes necessary to redetermine the wind-load stress. Investigation must be made of the effect with the wind on the left and right sides. As the truss is free to move on the right support, the reaction is theoretically, vertical; while at the left support the direction must be determined.

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[1] Assistance of the Timber Engineering Company is gratefully acknowledged.

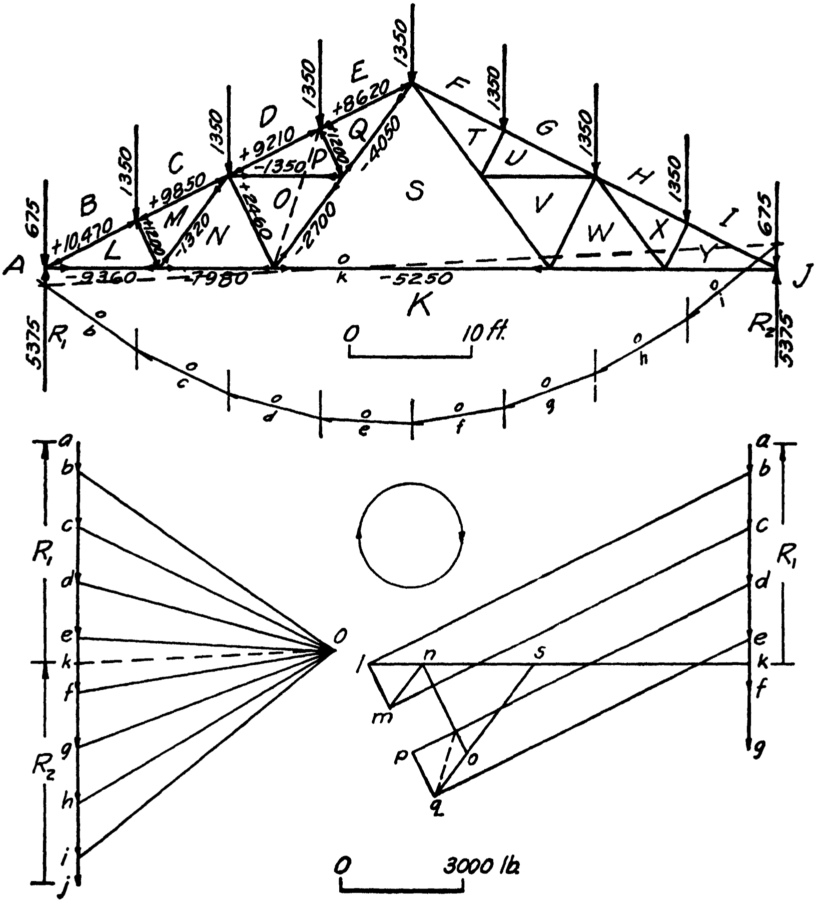


Fig. 32. Dead load for timber-connector truss.

The reactions may be found by moments or graphically. Both methods will be illustrated. Because the direction of the right reaction is known (vertical), the moments are taken about the left end (see Fig. 36).

Moments about A, Wind Right

Total upper chord moment arm = 60 × cos 26°34′ = 53.65 ft
Length of upper chord = 30 ÷ cos 26°34′ = 33.54 ft
Each panel length = 33.54 ÷ 4 = 8.38 ft

With the wind load acting from the right and moments about A, the first force, 1410 lb, has a moment arm of $53.65 - 33.54 = 20.11$ ft. To this is added each successive panel length.

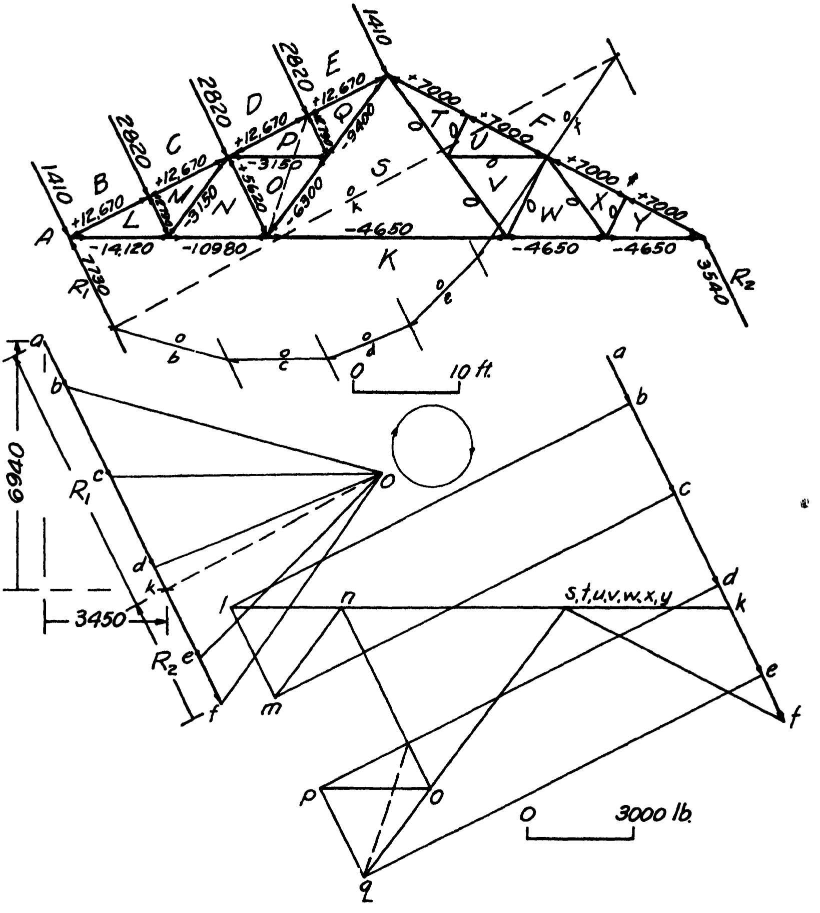


FIG. 33. Wind load for timber-connector truss.

$$60R_2' = 1410 \times 20.11 + 2820 \times 28.49 + 2820 \times 36.87 + 2820 \times 45.25 + 1410 \times 53.65$$

$$= 416{,}000$$

$$R_2' = 416{,}000 \div 60 = 6930 \text{ lb (graphically, 6920 lb)}$$

Reaction R_1' may be found as follows:

$$\text{Vertical component} = (1410 + 2820 + 2820 + 2820 + 1410) \times \cos 25°34 - 6930$$

$$= 11{,}280 \times \cos 26°34' - 6930 = 3170 \text{ lb}$$

Horizontal component $= 11{,}280 \times \sin 26°34' = 5040$ lb

$R_1' = \sqrt{5040^2 + 3170^2} = 5960$ lb (graphically, 5960 lb)

Wind from Left.—R_1 and R_2 are found graphically. A check on R_2 is as follows:

$$60R_2 = 2820 \times 8.38 + 2820 \times 16.76 + 2820 \times 25.14 + 1410 \times 33.54$$
$$= 189{,}130$$
$$R_2 = 189{,}130 \div 60 = 3150 \text{ lb (graphically, 3130 lb)}$$
$$R_1 = 8740 \text{ lb (graphically)}$$

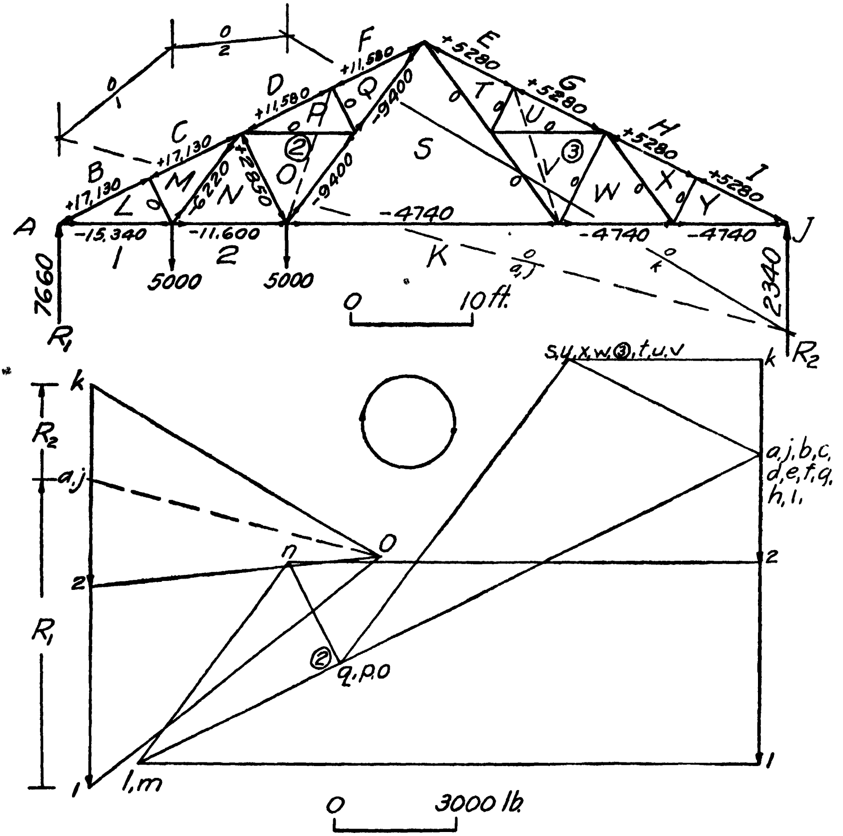
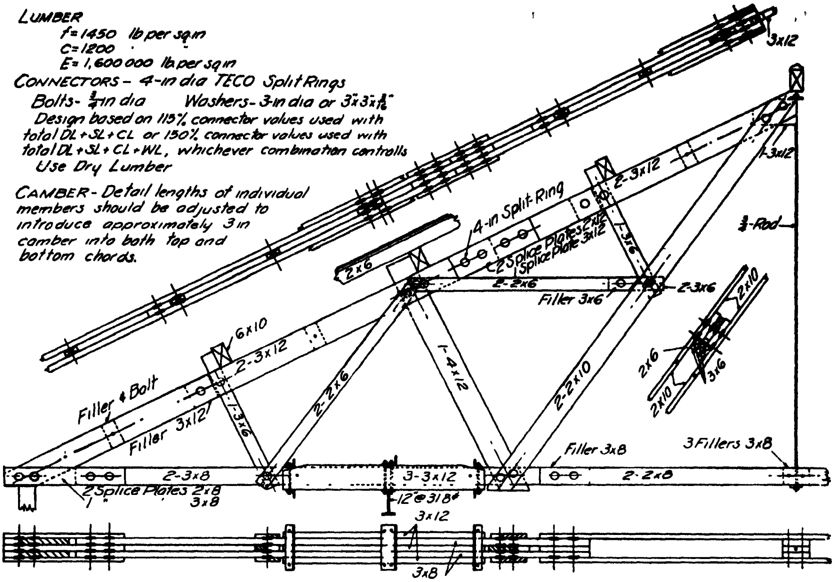


FIG. 34. Concentrated load for timber-connector truss.

Figure 36 shows the determination of the stresses. Table 14 gives the values of the stresses for the left half of the truss; the maximum from the two solutions is recorded.

The next step is to decide upon the structural shape to be used. Trusses are usually made of angles back to back. It is customary to use angles with unequal legs, the longer legs being in the vertical plane. In choosing the distance back to back, the ideal arrangement would be one where the rivet strength in the angles


LUMBER
f=1450 lb per sq in
C=1200 " "
E=1,600 000 lb per sq in
CONNECTORS – 4-in dia TECO Split Rings
Bolts – ¾ in dia Washers – 3-in dia or 3″x3″x$\frac{3}{16}$″
Design based on 115% connector values used with total DL+SL+CL or 150% connector values used with total DL+SL+CL+WL, whichever combination controlls
Use Dry Lumber

CAMBER – Detail lengths of individual members should be adjusted to introduce approximately 3 in camber into both top and bottom chords.

FIG. 35. Assembled timber-connector truss.

TABLE 14. STRESSES FOR STEEL TRUSS

Member	Wind	Dead load, snow load, concentrated load	Total stress	Length of member
BL	+12,880	+45,150	+58,030	8′4⅝″
CM	+12,880	+43,480	+56,360	8′4⅝″
DP	+12,880	+36,240	+49,120	8′4⅝″
EQ	+12,880	+34,650	+47,530	8′4⅝″
KL	−16,000	−40,400	−56,400	9′4⅜″
LM	+ 2,880	+ 3,210	+ 6,090	4′2⅜″
MN	− 3,200	− 9,770	−12,970	9′4⅜″
KN	−12,840	−32,960	−45,800	9′4⅜″
NO	+ 5,680	+ 9,430	+15,110	8′4⅜″
OP	− 3,200	− 3,610	− 6,810	9′4⅜″
PQ	+ 2,880	+ 3,210	+ 6,090	4′2⅜″
OS	− 6,330	−16,620	−22,950	9′4⅜″
QS	− 9,520	−20,230	+29,750	9′4⅜″
KS	− 6,480	−18,790	−25,270	22′5½″
KY	−10,810	−29,800	−40,610	9′4⅜″
Vertical reaction	6,930	22,055	28,985	20′
Horizontal component *R*	5,040	....	5,040	

(double shear and bearing) would closely approximate the enclosed bearing on the gusset plate. It is not usual practice to carry out the investigation to such an extent. Experience indicates that for light trusses ⅝-in. rivets and 5⁄16-in. gusset plates are suitable; for moderatly loaded trusses ¾-in. rivets and ⅜-in. gusset plates are suitable; and for heavy trusses ¾ to ⅞-in. rivets with ½ to ⅝-in. gusset plates are suitable. For the present example, ¾-in. rivets and ⅜-in. gusset plates will be used. This means that the angles, or other shape, if one becomes necessary, will be spaced ⅜ in. apart.

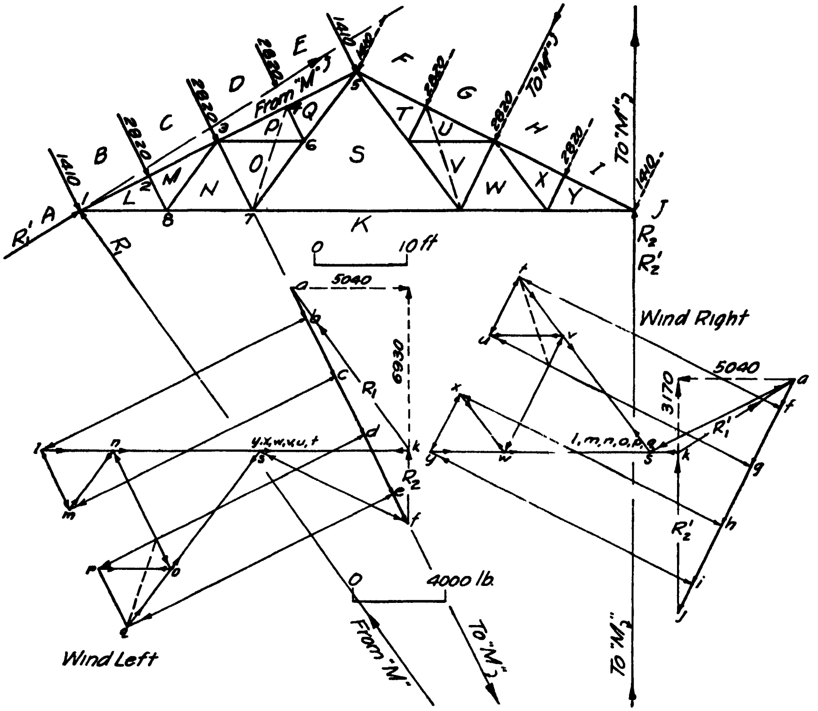


FIG. 36. Wind load for steel truss.

The other dimensions of the gusset plate should be just sufficient to meet the requirements of rivet pitch and end and edge distances. Unusual or difficult shapes for the gusset plate should be avoided.

For rivet gage, spacing, pitch, net section, clearance for driving and forming heads, and other design and fabricating data the "Steel Construction Handbook" should be consulted constantly.[1] In fact, steel design is out of the question without consulting this or a similar source. The data used in the calculations to follow were taken from the "Steel Construction Handbook."

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[1] "Steel Construction Handbook," American Institute of Steel Construction, New York.

Design of Top Chord. Using

$$\frac{P}{A} = 18{,}000 - 60\frac{l}{r}$$

where l/r is not to exceed 120 and P/A is less than 15,000 lb per sq in., we can choose a pair of angles $\frac{3}{8}$ in. back to back for the upper chord. Member *BL* has a stress of +58,030 lb and is 8 ft 4$\frac{5}{8}$ in. long.; this equals 101 in. For a trial, use $l/r = 120$; $r = {}^{101}\!/_{120} = 0.84$ in.

This is the minimum radius of gyration. Any value larger than 0.84 may be used, but the weight per foot for economic reasons should be kept at a minimum. Consulting the "Steel Construction Handbook" under two unequal angles, long legs back to back, with a $\frac{3}{8}$-in. spacing, we search for a pair of angles that appear to suit the requirements. The process is one of trial and error. Attention is called to the presence of *two* columns giving r values; one about the x axis, the other about the y axis. The smaller of these two values is used. Only the common thicknesses of angles appear in the table. For the complete list, the table on angles—unequal legs must be used. Data from this latter table are used in the table of two angles by interpolation. The radius of gyration about the x axis does not change regardless of the number of angles on the x axis; therefore, interpolation is necessary for only the y axis.

For mine plant structures it is the practice to use no material less than $\frac{5}{16}$ in. thick.

Getting back to the problem, two-4- by 3- by $\frac{5}{16}$-in. angles will be tried. About the x axis, $r = 1.27$ in.; about the y axis, r by interpolation is

$$\left(\tfrac{3}{8} - \tfrac{1}{4}\right):(1.31 - 1.29)::\left(\tfrac{5}{16} - \tfrac{1}{4}\right):x.$$
$$x = 0.01$$

and

$$r = 1.29 + 0.01 = 1.30$$

The above solution was made to illustrate the procedure; immediate inspection showed that r about x-x, 1.27 in., would be less than about y-y, which would fall between 1.29 and 1.31. The two angles have an area of $2 \times 2.09 = 4.18$ sq in.

$$\frac{P}{4.18} = 18{,}000 - 60 \times \frac{101}{1.27} = 13{,}230 \text{ lb per sq in.}$$

which is satisfactory, it being less than the 15,000 lb per sq in. limit imposed by the formula.

$$P = 4.18 \times 13{,}230 = 55{,}400 \text{ lb}$$

The selection is too small.

The next trial would be two- 4- by 3$\frac{1}{2}$- by $\frac{5}{16}$-in. angles. They weigh 15.4 lb per ft, which is less than any 4 by 3 choice. The least radius of gyration is still about the x axis and is 1.26 in. The area is 4.50 sq in. Substituting in the formula,

$$P = 59{,}400 \text{ lb}$$

This is a little excessive, but it is the closest choice that can be made.

No deduction is made for rivets in compression members so that the net area remains 4.50 sq in.

Lower Chord. The lower chord will be made of angles which are back to back with the long legs vertical, with the exception of panel KN which supports the concentrated load. For KN, two channels that are $\frac{3}{8}$ in. apart back to back will be used. Because the area for the rivet holes must be deducted from the tension members, we first determine the number of rows of rivets. The stress in KL is −56,400 lb. Trying $\frac{5}{16}$-in. thick angles, the various values for a $\frac{3}{4}$-in. rivet are (the hole is $\frac{1}{8}$ in. larger than the rivet and the rivets are power-driven)

$$\begin{aligned}
\text{Rivets in double shear} &= 2 \times 13{,}500 \times \frac{\pi}{4} \times \left(\frac{3}{4}\right)^2 \\
&= 11{,}930 \text{ lb} \\
\text{Bearing on } \tfrac{3}{8}\text{-in. gusset plate} &= \tfrac{3}{4} \times \tfrac{3}{8} \times 30{,}000 \\
&= 8440 \text{ lb} \\
\text{Bearing on two } \tfrac{5}{16}\text{-in. angles} &= 2 \times \tfrac{3}{4} \times \tfrac{5}{16} \times 24{,}000 \\
&= 11{,}250 \text{ lb}
\end{aligned}$$

The second condition is the weakest; therefore

$$\text{Number of rivets} = 56{,}400 \div 8{,}440 = 6.7, \text{ or } 7$$

For a single row of rivets, the minimum pitch (spacing center to center) is commonly taken at three times the diameter of the rivet. For seven rivets the gusset plate would not prove excessively long.

At the peak of the upper chord the stress is +47,530 lb,

$$\text{Number of rivets} = 47{,}530 \div 8440 = 5.6, \text{ or } 6$$

As the design will assume that the horizontal, or outstanding, legs of the angles are joined with a $\frac{3}{8}$-in. splice plate, area will have to be deducted for an additional two rivets.

Area of 4 holes $= 4 \times (\frac{3}{4} + \frac{1}{8}) \times \frac{5}{16} = 1.10$ sq in.
Net area $= 56{,}400 \div 18{,}000 = 3.13$ sq in.
Gross area $= 3.13 + 1.10 = 4.23$ sq in.

Two- 4 by 3 by $\frac{5}{16}$-in. angles have $2 \times 2.09 = 4.18$ sq in. and weigh 14.4 lb per ft. They will be used at KL.

For panel KN, the stress is −45,800 lb and the length is 9 ft $4\frac{3}{8}$ in. The panel may be considered as acting like a simple beam with the 10,000-lb load concentrated at the center.

$$\begin{aligned}
M = \frac{1}{4}Pl \times 12 &= \frac{1}{4} \times 10{,}000 \times 9.365 \times 12 \\
&= 281{,}000 \text{ in.-lb} \\
&= \frac{sI}{c} = 18{,}000 \times \frac{I}{c} \\
281{,}000 &= 18{,}000 \times \frac{I}{c} \\
\frac{I}{c} = (281{,}000 \div 18{,}000) \times \frac{1}{2} &= 7.8 \text{ in.}^3 \text{ for one channel}
\end{aligned}$$

From the section on channels—American Standard, $I/c = S$ (section modulus) about x axis—the nearest choice is $S = 7.7$ in.^{3} for one 7-in. channel at 14.75 lb.

This is a little too small; and also no allowance has been made for the weight of the channels. The next size, 7 in. at 17.25 lb, $S = 8.6$ in.^{3}, will be tentatively selected.

Member *KN* is subjected to both tension and bending. The material selected should be checked for the simultaneous action of both stresses.

The bending moment resulting from the weight of the channels is

$$\text{Weight} = 2 \times 17.25 \times 9.365 = 324 \text{ lb}$$
$$M = \tfrac{1}{8} \times wl \times 12 = \tfrac{1}{8} \times 324 \times 9.365 \times 12 = 4560 \text{ in.-lb}$$

Total moment on the channels = 281,000 + 4560 = 285,560 in.-lb

This member is subjected to combined bending and direct stress. The size selected should be checked for this loading.

Total stress in *KN* is −45,800 lb. Part of this, −11,600 lb, must be deducted as it results from the concentrated load.

$$\text{Net load} = -45{,}800 - (-11{,}600) = -34{,}200 \text{ lb direct stress}$$

From $M = sI/c$, $s = Mc/I$, pound per square inch unit stress resulting from the bending stress; and

$$\text{Unit stress, lb per sq in. for direct loading} = \frac{\text{direct stress}}{\text{area of member}}$$

Substituting in the formulas for the two 7-in. channels at 17.25 lb

$$\begin{aligned} I &= 2 \times 30.1 = 60.2 \text{ in.}^4 \\ \text{Area} &= 2 \times 5.05 = 10.1 \text{ sq in.} \\ c &= 7 \div 2 = 3.5 \text{ in.} \\ s &= \frac{285{,}560 \times 3.5}{60.2} = 16{,}600 \text{ lb per sq in.} \\ \text{Direct load} &= \frac{-34{,}200}{10.1} = -3390 \text{ lb per sq in.} \end{aligned}$$

On the upper edge (compression) of the channels, the unit compressive stress is 16,600 − 3390 = 13,210 lb per sq in. On the lower edge, (tension) the unit tensile stress is −16,600 + (−3390) = −19,990 lb per sq in.

The allowable unit stress in bending is 18,000 lb per sq in. with a 5 per cent leeway. With the channel selected it is plus 10 per cent over the allowable limit; a larger selection must be made. The next best choice is two- 8-in. channels at 18.75 lb. Using these, the direct load is −3220 lb per sq in., which gives a total tension of −16,270 lb per sq in. These channels are satisfactory for member *KN*.

Joint 7. The design of this joint (see Fig. 36) will include provision for attaching the lateral and diagonal bracing. The joint is made up of the following members: *KN*, which has been selected; *NO*, which is +15,110 lb; *OS*, which is −22,950 lb.; and *SK*, which is −25,270 lb.

Member SK. A tie plate will be attached to the outstanding legs of the member and to the bottom flanges of the channels. To this plate will also be attached the lateral and diagonal bracing. A ⅜-in. plate is used, and the rivets are in single shear and bearing.

Single shear for a ¾-in. rivet = 5965 lb

Bearing in single shear on a ⅜-in. plate = ¾ × ⅜ × 24,000 = 6750 lb

The safe strength is in single shear.

Net area for $SK = 25{,}270 \div 18{,}000 = 1.4$ sq in.
Four lines of rivet holes $= 4 \times 0.273 = 1.092$ sq in.
Gross area $= 1.4 + 1.09 = 2.49$ sq in.

Two $2\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{5}{16}$-in. angles have an area of $2 \times 1.47 = 2.94$ sq in. This is the smallest combination because of the edge limitation imposed by the use of $\frac{3}{4}$-in. rivets.

We may assume that half the stress in SK is transmitted by the outstanding legs.

Stress carried by each set of rivets $= \frac{1}{2} \times 25{,}270 = 14{,}635$ lb
Rivets in upstanding legs $= 14{,}635 \div 8440 = 1.7$, or 2
Rivets in tie plate and outstanding legs $= 14{,}635 \div 5965 = 2.5$, or 4 (two rivets in each leg)

Member KN. This member is composed of two- 8-in. channels at 18.75 lb. The channel flange is 2.53 in. wide. This is just sufficient to take a $\frac{3}{4}$-in. rivet. Because of the load which KN may have to support, rivets will be provided in the channels to take the entire stress.

$$\text{Number of rivets} = 45{,}800 \div 8440 = 5.4, \text{ or } 6$$

The channel flanges will have two rivets each, as previously found for SK. Two rows of three rivets each will be used in the channel webs.

The concentrated load on the channels is 10,000 lb; therefore the reaction at the joint of 5000 lb equals the vertical shear Less than one rivet is needed to meet this; the six rivets already found are satisfactory.

Member NO. The length of NO is 8 ft $4\frac{5}{8}$ in , which equals 101 in.; the stress is +15,110 lb.

$$r = {}^{101}\!/_{120} = 0.84 \text{ in.}$$

Two 3-by 2- by $\frac{5}{16}$-in. angles will be checked for their suitability. These angles are not listed in the table for two angles back to back. A moment's inspection of the data on each side of this size shows that the least r will be about the xx axis; therefore the table for single angles may be used. Two 3- by 2- by $\frac{5}{16}$-in. angles have an area of $2 \times 1.47 = 2.94$ sq in., and $r = 0.95$ in.

$$\frac{P}{2.94} = 18{,}000 - 60 \times \frac{101}{0.95} = 11{,}620 \text{ lb per sq in.}$$
$$P = 2.94 \times 11{,}620 = 34{,}200 \text{ lb}$$

This is more than twice the stress. According to the specifications, secondary members may have l/r up to 200. Trying two- $2\frac{1}{2}$- by $1\frac{1}{2}$- by $\frac{5}{16}$-in. angles, area $= 2 \times 1.15 = 2.30$ sq in., and $r = 0.79$, the $l/r = 128$, and $P = 23{,}800$ lb.

$$\text{Number of rivets} = 15{,}110 \div 8440 = 1.8, \text{ or } 2$$

Member OS and QS. The member will be continuous from the peak to the lower chord. The maximum stress is $QS = -29{,}750$ lb.

Net area $= 29{,}750 \div 18{,}000 = 1.65$ sq in.
Gross area $= 1.65 + 2 \times 0.273 = 2.20$ sq in.

Two 2½- by 1½- by $\frac{5}{16}$-in. angles provide 2.30 sq in.

Rivets at the peak = 29,750 ÷ 8440 = 3.5, or 4
Rivets at lower chord (*OS*) = 22,950 ÷ 8440 = 2.7, or 3

Figure 37 shows the bracing detail.

Joint 8. Members making up this joint are *KL* = −56,400 lb.; *LM* = +6090 lb; 4 ft 2⅜ in. = 50 in.; *MN* = −12,970 lb; *NK* = −45,800 lb.

Member KL. As previously found, *KL* consists of two 4- by 3- by $\frac{5}{16}$-in. angles. One-half the stress is transmitted by the ⅜-in. splice plate.

Stress = ½ × 56,400 = 28,200 lb

Rivets in splice plate (outstanding leg) = 28,200 ÷ 5965
= 4.7, or 6

There are three rivets in each leg.

Rivets in upstanding legs = 28,200 ÷ 8440 = 3.3, or 4

Member LM. The stress is +6090 lb; the length is 50 in. Two 2½- by 1½- by $\frac{5}{16}$-in. angles are the smallest available.

Rivets = 6090 = 8440 = 0.7, or 2

Two rivets are used because less than two are not considered good practice.

Member MN. Gross area required is 1.27 sq in. Two 2½- by 1½- by $\frac{5}{16}$-in. angles provide 2.30 sq in.

Rivets = 12,920 ÷ 8440 = 1.5, or 2

Member NK. As found at *joint* 7, six rivets are required for the channel webs. The flanges will take three each, as found for member *KL.*

Joint 6. The members are *OP* = −6810 lb, *PQ* = +6090 lb, 4 ft 2⅜ in. = 50 in.; *QS* = −29,750 lb; and *SO* = −22,950 lb.

Member OP. Two 2½- by 1½- by $\frac{5}{16}$-in. angles with two rivets are needed.

Member PQ. This member is the same as *LM.*

Member QS and SO. See *joint* 7.

Peak Joint. The maximum stress is *EQ* = +47,530 lb. As previously decided, the upper chord will be made of two 4- by 3- by $\frac{5}{16}$-in. angles.

Rivets = 47,530 ÷ 8440 = 5.6, or 6

The rivets for *QS* have already been found.

The sag member, connecting the peak with the lower chord, will be two 2½- by 1½- by $\frac{5}{16}$-in. angles with two rivets in each end.

Member KY. This member has a stress of −40,610 lb. It will be made so as to connect with *KW.*

Net area = 40,610 ÷ 18,000 = 2.25 sq in.

A deduction of four rivets holes, one in each leg of the two angles, must be allowed for.

Gross area = 2.25 + 4 × 0.273 = 3.34 sq in.

Two $3\frac{1}{2}$- by $2\frac{1}{2}$- by $\frac{5}{16}$-in. angles provide 3.56 sq in. From joint 7 to the heel the lower chord will be of this size. At joint *SVWK*, a splice plate will be connected to the outstanding legs for attaching lateral and diagonal bracing.

Rivets at the heel connection are equal to 40,610 ÷ 8440 = 4.8, or 5. Seven will be used to correspond with the left heel joint.

Other Connections. Figure 37 shows the rivets in the ends of members not calculated. As but half of the truss is shown, mention should be made of the rivets at the joints *KSVWK* and *KWXYK*. The member *KW* is made of angles, not channels which were used for the left side of the truss. Members making up these joints will have the same number of rivets as previously determined for the left-hand side.

Connection to Wall. The joint shown in Fig. 28*a* will be used. It consists of the gusset plate, two sets of chord angles, the sole-plate angles, the sole plate, the bearing plate, and the anchor bolts. To allow for expansion, the anchor boltholes in the sole plate on the free end (right end) of the truss will be slotted. For the expansion of a steel truss this size, an allowance of $\frac{1}{8}$ in. per 10 ft of length is satisfactory. On the left end but one plate, the bearing plate, is necessary. Bearing on each end of the truss will be based on the maximum reactions. The concrete has a bearing value of 400 lb per sq in.

$$\text{Bearing area} = 28{,}985 \div 400 = 72.3 \text{ sq in.}$$

The wall is 8 in. thick. The width of the bearing plate, from the previous discussion, is

$$\begin{aligned}\text{Width} &= 8 - 2 \times \tfrac{1}{2} = 7 \text{ in.}\\ \text{Length} &= 72.3 \div 7 = 10.3 \text{ in.}\end{aligned}$$

If 4- by 4- by $\frac{1}{2}$-in. angles are used for the shoe angles, the width of the framework is $4 + 4 + \frac{3}{8} = 8\frac{3}{8}$ in. To provide space for the anchor bolts, a plate 13 in. long will be used. The plates are 8 by 13 in., but the effective size is 7 by 13 in. The thickness of one plate is found from

$$t = \sqrt{\frac{3wa(a - x)}{4f}} \quad \text{in.}$$

$$\text{Effective area} = 7 \times 13 = 91 \text{ sq in.}$$

$$w = 28{,}985 \div 91 = 318 \text{ lb per sq in.}$$

$$t = \sqrt{\frac{3 \times 318 \times 13 \times (13 - 8\frac{3}{8})}{4 \times 18{,}000}} = 0.89, \text{ or } 1 \text{ in.}$$

Both plates are 1 in. thick.

Anchor Bolts. The horizontal component of the wind load is 5040 lb. If two bolts are used, the stress in each bolt is $\frac{1}{2} \times 5040 = 2520$ lb.

$$\text{Area for shear} = 2520 \div 10{,}000 = 0.252 \text{ sq in.}$$

From Table 7, a $\frac{3}{4}$-in. bolt has a net area of 0.302 sq in.

Length of Bolts. The bolts will be set into the concrete a depth sufficient to develop their full strength by adhesion with the concrete.

One $\frac{3}{4}$-in. bolt develops $18{,}000 \times 0.302 = 5440$ lb.

From page 103,

$$x = \frac{f_s \pi i^2/4}{\pi d u} \quad \text{in. of embedment}$$

$$\frac{f_s \pi i^2}{4} = 5440 \text{ lb}$$

$$x = \frac{5440}{\pi \times \frac{3}{4} \times 80} = 28.6 \text{ in.} \qquad \text{depth of embedment}$$

They will be embedded 30 in. and will have a total length of 34 in.

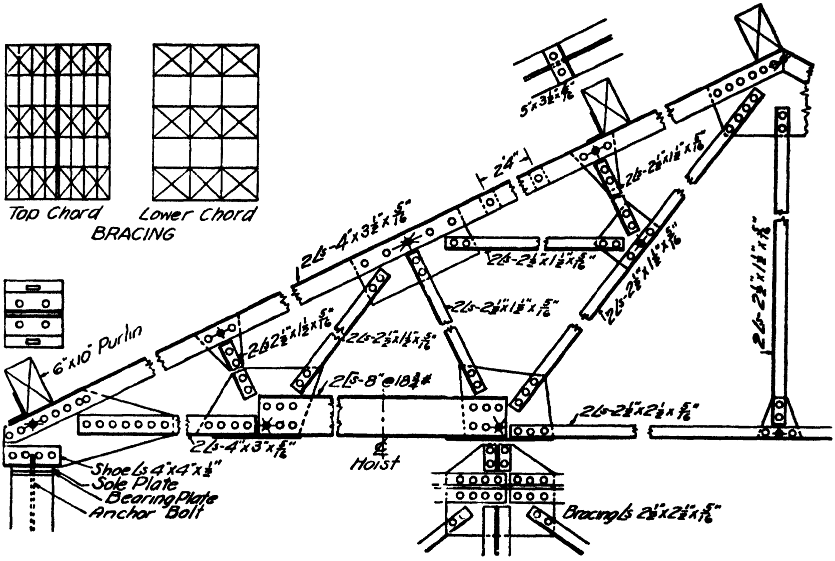

FIG. 37. Completed steel truss.

With an expansion of $\frac{1}{8}$ in. per 10 ft, the maximum movement (expansion or contraction) is

$$\tfrac{1}{8} \times \tfrac{60}{10} + \tfrac{1}{8} \times \tfrac{60}{10} = 1\tfrac{1}{2} \text{ in.}$$

The slot in the sole plate will be $2\frac{1}{2}$ by $\frac{7}{8}$ in. on the free end. On the anchored end $\frac{7}{8}$-in. holes in the bearing plate are sufficient.

To fasten the shoe angle to the gusset plate,

$$28{,}985 \div 8440 = 3.3, \text{ or 4 rivets are needed}$$

Two countersunk rivets in each outstanding leg of the bearing angle will provide the connection to the sole plate or bearing plate.

The essential details are shown in Fig. 37. In some instances more rivets are used at certain joints than are required by the calculations. This provides symmetry about the intersection of the center lines and avoids difficult shapes for the gusset plates. To prevent eccentricity at a connection, the rivets should, as nearly as possible, be equally spaced about the center lines.

STRESSES IN A KNEE-BRACED TRUSS

The type of truss now to be discussed is that ordinarily used where columns are used for supports. If a building is so designed that the action of the wind on the sides is transmitted to the columns supporting the truss, angle or knee braces must be placed, connecting the columns and lower or upper chord of the truss. In Fig. 38 is shown a knee-braced truss. The various dimensions are given in the drawing. The dead load and snow load on a truss of this design are determined as if the braces were absent (redundant). Such an assumption will cause but a slight error in the stresses.

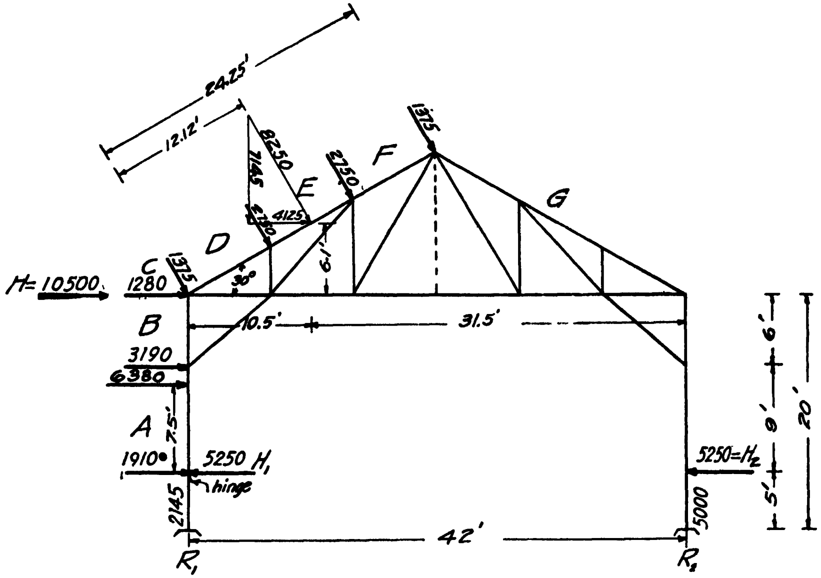

Fig. 38. Layout for knee-braced truss.

The wind load, after resolving normal to the roof, is 24.3 lb per sq ft. The total pressure is 24.25 × 14 × 24.3 = 8250 lb (the trusses are spaced 14 ft apart). This is distributed between the three panels.

$$\text{The forces } CD \text{ and } FG = {}^{8250}\!/_{3} \times {}^{1}\!/_{2} = 1375 \text{ lb.}$$

At DE and EF the forces are ${}^{8250}\!/_{3} = 2750$ lb.

As the truss under consideration is mounted on columns fixed[1] at the base (anchor bolts), the point of contraflexure (known as the hinge point) must be found. This point is considered as acting like a hinge. Various formulas have been proposed for determining the hinge point. Practice indicates that it is located about one-third of the distance between the bottom of the column and the bottom of the knee brace, measured from the base of the column. This distance is taken to the nearest foot. From the dimensions given in Fig. 38, this is found to be

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[1] There are two other possibilities: pin-connected and free. For a discussion of these the reader is referred to advanced texts on knee-braced trusses. See Ketchum's "Design of Mill Buildings," or McCullough's book.

$$\frac{14}{3} = 5 \text{ ft}$$

The reactions R_1 and R_2 may be determined either graphically or by moments. Moments will be used since less confusion will probably result. The wall area supported by each column is (remembering that below the hinge point no area is considered)

$$30.4 \text{ lb} \times 14 \times (20 - 5) = 6380 \text{ lb}$$

The resultant of this acts at a point midway between the hinge and the lower chord ($20 - 5 \times \frac{1}{2} = 7.5$ ft). The resultant of the wind on the roof is at the mid-point of the upper chord and is equal to 8250 lb. The vertical component of this force is $8250 \times \cos 30° = 7145$ lb, and the horizontal component is

$$8250 \times \sin 30° = 4125 \text{ lb}.$$

To make a solution of the problem possible, it must be assumed that the total horizontal force resisting the wind pressure is borne equally by the two columns and acts at their hinges. The total wind pressure is made up of the horizontal component on the roof and the pressure on the wall, and is

$$H = 6380 + 4125 = 10{,}505, \text{ or } 10{,}500 \text{ lb}$$

The reaction of H at the hinges is

$$H_1 = H_2 = \frac{10{,}500}{2} = 5250 \text{ lb}$$

The reactions are found by taking moments about the hinges. Taking moments about R_2, we find

$$\begin{aligned} 42R_1 &= 7145 \times 31.5 - 4125 \times 21.1 - 6380 \times 7.5 \\ R_1 &= \frac{7145 \times 31.5 - 4125 \times 21.1 - 6380 \times 7.5}{42} \\ &= 2145 \text{ lb} \end{aligned}$$

Taking moments about R_1,

$$\begin{aligned} 42R_2 &= 7145 \times 10.5 + 4125 \times 21.1 + 6380 \times 7.5 \\ R_2 &= 5000 \text{ lb} \end{aligned}$$

The sum of the two reactions should check the vertical component of the forces acting on the roof.

The pressure on the wall (the wall being considered as two panels) is distributed as follows: one from the hinge to the bottom of the knee brace; the other from the bottom of the knee brace to the lower chord. There are then forces acting at the hinge, the knee brace, and the top of the column. They are

Top of column $= BC = \frac{6}{2} \times 14 \times 30.4 = 1280$ lb
Foot of knee brace $= AB = \frac{6}{2} \times 14 \times 30.4 + \frac{9}{2} \times 14 \times 30.4 = 3190$ lb
Hinge $= \frac{9}{2} \times 14 \times 30.4 = 1910$ lb

The forces exerted by the column on the truss, under the conditions assumed, can now be found. The forces to be determined are those acting at the top of the columns and those acting at the foot of the knee braces. Let F_1 and F_2, and F'_1 and F'_2 be these forces acting on the windward and leeward columns, respectively. The forces are actually resisting the stresses set up; therefore in determining them by moments, as we shall do, their direction is taken acting opposite to that shown in Fig. 40. Figures 39*a* and *b* show the determination of the forces.

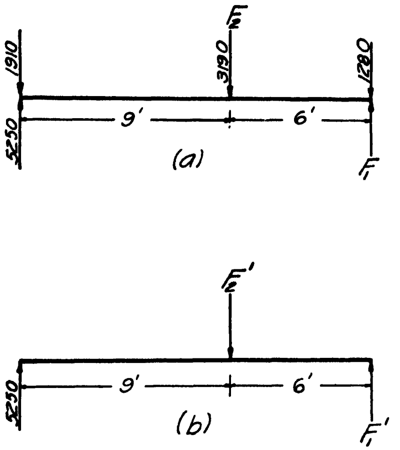

FIG. 39.

From Fig. 39*a*, taking moments about F_2,

$$6(F_1 - 1280) = (5250 - 1910) \times 9$$
$$F_1 = 6290 \text{ lb}$$

Taking moments about F_1,

$$6(F_2 + 3190) = (5250 - 1910) \times 15$$
$$F_2 = 5160 \text{ lb}$$

From Fig. 39*b*, taking moments about F'_2,

$$6F'_1 = 5250 \times 9$$
$$F'_1 = 7875 \text{ lb}$$

Taking moments about F'_1,

$$6F'_2 = 5250 \times 15$$
$$F'_2 = 13{,}125 \text{ lb}$$

With F_1, F_2, F'_1, and F'_2 known, a graphical solution of the wind-load stress in the truss may be made. This solution is shown in Fig. 40. The external forces, beginning with *AB*, are laid out in a clockwise manner, thus giving the load line *abcdefgb′a′na*. Beginning at the left end, *bo* intersects *on*; *di* intersects *io*; *ej* intersects *ji*; *jk* intersects *kn*; etc. When the middle of the truss has been reached it is best to start at the right end and to carry on in a similar way. The check on the drafting will be the line *l′m*, *m* falling on the *m* established by the work carried from the left end.

The sense of the stresses is found in exactly the manner previously discussed (page 10), remembering to follow around the point in a clockwise direction. As we are interested in the column, only the stresses shown in Fig. 40 have been assigned values.

Selection of Columns. The selection of the columns must be considered from two points: (1) bending stress caused by the wind and (2) direct compression caused by the dead load, snow load, and wind load. For our purpose, we shall assume that the dead load plus the snow load equals 60 lb per sq ft of horizontal projection. Their reactions are

$$R_1 = R_2 = \frac{60 \times 42 \times 14}{2} = 17{,}640 \text{ lb}$$

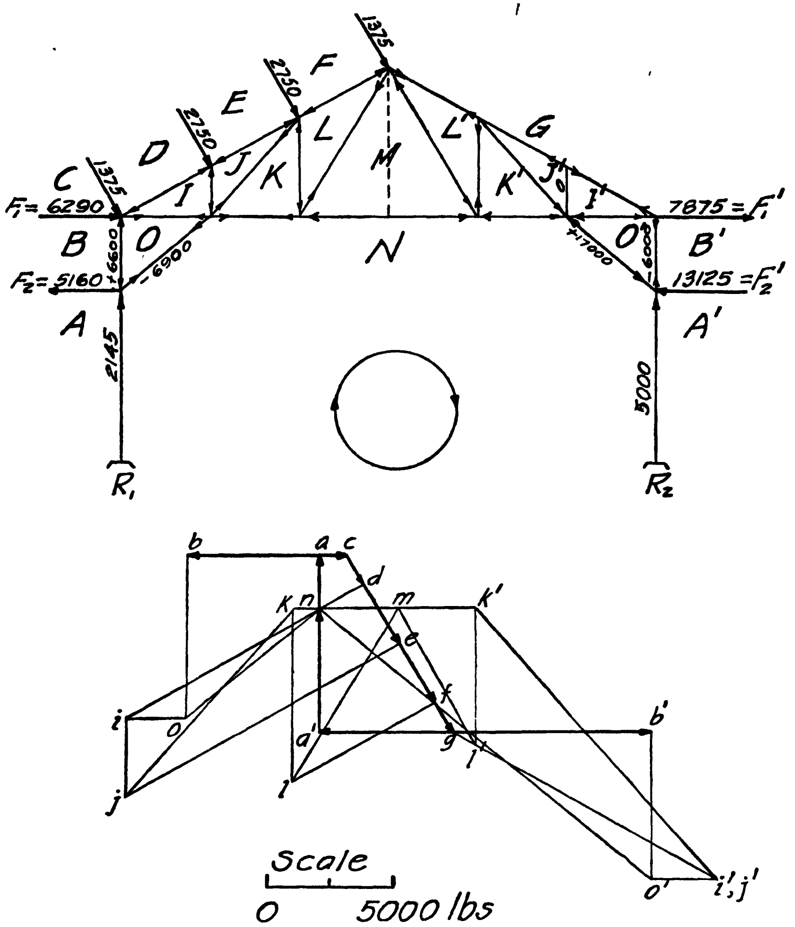

FIG. 40. Wind-load stresses in knee-braced truss.

The bending moments in the columns are

Windward Columns. Moment at the bottom of the column is

$$[(5250 \times 5) + (1910 \times \tfrac{5}{2})] \times 12 = 372{,}500 \text{ in.-lb}$$

At the bottom of the knee brace the moment is

$$[(5250 \times 9) - (1910 \times 9)] \times 12 = 360{,}500 \text{ in.-lb}$$

Leeward Columns. The moment at the bottom of the column is

$$5250 \times 5 \times 12 = 315{,}000 \text{ in.-lb}$$

Moment at the bottom of the knee brace is

$$5250 \times 9 \times 12 = 567{,}000 \text{ in.-lb}$$

The maximum moment is found to be at the bottom of the leeward knee brace. The column must be designed to resist that value. The selection of the column is based on the following formula:[1]

$$A = \frac{P}{f_c} + \frac{Mc}{fr^2}$$

where A = area, sq in., of the column to resist bending and direct stress
P = axial load (dead load, snow load, and wind load)
f_c = working stress as given by the column formula
$= 18{,}000 - 60\frac{l}{r}$
M = bending moment, in.-lb
c = distance from center of column to extreme outer fiber
f = maximum stress on extreme fiber = 18,000 lb per sq in.
r = radius of gyration of the section about the axis 1-1 (see Fig. 41)

A column section made of angles laced together will be used (see Fig. 41). This section permits simple and easy connections to the knee brace and the truss. The section should be quite wide in the plane of the truss to resist the bending moment. It must have a width along aixs 1-1 so that the l/r ratio is not greater than 100. If the column is not fastened rigidly at the bottom by anchor bolts or embedment and at the top by an eave strut, l equals the full length of the column.[2] When the column is connected as indicated above, l equals one-half the column length.[2] We shall assume that the column is fastened at top and bottom. The least allowable radius about 2-2 is

$$r_{2\text{-}2} = \frac{1}{2} \times \frac{20 \times 12}{100} = 1.2 \text{ in.}$$

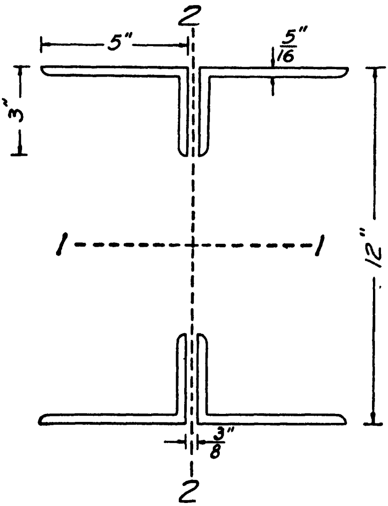

FIG. 41.

As a trial, four 5- by 3- by $\frac{5}{16}$-in. angles, spaced $\frac{3}{8}$ in. apart, will be selected. Figure 41 shows the layout. From the handbook the radius of gyration about axis 2-2 is found to be

$$r_{2\text{-}2} = 2.47 \text{ in.}$$

The radius of gyration about 1-1 may be found from the expression

$$r_{1\text{-}1} = \sqrt{r^2 + d^2}$$

where r = the radius of gyration of one angle about an axis parallel to the long leg
d = distance from center of section to center of gravity of the angle

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[1] HOOL and KINNE, *op. cit.*, pp. 212, 230.

[2] *Ibid.*, p. 231.

TABLE 15. SIZE OF LATTICE BARS AND STAY PLATES TO BE USED WITH LATTICED COLUMNS

Depth of channel, in.	Dimension of lattice bars, in.		Wt. of lattice bars, lb. per ft.	Center of hole to end of bar (*a*), in.	Distance, center to center, of rivets (*d*)		Minimum size of stay plates at end of column, in.			Wt. of minimum stay plate, lb.	Diameter of rivets, in.
	W	Thickness			Max.	Min.	*b*	Thickness	*l*		
6	1½	¼	1.28	1⅛	0′ 11½″	6⅝″	8¼	¼	7½	4 38	⅝
7	1¾	¼	1.49	1⅛	1′ 1½″	7⅝″	9¼	¼	10	6 55	⅝
8	2	⁵⁄₁₆	2.12	1¼	1′ 3″	8¹¹⁄₁₆″	10½	⁵⁄₁₆	9	8 37	¾
9	2	⁵⁄₁₆	2.12	1¼	1′ 4½″	9½″	11¼	⁵⁄₁₆	12	11.95	¾
10	2	⅜	2.55	1¼	1′ 6½″	10¹¹⁄₁₆″	12¼	⅜	12	15.62	¾
12	2¼	⅜	2 87	1⅜	1′ 10½″	13″	14¼	⅜	15	22.73	¾
15	2½	⅜	3.19	1½	2′ 2½″	15⁵⁄₁₆″	16¼	⅜	15	25.90	¾

For a 5- by 3- by 5/16-in. angle

$$r = 0.85 \text{ in.} \qquad \text{and} \qquad d = \tfrac{12}{2} - 0.68 = 5.32 \text{ in.}$$

$$r_{1\text{-}1} = \sqrt{0.85^2 + 5.32^2} = 5.39 \text{ in.}$$

The value for f_c is based on the column length between the hinge point and the top of the column. In the present problem, $l = 15$ ft.

$$f_c = 18{,}000 - 60\,\frac{15 \times 12}{5.39} = 15{,}900 \text{ lb per sq in.}$$

P = dead load + snow load + vertical component of the wind-load reaction
= 17,640 + 5000 = 22,640 lb

The area of the section to withstand the loading is

$$A = \frac{P}{f_c} + \frac{Mc}{fr^2} = \frac{22{,}640}{15{,}900} + \frac{567{,}000 \times 6}{18{,}000 \times 5.39^2} = 7.91 \text{ sq. in.}$$

The area of one 5- by 3- by 5/16-in. angle = 2.40 sq in.

$$4 \times 2.40 = 9.60 \text{ sq in.}$$

The section chosen, four 5- by 3- by 5/16-in. angles, is large enough.

The section should be checked in the plane of the axis parallel to the face of the long leg.

$$f_c = 18{,}000 - 60\,\frac{20 \times 12}{2.47} = 12{,}170 \text{ lb per sq in.}$$

$$A = \frac{22{,}640}{12{,}170} = 1.86 \text{ sq in. are required}$$

The section as selected is ample.

TABLE 16. LATTICE BARS FOR CHANNEL AND ANGLE COLUMNS

Section	Minimum width, in.	Diameter of rivet, in.
15-in. channels or 3½-in. angles .	2½	⅞
12-, 10-, and 9-in. channels or 3-in. angles	2¼	¾
8- and 7-in. channels or 2½-in. angles....	2	⅝
6- and 5-in. channels or 2-in. angles.....	1¾	½

The size of the lattice bars may be obtained from Table 16. Table 15 and Fig. 42 give data on lacing channels.[1]

Inclination of lattice bars should not be less than 45 deg. The thickness of a single lattice is to have a minimum value of 1/40 the distance between rivets; the double lattice is to have a value 1/60 of the distance.

From an inspection of Fig. 40, we find that some of the members of the truss must be selected for both tension and compression. This point must not be ignored in the actual design of the truss.

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[1] "Cambria Steel Handbook," 1904.

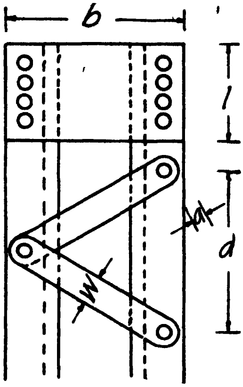

FIG. 42. Lattice bars and stay plate for latticed columns. (*Cambria Steel Handbook.*)

FOUNDATIONS FOR COLUMNS

The design of individual footings or piers for columns is a subject on which there is none too much information. Practically all the methods in use require a good "guess." In general such designs may be approached from two angles: one in which punching shear and bending are considered; and the other in which the footing is selected so as to make the consideration of shear or moments, in most cases, unnecessary. Talbot, at the University of Illinois, performed a series of tests on concrete footings, both plain and reinforced, and derived a formula that agrees closely with the test results.[1] It gives the maximum bending moment in the reinforcing steel in the footing. The formula is

$$M = (\tfrac{1}{2}ac^2 + 0.6c^3)w$$

where a = one dimension of the object (column, pier, etc.) resting on footing

c = offset from face of column to extreme edge of footing

w = unit bearing pressure on soil or other material supporting footing

The above formula is for square footings, rectangular footings will be treated later (see page 148).

After finding the bending moment, the total area A_s, of reinforcing steel required to resist the moment may be found from

$$A_s = \frac{M}{f_s jd}$$

With A_s known, reinforcing rods of proper diameter and number are selected to meet the requirements.

Bond Resistance. The effect of the bond resistance of reinforcing steel on the strength of concrete footings is of considerable importance.[2]

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[1] *Univ. of Ill. Eng. Exp. Sta. Bull.* 67.

[2] URQUHART and O'ROURKE, "Design of Concrete Structures," 1st ed., p. 45, McGraw-Hill Book Company, Inc.

HOOL and KINNE, "Foundations, Abutments and Footings," 1st ed., p. 219, McGraw-Hill Book Company, Inc.

Univ. of Ill. Eng. Exp. Sta. Bull. 71

Adhesive resistance between the reinforcing steel and the concrete is one of the principal considerations in reinforced-concrete work. Many tests performed at the University of Illinois Experiment Station show (among other things considered) that bond stress caused by adhesion is quite large. To help effect this resistance, at least 3 in. of concrete should be placed under the bars. The required bond resistance is usually obtained by using rods ¾ in. or less in diameter.

Bond Stress. It is customary to base the bond stress on the vertical shear developed at a section at the face of the footing. The vertical shear is found by

$$V = \tfrac{1}{4}(l^2 - a^2)w = (ac + c^2)w$$

and the bond stress is

$$u = \frac{V}{\Sigma ojd}$$

where V = vertical external shear
l = length of footing
a = one dimension of column
c = offset from face of column to extreme edge of footing
w = unit bearing pressure on soil or other material supporting footing
Σ = number of reinforcing rods used
o = circumference of one rod
j = ratio of lever arm of resisting couple to depth d
d = depth from surface of footing to reinforcing steel
f_s = allowable tensile stress of steel = 16,000 lb per sq in.

The value 0.875 is usually assigned to j. The allowable bond stress u must be kept under 80 lb per sq in. for 1:2:4 concrete.[1]

Plain Concrete Footings. If footings without reinforcement are used, the following procedure is considered permissible:[2] Bending stresses and shear need not be calculated, although it is advisable to check the depth thus found for punching shear. The design is based on the assumption that the load is transmitted through the footing at an angle of 60 deg with the horizontal, measured from the face of the column to the base of the footing (see Fig. 43). The area of the top of the footing is taken at not less than twice the area of the column base plate. The area of the bottom of the footing is found from the allowable bearing pressure of the soil and the total load on the column. The weight of the footing must also be considered in determining the bearing area. It may at first have to be approximated and later corrected.

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[1] If deformed bars are used, u = 100 lb per sq in.

[2] HOOL and KINNE, "Foundations, Abutments and Footings," p. 216.

The depth of the footing is found by multiplying one-half the base of the footing minus one-half the width of the column base by the tangent of 60 deg (see Fig. 43).

$$d = \frac{(b - a)}{2} \tan 60^\circ$$

where b = width of footing base
a = width of column base

If the above procedure is followed, the allowable punching shear of 120 lb per sq in. will practically never be exceeded. To determine the depth for resisting punching shear, the total load on the column must be reduced by an amount equal to the allowable unit bearing pressure of the soil times the area of the column base. The adjusted load is then divided by the periphery of the column times the allowable unit stress in shear. (Allowable shear equals 120 lb per sq in. for 1:2:4 concrete.) The result is the depth of concrete necessary to resist shear. The load producing punching shear may be expressed by

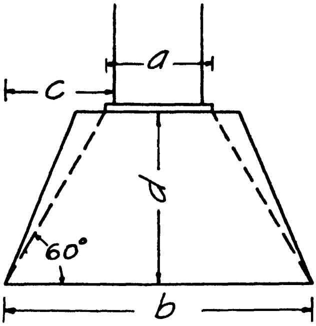

Fig. 43. Design of plain concrete footing.

$$\frac{\text{Area of footing} - \text{area of column base}}{\text{Area of footing}} \times \text{column load}$$

SELECTION OF FOOTINGS FOR THE KNEE-BRACED TRUSS

Two types of footings may be designed for the column supporting the knee-braced truss previously examined: plain concrete and reinforced concrete.

Plain Concrete. On page 94, we found that four 5- by 3- by $\frac{5}{16}$-in. angles laced together were required. They were arranged so that the maximum moment of inertia is in the plane of the truss. The columns are fastened to the footing by anchor bolts. For this purpose, the design at the bottom of the column will be similar to that shown in Fig. 44. The column angles are fastened to the base plate by means of angles riveted by their vertical legs to the column angles. For this connection, 5- by 5- by ¾-in. angles are sufficient. The dimensions of the base plate may now be found. The column is 12 in. deep in the plane of the truss. To this must be added the width of two 5-in. angles.

Length of the base plate $= 12 + 2 \times 5 = 22$ in.
Width of the base plate $= 5 + \frac{3}{8} + 5 = 10\frac{3}{8}$, or 10.5 in.
Periphery of the plate $= 2 \times 10.5 + 2 \times 22 = 65$ in.
Area $= 10.5 \times 22 = 231$ sq in. $= 1.6$ sq ft

If the bearing pressure of the soil is taken at 4000 lb per sq ft, the area to sustain a load of 22,640 lb is

$$\frac{22{,}640}{4000} = 5.66 \text{ sq ft}$$

The top of the footing will be made rectangular, being proportioned to the dimensions of the base plate. Its area is to be at least twice the area of the base plate and is $2 \times 1.6 = 3.2$ sq ft. The dimensions of the top are found as follows:

Let y = greater dimension
x = lesser dimension

then

$$y:\frac{22}{12}::\sqrt{3.2}:\sqrt{1.6}$$
$$y = 18.3\sqrt{2} = 2.59 \text{ ft}$$
$$x:\frac{10.5}{12}::\sqrt{3.2}:\sqrt{1.6}$$
$$x = 0.875\sqrt{2} = 1.24 \text{ ft}$$

Proportioning the bottom of the footing in a similar way,

$$y:1.83::\sqrt{5.66}:\sqrt{1.6}$$
$$y = 3.44 \text{ ft}$$
$$x:0.875::\sqrt{5.66}:\sqrt{1.6}$$
$$x = 1.65 \text{ ft}$$

The depth of the footing, using the 1.83-ft side of the base plate and the 3.44-ft side of the bottom of the footing, is

$$d = \frac{(3.44 - 1.83)}{2} \tan 60° = 0.81 \times 1.732 = 1.4 \text{ ft}$$

Checking the footing for punching shear, we get

$$\frac{5.66 - 1.6}{5.66} \times 22{,}640 = 16{,}250 \text{ lb}$$

the corrected load.

$$\frac{16{,}250}{65 \times 120} = 2.1 \text{ in.}$$

depth of footing to resist punching shear. The depth of the footing will be 1 ft 5 in.

In the above calculations no account was taken of the weight of the footing in determining the bearing area. The volume of the frustum of the pyramid (footing) just determined is

$$\frac{1.4}{3}(5.66 + 3.2 + \sqrt{5.66 \times 3.2}) = 6.1 \text{ cu ft}$$

Using for the weight of concrete, 145 lb per cu ft, the footing as determined weighs $145 \times 6.1 = 885$, or 900 lb. The weight of the footing in the present example will have but very little effect on the bearing area and size of the footing. A recalculation including the weight need not be made.

Reinforced-concrete Footing. There would be no advantage in reinforcing a footing as small as the above. The design of reinforced footings will be postponed until headframes are discussed.

Anchor Bolts[1]

Anchor bolts are placed in the plane of the moment to be resisted. Not less than two bolts should be used (one on each side), even though the moment is small. For large moments, the proper number of bolts are determined. The assumption is made that any tension resulting from an overturning effect on the column is resisted entirely by the bolts. Figure 44 shows the column with bolts and dimensions in the plane of the moment.

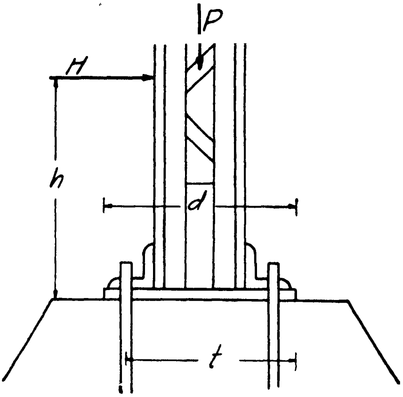

FIG. 44. Design of anchor bolts.

For the knee-braced truss shown in Fig. 38, the values are

$P = (5000 + 17{,}640) = 22{,}640$ lb
$H = 5250$ lb
$h = 5$ ft
$d = 22$ in.
$t = 19.75$ in.

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[1] HOOL and KINNE, "Steel and Timber Structures," p. 235.

With the above data the maximum compressive stress on the foundation and the stress in the anchor bolts may be found. The forces acting on the bottom of the column are caused by the stress P and the moment M, resulting from H. The load is then considered as acting at a distance e from the center of the column. The load must have a moment arm e of such a length as to make its moment equal to that caused by H. Therefore

$$e = \frac{M}{P}$$

where M = moment caused by H about the base of column

The compressive stress on the footing is given by

$$p = \frac{P}{bd}\left(1 + \frac{6e}{d}\right)$$

where p = maximum compressive stress on footing, lb per sq in.
b = short dimension of column base
d = long dimension of column base
P and e as given above

The stress in the anchor bolts is found from the following expression:

$$T = \frac{Pd}{24e}\left(\frac{6e}{d} - 1\right)^2$$

By substituting the values found in the knee-braced truss in the above formula, the compressive stress on the footing can be found and the anchor bolts can be selected.

$$M = 5250 \times 5 \times 12 = 315{,}000 \text{ in.-lb}$$

$$e = \frac{315{,}000}{22{,}640} = 13.9 \text{ in.}$$

$$p = \frac{22{,}640}{10.5 \times 22}\left(1 + \frac{6 \times 13.9}{22}\right) = 98.1 \times 4.8 = 471 \text{ lb per sq in.}$$

An allowable stress in 1:2:4 concrete of 650 lb per sq in. is permissible, so that 471 lb is safe.

$$T = \frac{22{,}640 \times 22}{24 \times 13.9}\left(\frac{6 \times 13.9}{22} - 1\right)^2$$
$$= 11{,}700 \text{ lb}$$

The working stress in anchor bolts may safely be taken at 10,000 lb per sq in. The area required is

$$\frac{11{,}700}{10{,}000} = 1.17 \text{ sq in.}$$

A 1½-in. bolt has a net area of 1.294 sq in., which will do.

Embedment of Bolts in Concrete. If the bolts do not have anchor plates on their lower ends, their length must be such that the total bond stress between the bolts and the concrete is sufficient to prevent their being pulled out. The depth of the embedment should be such as to develop the full strength of the bolt. The desired length in the concrete may be found from[1]

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[1] URQUHART and O'ROURKE, "Design of Concrete Structures," 4th ed. p. 53, McGraw-Hill Book Company, Inc., New York.

$$x = \frac{f_s \frac{\pi i^2}{4}}{\pi du} \text{ in.}$$

where x = depth of embedment in concrete, in.
f_s = allowable unit tensile stress in steel
i = diameter of net section of bolt, in.
d = gross diameter of bolt
u = allowable unit bond stress between steel and concrete
$\pi i^2/4$ = net area

Values[1] for u may be obtained (for plain rods) by multiplying the ultimate compressive strength of the concrete by 0.04. For 1:2:4 concrete, 80 lb per sq in. is usually taken. For bolts used as anchors, f_s equals 10,000 lb per sq in.

$$x = \frac{10{,}000 \times 1.294}{\pi \times 1\frac{1}{2} \times 80} = 34.6 \text{ in.}$$

This is 17.8 in. longer than the depth of the footing (1.4 ft) originally found. We shall therefore make the embedded length the same as the footing depth and connect the lower ends of the two bolts with a plate, or hooks may be placed on the embedded ends of the bolts.

In practice, it is customary to assume the depth of embedment necessary to develop the full strength of the bolt as being equal to 20 times the diameter of the bolt.[2] On this assumption we should need 20 × 1½ = 30 in.

The size of the anchor plate can be calculated in the following manner: The plate is considered as acting like a cantilever beam. The moment[3] at the edge of the bolthole is

$$M = \frac{Wy^2}{2(2y + a)} \text{ in.-lb}$$

The thickness of the plate necessary to resist this moment is[1]

$$t = \sqrt{\frac{6M}{fb}} \text{ in.}$$

where W = load on the rod or bolt
y = distance from edge of the hole to edge of the plate; if the longer distance is within 1.25 times the shorter, the two are averaged; otherwise the longer measurement is used
a = diameter of hole (punched ¼ in. greater than diameter of bolt)
f = fiber stress in bolt
b = width of plate after deducting diameter of hole

It was previously decided to embed the bolts 1.4 ft, which is the depth of the footing. The total bonding resistance for this depth is

$$u = 1.4 \times 12 \times \pi \times 1.5 \times 80 = 6340 \text{ lb}$$

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[1] *Ibid.*, p. 52.

[2] HOOL and KINNE, "Steel and Timber Structures," p. 236.
PEELE (ed), *op. cit.* Sec. 43, p. 27.

[3] MCCULLOUGH, *op. cit.*, p. 150.

The force acting on the bolt is 11,580 lb. A plate to make up the difference (11,580 − 6340 = 5240 lb) is required. To select the plate a cut-and-try process must be followed; *i.e.*, dimensions for the plate are selected and tried, and, if they are not suitable, another try is made. The metal in the plate should extend at least ¾ in. on each side of the edge of the bolthole. We shall use this to try a minimum width of the plate. The length of the plate may be taken at one-half the distance between the centers of the bolts and is

$$\frac{22 - 2(0.75 + 2.0)}{2} = 8.25 \text{ in.}$$

The edge of the boltholes in the angles at the base of the column is 1.5 in. from the edge of the angles. The bolthole is ¼ in. larger than the bolt, and the edge of the plate is ¾ in. greater (on each side) than the diameter of the hole. The width is

$$1.5 + 0.25 + 2(0.75) = 3.25 \text{ in.} = b.$$

$$\text{Gross area of plate} = 8.25 \times 3.25 = 26.8 \text{ sq in.}$$

$$\text{Net area} = 26.8 - \frac{\pi \times 1.75^2}{4} = 26.8 - 2.3 = 24.5 \text{ sq in.}$$

$$\text{Compressive stress on concrete} = \frac{5240}{24.5} = 214 \text{ lb per sq in.}$$

which is safe for 1:2:4 concrete.

$$M = \frac{5240 \times \left(8.25 - \frac{1.75}{2}\right)^2}{2\left[2\left(8.25 - \frac{1.75}{2}\right) + 1.75\right]} = \frac{5240 \times 54.4}{2 \times 16.5} = 8630 \text{ in.-lb}$$

$$t = \sqrt{\frac{6 \times 8630}{16{,}000 \times 1.5}} = \sqrt{2.18} = 1.48 \text{ in.}$$

A 1½-in. plate will be necessary. It will have a projection of 6 in. beyond the bolts. This gives a plate the length of which is 16.5 + (2 × ¾) + (2 × 6) = 30.0 in. As a precaution, especially when an exceptionally heavy force is acting on the bolt, the concrete should be checked for shear. The same procedure as for punching shear in column footings would be used. In the above case, shear has very little effect.

CHAPTER IV

HEADFRAMES

The selection of a headframe depends on a great many things of more or less importance. Probably the best way to emphasize these points is by means of an itemized list. This list follows:[1]

1. Ore reserves
2. Daily tonnage
3. Hoisting speed
4. Method of hoisting ore or waste
    a. Skips
    b. Cage and car
5. Method of handling ore at shaft collar
6. Height of headframe
7. Material of which headframe is made
    a. Wood
    b. Steel
    c. Concrete
8. Topography
9. Ground subsidence (not necessarily in the immediate vicinity of the shaft)
10. Foundations
11. Method of dumping skips
12. Future development
13. Type of headframe
    a. A-type
    b. Four- or six-post type
14. Type of hoist
    a. Drum
    b. Reel
15. Shaft
    a. Shape
        (1) Rectangular
        (2) Square

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[1] KETCHUM, M. S., "The Design of Mine Structures," 1st ed., McGraw-Hill Book Company, Inc., New York.

CALHOUN, A. B., Fundamental Hoisting Problems, Part I, *Eng. Mining J.*, **122,** 485 (1926). Part II, 122, 524 (1926).

HARRER and FARNHAM, Ore Hoisting in the Butte District, *Eng. Mining J.*, **129,** 44 (1930).

BURR, F. L., The Design of Head Frames, *Eng. Mining J.*, 103, 611 (1917).

PEELE, R. (ed), "Mining Engineers' Handbook," 3d ed. Sec. 12, p. 61, John Wiley & Sons, Inc., New York.

        (3) Circular
        (4) Elliptical
    b. Arrangement of compartments

16. Crushing equipment, ore or waste bins, etc., supported by the headframe
17. Location of hoist with reference to headframe
18. Height at which ore is discharged on headframe
19. Inclination of shaft
20. Position of sheaves
21. Effect of wind
22. Ultimate depth of shaft
23. Nature and condition of shaft and surrounding ground near and at the collar of the shaft

The above list will give a general idea as to the considerations that influence the design. All headframes will involve most of the above items and possibly some that are not listed. However, each structure will have peculiarities applying to it alone, which rather effectively prevents standardization.

A detailed discussion of all the factors listed is unnecessary since the part that each plays is more or less self-evident. The salient points of some of them will be briefly taken up.

Hoisting Speed. The height of the headframe is, to a limited extent, dependent on the speed with which the material is hoisted. This is more apt to be true with shafts less than 500 ft in depth than with those deeper than 500 ft, when a large daily tonnage is handled. A large tonnage requires a high hoisting speed, with rapid acceleration and retardation. It is not desirable in all cases to confine the entire retarding period to the shaft; hence an increase in height of the headframe is necessary.

Methods of Hoisting. We shall consider but two methods of removing material: (1) skip and (2) cage and car. For large tonnages at metal mines the skip is used almost exclusively. It is not within the scope of this book to present the economical reasons for the choice between skips and cages; any good textbook on mining may be consulted.[1] The over-all length of skips in many cases is 20 ft or more. With skip hoisting, the skips dump directly into large storage bins or into a small receptacle that functions as a transfer bin. In many cases the headframe must be high enough to give the desired storage capacity and in addition clearance for discharging the contents of the bin into railroad cars. An overwinding clearance must be allowed for. It is measured from the rope socket, when the skip is in the dumping position, to the contact of the rope with the sheave. This distance depends on the hoisting speed, being greatest for the higher speeds. In general it should not be less than two-thirds the circumference of the drum. If cage and car are used, and are landed at the shaft collar, the headframe need not be so high as when the landing is above the collar to permit storage or ore-dressing treatment. Additional height would ordinarily be required for two or three deck cages.

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[1] YOUNG, G. J., "Elements of Mining," 4th ed., McGraw-Hill Book Company, Inc., New York.

LEWIS, R. S., "Elements of Mining," 2nd ed., John Wiley & Sons, Inc., New York.

Height of Headframe. The height of the structure depends on the requirements necessary to meet the demands of the following: hoisting speed; handling of ore and waste at the surface; material of construction; location of hoist; future deepening of shaft; type of hoist (drum or reel); storage bins; crushing equipment; etc. If possible, the headframe should be of such a height that the rope from the hoist drum to the sheave makes an angle of about 45 deg with the horizontal; and the resultant from the part of the rope in the shaft and the exterior part of the rope falls within the back post. If the resultant falls outside the post, the headframe must be anchored against overturning, which may be inconvenient. More will be said about the resultant under the section on the design of a headframe.

Type of Headframe. There are in general three types of headframes in use: the A-type, four-post type, and six-post type. Most metal-mine headframes are of the A-type. Four- and six-post headframes are installed mostly at coal mines. The number refers to the front posts and not to the back posts or braces. The choice between the A-frame and others depends almost entirely on the shaft layout. A rectangular shaft with all the compartments side by side can be equipped with the A-type. Square shafts, divided into equally or unequally dimensioned compartments, are usually equipped with the four-post frame, although the A-type in exceptional cases may be used. Circular and elliptical shafts are, so far as the author knows, equipped with four- or six-post headframes.

The stresses in the A-frame are easily solved either graphically or algebraically by considering only one diagonal bracing member in each panel. The four- and six-post types are statically indeterminate, and are therefore more difficult of solution.

Headframes may be made of wood, steel, or reinforced concrete. No discussion of the last will be made here. For such information the reader is referred to an article appearing some years ago in one of the technical journals.[1] The principle involved in the design of wood or steel headframes is the same. Different structural formulas are of course used. Wooden headframes are usually not so high as the steel frames, nor are they as a rule subjected to as high hoisting speed or required to supply as great a tonnage. They may or may not be as permanent as steel, depending on the weather and the condition of the mine air, if the hoisting shaft is upcast. Wooden headframes for coal mines should be avoided as far as possible because of fire hazards. The material is of built-up sections (laminated), or one piece may be used.

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[1] Burr, *op. cit.* p. 611.

The structural material for steel headframes may be made of built-up sections (Z bars, plate or latticed angles, etc.) or rolled sections (H beams, laced channels, wide flange beams, etc.). The bracing is usually channels or angles. Steel headframes must be kept well painted to prevent too rapid deterioration from noxious mine gases.

Type of Hoist. The selection of the hoist affects the design of the headframe indirectly. Its location with respect to the distance from the shaft affects the life of the hoisting rope and the position of the resultant with reference to the back posts. If a cylindrical hoist is placed too close, the bending stress in the rope becomes excessive. On the other hand, a reel with a flat rope may be placed much closer because the bending stresses are less. The magnitude of the fleet angle depends upon the distance the hoist is from the sheave and width of the drum face. The fleet angle is the angle between the center line of the sheave and drum and the rope when it has reached the end of the drum. It should be not greater than 1°30′; otherwise the wear on the sheave and rope is prohibitive. If a reel is used, there is no fleet angle. In general, the round rope is more economical than the flat rope; however, in some instances because of topographic conditions, the reel and flat rope must be used. The flat rope has been used on some underground installations to reduce the drifting and raising necessary to cut out the hoist station and cable way.

Storage Bins and Other Equipment on the Headframe. While no examples will be offered in this dissertation on this subject, a few words concerning it will not be out of place. The weight of the storage bins, with contents, and the weight of the crushing equipment must be added to the weight of the headframe for the determination of dead-load stresses. In addition, at least 25 per cent of the weight of ore and crusher should be added to take care of impact and vibration. If the bins and crushing equipment can be placed on supports separate from the headframe, a less costly structure will probably result.

Ultimate Depth of Shaft. If hoisting is to be carried on from levels deeper than those in operation at the time the headframe is erected, such increase in depth should be considered in the design. Economically, the first cost of an oversized headframe must be compared with the interest on such an investment for the time that elapses before hoisting from the deeper levels commences. The difference between the two designs will be mainly one of rope size, skip-dumping conditions, and hoisting speed. An increase in height of frame and size of structural steel for the members will result.

SELECTING THE HOISTING ROPE

The choice of a rope depends on factors, the exact effect of which are difficult to determine. There is considerable difference of opinion as to the extent to which some of these should be considered. Involved in the selection of the rope are the following:

1. Factor of safety
2. Bending stresses
3. Load (weight of skip and contents, or cage and car with contents)
4. Weight of rope
5. Acceleration
6. Starting and stopping stress
7. Conditions under which the rope is used

Aside from the statement that 6 by 19 ropes are generally used for hoisting, the author does not feel that it is necessary to go into a discussion on the relative merits of the various types of wire ropes.

The Young Committee[1] in their report to the Minister of Mines of Ontario offered the following conclusions, among many others, regarding hoisting ropes:

> 1. The flat-strand rope is more vulnerable to corrosion than the round-strand rope.
>
> 2. Factors of safety for hoisting ropes should be fixed with due regard to the uncalculated bending, torsional, and shock stresses. The Committee sees no advantage in the use of the so-called "capacity factor," in that it is based on the erroneous assumption that the weakest section of the rope is at or near the capel, *i.e.*, the point of its attachment to the conveyance.

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[1] Report of the committee appointed by the government of the Province of Ontario to enquire into matters arising out of the accident at the Paymaster Consolidated Mines, Ltd., Feb. 2, 1945. Dean C. R. Young, chairman, Ontario Department of Mines, Toronto, Canada, Mar. 28, 1946.

3. Corrosion is by far the most common cause of rope failure in Canada. It is often unaccompanied by any readily detected external impairment except surface wear.

4. The Committee is of the opinion that when abrasive wear on outside wires has removed 20 per cent of their area the rope should be discarded.

5. The weakest section of a rope may be at almost any point in its length, particularly in the long stretch between the sheave and the conveyance when the latter is at the bottom of the shaft. Theories based on assumption that it is at the capel are consequently without sound foundation.

6. Cutting off lengths of rope at the drum end to change the crossover points is to be commended.

7. The Committee is of the opinion that the device of turning a rope end for end is undesirable, in that it is likely to result in possible weak spots being placed where they cannot readily be detected.

RECOMMENDATIONS

8. That steps be taken by the Department of Mines to promote the use of drums and head sheaves of a diameter not less than 80 to 100 times the rope diameter in future installations, or modifications of existing ones.

9. That the Department of Mines investigate, or cause to be investigated, the desirability and feasibility of incorporating electrical conductors in hoisting ropes for the purpose of communication between the cage and the surface and of applying the brakes automatically in the case of rope failure.

In addition the Committee recommends the use of only one layer of rope on the drum; at the most two layers might be used. Corrugations occurring on ungrooved drums should occasionally be ground off.

Factor of Safety. It is a generally accepted rule that the factor of safety should depend on the depth of the shaft. In shallow shafts there is not enough rope hanging in the shaft to take full advantage of the inherent stretch in the cable. For very deep shafts, the stretching of the rope safely overcomes shocks resulting from sudden application of the brakes, or starting with excessive slack. The values in Table 17 from a U.S. Bureau of Mines[1] report have been quite commonly accepted as containing minimum safe values. Some companies specify even greater factors.

Table 17 applies to men only, no restriction or recommendation being offered for the hoisting of ore. A factor of safety of 3.5 to 4 is usually used for ore. Also bending stresses are disregarded if the values in Table 17 are used. If bending stresses are considered, a

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[1] Rules and Regulations for Metal Mines, *U.S. Mines Bull.* 75, p. 112.
RAYMOND, R. M., Safety Practice for Hoisting Ropes, *Trans. AIMME*, **68**, 171.

TABLE 17. FACTORS OF SAFETY FOR HOISTING ROPES FOR VARIOUS DEPTHS OF SHAFTS

Length of rope, ft.	Minimum safety factor for new rope	Minimum safety factor when rope must be discarded	Percentage reduction
500 or less	8	6.4	20
500 to 1000	7	5 8	17
1000 to 2000	6	5.0	16.5
2000 to 3000	5	4.3	14
3000 and over	4	3.6	10

minimum factor of safety of 3 may be used.[1] It would be better to use 4 or 5, as will be seen later.

Drums and sheaves with a smaller diameter than that recommended by the rope manufacturers should not be used. They give the bare minimum. If anything a higher ratio should be used. A recommended ratio is 60 to 100 times the diameter of the rope; the minimum value for smaller ropes and the maximum for the larger ropes.[2]

Table 18 gives permissible hoisting speed when the safety factors given in Table 17 are used. It is from the U.S. Bureau of Mines *Bulletin* previously quoted.

TABLE 18. PERMISSIBLE HOISTING SPEEDS WHEN SPECIFIED SAFETY FACTORS ARE USED

Length of Rope in Shaft, Ft	Maximum Hoisting Speed, Ft per Min
500 or less	1200
500–1000	1600
500–1500	2000
1500–2000	2000
2000–2500	2250
2500–3000	2500
3000–3500	2750
3500–4000	3000
4000–4500	3250
4500–5000	3500

Bending Stresses. There is a considerable difference of opinion as to just what extent the bending stresses should be considered. Many feel that its determination should be ignored, and an increase in the factor of safety be made to offset it. If the diameters of the sheaves and drums are made from 90 to 112 times the diameter of the rope the wear on the rope from bending is practically nil. The following formula[1] may be used for calculating the bending stress in a 6 by 19 rope.

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[1] KETCHUM, M. S., "The Design of Mine Structures," p. 26.

[2] RAYMOND, *loc. cit.*

$$S_b = 288{,}000 \times \frac{d^3}{D}$$

where S_b = total bending stress, lb
d = diameter of rope, in.
D = diameter of sheave, in., or drum if it is smaller than sheave

The bending stress thus determined is added to the other factors making up the working load for determining the rope size.[2]

Load. This refers to the hanging load in the shaft. It consists of the weight of the skip and contents, or if a cage is used, of the cage and car (or cars) with contents. A fairly close approximation of the weight of the skip or cage is necessary for determining the stresses in the headframe and for selecting the proper rope. In the case of a service cage, the maximum weight resulting from handling locomotives, drill steel, etc., must be known and the rope determined on this basis. Tables 37*a*, *b*, and *c* and 22 give information on the weight and capacity of various skips and cages.[3]

Weight of Rope. For determining the size of the rope only the portion hanging in the shaft is considered. This distance is measured from the sheave to the lowest point to which the rope is lowered. quite often the length of the rope used is made equivalent to the depth of the shaft without any serious error. Table 19 lists the three better grades of steel hoisting ropes.

Acceleration. Many engineers disregard the effect of acceleration stresses in selecting a rope. Actually its value in pounds should be calculated and added to the other forces acting on the rope. High acceleration sets up a considerable stress in the rope. The formula commonly used is

$$F = ma = \frac{W}{g} a \qquad \text{lb}$$

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[1] PEELE, *op. cit.*, Sec. 12, p. 24.

[2] BOOMSLITER, G. P., Acceleration Stresses in Wire Hoisting Rope, *Trans. AIMME*, **75**, 87.

[3] CALHOUN, *op. cit.*, Part II.

TABLE 19. STANDARD HOISTING ROPES*
6 Strands, 19 Wires to the Strand, 1 Hemp Core,

Diameter, in.	Approx. cir., in.	Approx. wt. per ft, lb	Extra strong cast steel		Plow steel		Special†	
			Bkrg. stress, tons	Cost‡ per foot	Bkrg. stress, tons	Cost‡ per foot	Bkrg. stress, tons	Cost‡ per foot
$2\frac{3}{4}$	$8\frac{5}{8}$	12.10	234	$2.55	256	$3.00	294	$3.45
$2\frac{1}{2}$	$7\frac{7}{8}$	10.00	195	2.10	214	2.50	246	2.80
$2\frac{1}{4}$	$7\frac{1}{8}$	8.10	160	1.70	176	2.00	202	2.50
$2\frac{1}{8}$	$6\frac{5}{8}$	7.22	143	1.52	157	1.79	181	2.15
2	$6\frac{1}{4}$	6.40	127	1.34	140	1.58	161	1.85
$1\frac{7}{8}$	$5\frac{3}{4}$	5.63	112	1.25	123	1.46	142	1.75
$1\frac{3}{4}$	$5\frac{1}{2}$	4.90	98	1.10	108	1.30	125	1.60
$1\frac{5}{8}$	5	4.23	85	0.94	94	1.08	108	1.30
$1\frac{1}{2}$	$4\frac{3}{4}$	3.60	72.5	0.80	80.5	0.93	92.5	1.10
$1\frac{3}{8}$	$4\frac{1}{4}$	3.03	61.5	0.68	68	0.79	78.5	0.90
$1\frac{1}{4}$	4	2.50	51	0.56	56.5	0.65	65	0.75
$1\frac{1}{8}$	$3\frac{1}{2}$	2.03	41.5	0.46	46	0.54	53	0.62
1	3	1.60	33	0.37	36.5	0.43	42	0.50
$\frac{7}{8}$	$2\frac{3}{4}$	1.23	25.4	0.29	28	0.345	32.2	0.39
$\frac{3}{4}$	$2\frac{1}{4}$	0.90	18.7	0.24	20.6	0.28	23.7	0.31
$\frac{5}{8}$	2	0.63	13.1	0.18	14.4	0.21	16.6	0.225
$\frac{9}{16}$	$1\frac{3}{4}$	0.51	10.6	0.1575	11.7	0.1825	13.5	0.19
$\frac{1}{2}$	$1\frac{1}{2}$	0.40	8.5	0.1375	9.4	0.16	10.8	0.17
$\frac{7}{16}$	$1\frac{1}{4}$	0.31	6.6	0.12	7.3	0.14	8.4	0.155
$\frac{3}{8}$	$1\frac{1}{8}$	0.28	5.0	0.11	5.5	0.13	6.3	0.145
$\frac{5}{16}$	1	0.16	3.5	0.1075	3.9	0.1225	4.5	0.135
$\frac{1}{4}$	$\frac{3}{4}$	0.10	2.3	0.105	2.5	0.12	2.9	0.13

* Taken from Roebling's Sons Catalogue, Pacific Coast ed., Jan. 1, 1930; and American Steel and Wire Company Catalogue, May, 1931.

† Roebling's "Blue Center" and American Steel and Wire Company's "Monitor Silver Strand."

‡ The costs of these ropes are prewar.

where F = force exerted on rope, lb
W = weight of hanging load plus weight of rope
a = acceleration, ft per sec^2
g = acceleration of gravity = 32.2 ft per sec^2

As an example, we shall select a rope to meet the following conditions:

Depth of shaft = 1000 ft
Weight of skip and contents = 10,000 lb

Acceleration = 6 ft per sec²

$$F = \frac{10{,}000}{32.2} \times 6 = 1863 \text{ lb}$$

$$10{,}000 + 1863 = 11{,}863 \text{ lb}$$

As the shaft is 1000 ft deep, the factor of safety from Table 17 is 7.

$$11{,}863 \times 7 = 83{,}041 \text{ lb}$$

Consulting Table 19, we find that the 1-in. special brand of rope has a breaking strength of 42 tons. The weight of the rope has not been considered, and therefore we must choose a larger size. The 1⅛-in. rope has a breaking strength of 53 tons, and weighs 2.03 lb per ft.

$$\begin{aligned} 1000 \times 2.03 &= 2030 \text{ lb} \\ F &= \frac{2030}{32.2} \times 6 = 378 \text{ lb} \\ 2030 + 378 &= 2408 \text{ lb} \\ 11{,}863 + 2408 &= 14{,}271 \text{ lb} \qquad \text{the total force to be allowed for} \\ 14{,}271 \times 7 &= 99{,}904 \text{ lb, or } 49.95 \text{ tons} \end{aligned}$$

which is safe.

If the bending stress is considered and a factor of safety of 4 used, the following is obtained. Trying a 1⅛-in. rope, the bending stress is

$$S_b = 288{,}000 \times \frac{1.125^3}{72} = 5697 \text{ lb}$$

where 72 = 1⅛ × 64 = the diameter of the sheave.

$$\begin{aligned} 11{,}863 + 2408 + 5697 &= 19{,}968 \text{ lb} \\ 19{,}968 \times 4 &= 79{,}872 \text{ lb, or } 39.9 \text{ tons} \end{aligned}$$

Or the factor of safety, if the 1⅛-in. rope were used, would be

$$\frac{53 \times 2000}{19.968} = 5.3$$

which would not warrant a reduction in the rope size.

To substantiate this statement, reference is made to experiments performed by Boomsliter[1] on a coal-mine hoisting rope and also in the laboratory. From these experiments he deduced the following formula (among others):

$$S = W(1.5 + 0.1f)$$

where S = maximum stress due to acceleration and weight
W = weight of skip plus load, or cage, car, and load
f = acceleration, ft per sec²

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[1] BOOMSLITER, *op. cit.*, p. 74.

The weight of the rope is not considered as it is taken care of in the formula. Substituting values, we find

$$S = 10{,}000(1.5 + 0.1 \times 6) = 21{,}000 \text{ lb}$$

An addition of 2.5 per cent of W is added for friction.

$$10{,}000 \times 2.5\% = 250 \text{ lb}$$

The total stress is

S	21,000 lb
Friction	250 lb
Bending	5,697 lb
Total	26,947 lb

The factor of safety is

$$\frac{53 \times 2000}{26{,}947} = 3.93$$

which is somewhat low, as Boomsliter uses a factor of safety of 4.

From the manner in which he measured the stresses in the rope under actual hoisting conditions it would seem that friction should not be considered. It would automatically be taken care of in the formula. If friction is left out, the factor of safety becomes 4. From this it appears that the method, $F = (W/g)a$, first followed is satisfactory as long as the factor-of-safety values recommended in Table 17 are adhered to, and as long as the acceleration values in Table 20 are not exceeded.[1]

Ketchum[2] uses the following formula for determining the stress due to starting the cage suddenly.

$$K = 2W + R + F$$

where K = stress at sheave at instant of picking up load, lb
W = gross load, lb
R = weight of rope, lb
F = friction, lb, equals $(W + R)f$, where f = 0.01 to 0.02 for vertical shafts and 0.02 to 0.04 for inclined shafts

Starting and Stopping Load. The stresses developed through suddenly starting the load, especially when slack is in the cable, are extraordinarily high; and their exact value cannot be determined. Danger from starting with slack in the rope is far greater for shallow shafts than for deep ones when most of the rope is out. Table 17 to a certain extent takes care of starting conditions. The best remedy is to eliminate the practice of too sudden starting and of starting with

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[1] "Rules and Regulations for Metal Mines," p. 113.

[2] KETCHUM, M. S., "The Design of Mine Structures," p. 25.

TABLE 20. RATES OF ACCELERATION FOR GIVEN HOISTING SPEEDS WHEN FACTORS OF SAFETY IN TABLE 17 ARE USED

Speed, ft. per min.	Maximum acceleration, ft. per sec. per sec.	Minimum time of acceleration in which to attain speed corresponding thereto, sec.
500 or less	4.16	2
750	4.16	3
1000	5.55	3
1250	5.95	3½
1500	6.25	4
2000	8.33	4
2500	8.33	5
3000	8.33	6
3500	8.33	7

slack. If the duty cycle is correctly figured out, and the hoist properly powered, there is no occasion for excessive abuse of the rope.

Conditions under Which the Rope Is Used. Extra rope size may sometimes be called for because of the acid conditions of the mine air, which in many cases rapidly causes deterioration. This can, within limits, be prevented by properly coating the rope with a preservative. A somewhat larger rope size would, as a rule, be desirable to combat such deterioration, rather than the calculated size with a resulting greater frequency of rope changes. Replacing the rope means not only delays in making the change, but also delays and bother spotting the cage or skip at the desired point until the rope has adjusted itself. This sometimes runs over a period closely approaching the life of the rope.

Sheave and Drum Size. The diameter of sheave and drums should not be less than the minimum specified by rope manufacturers. For the large-sized ropes, the maximum values should be used. When at all possible, the sheave and the drum should be the same diameter. This will give the same bending stress for each. Because of a change in the direction of the bending action, underwound ropes have a shorter life than overwound. To equalize the duration of the two, the mode of winding should be interchanged occasionally.[1] The ideal sheave would be one of such a large diameter that the bending would be gradual. If, however, too large a wheel is used, it does not come to rest quickly enough when the hoist is stopped. The result is that the sheave wears out its tread and the rope; also the inertia of a large sheave makes it slow in reacting to speed changes.

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[1] On the new main hoist of the Hecla Mining Company, Burke, Idaho, a 6 by 21 rope is used on the underwind drum, and a 6 by 19 rope on the overwind drum. This hoist has a continuously rated 2100-hp motor. *U.S. Bur. Mines Inf. Circ.* **6232** (1930).

The most desirable drum would be one that would wind all the rope required in one layer. There are numerous reasons why this cannot always be done. If the shaft is very deep, a drum to handle all the rope in one layer would have a very large inertia. This would demand a very high peak for starting, which in most cases would be prohibitive. We shall go into more detail on this subject in the section on hoist selection.

Weight of Cages, Skips, Cars, and Sheaves. For purposes of designing the headframe and selecting the rope, Tables 37*a*, *b*, *c*, and *d* may be consulted. Table 22 gives further information. In Table 21 is given the weight of sheaves.

DESIGN OF A HEADFRAME

We shall now take up the actual design of a simple headframe. The forces acting on the frame are

1. The dead load (weight of the frame plus sheaves)
2. The live load acting under various conditions of loading
3. The wind pressure acting horizontally

TABLE 21. HEAVY-DUTY SHEAVES FOR ROUND ROPE
Wellman Engineering Company

Diameter, ft.	Style of tread	Maximum diameter of rope, in.	Journals		Weight, lb.	
			Diameter, in.	Length, in.	Sheave only	Sheave, shaft, and two flat boxes
2	Cast	1	2⁷⁄₁₆	7	250	370
3	Cast	1	3½	8	350	607
4	Cast	1	4	8	600	961
5	Wrought	1	4½	8	1,050	1,467
6	Wrought	1	4½	10	1,500	1,973
7	Wrought	1⅛	5½	10	2,000	2,764
8	Wrought	1¼	6	12	2,700	3,914
10	Wrought	1⅜	7	13	3,500	5,160
12	Wrought	1½	8	13	6,100	8,070
14	Wrought	1¾	9	16	8,600	11,580

The stresses in the headframe result from

1. The location of the hoist
2. The dimensions of the headframe
3. The weight of the load hoisted (skip, contents and rope)
4. The wind pressure
5. The design of the headframe itself

Dead Loads. The dead load consists of the weight of the headframe, sheaves, ore bins and contents, etc. Only the headframe and sheaves will be considered. It is customary to estimate the weight of the headframe from some similar structure. After completing the design the weight is calculated; if it differs radically from the assumed weight, the problem is again worked out. Table 22 gives data from which the weight of a proposed structure may be estimated. Table 22 was compiled from the following sources: The first 24 shafts are in Butte, Montana.[1]

Live Loads. The live loads are caused, so far as we are concerned, by the hoisting operations. The minimum live load results from the two empty skips; the maximum when the skip sticks in the shaft and the rope breaks. The maximum live load will, of course, depend on the breaking strength of the rope. There is also an intermediate load, which is caused by the loaded skip. These three conditions are usually combined into three cases[2] for calculating live-load stresses. Some companies consider only the breaking strength of the rope. We shall consider the three cases given by Ketchum.

Wind Pressure. The wind is usually taken as acting horizontally with a pressure of 30 lb per sq ft. It should be estimated from information available on the locality in which the mine is situated. The framework is ordinarily assumed to be boxed in, so that the exposed area is found from the over-all dimensions of the headframe. If the headframe is not considered boxed in, 50 lb per sq ft is used. There are three directions in which the wind is considered as acting: against the front transverse frame; against the back transverse frame; and against the A-frame. The pressure on the panels is taken as being applied at the panel points.

Location of Hoist. The location of the hoist drums determines the position of the resultant. Theoretically, the ideal position of the resultant would be coincident with the back post. Actually, an element of risk would enter into such a design. There is more or less

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[1] The data are from Ore Hoisting in the Butte District, *Eng. Mining J.*, **129,** 444 (1930). The rest are from Ketchum's "The Design of Mine Structures," p. 114.

[2] KETCHUM, M. S., "The Design of Mine Structures," p. 43.

TABLE 22. GENERAL DATA FOR VARIOUS SHAFTS

Shaft	Weight, lb.		Ore capacity of skip, tons	Depth of shaft, ft.	Ore-hoisting speed, ft. per min.	Height of headframe, ft.	Weight of headframe, lb.	Diameter of sheave wheels, ft.	Size and type of hoisting rope in. × in.	Tons per day
	Cage	Skips								
Anaconda	4,000	8,500	5	2,800	2,200	58 82	74,700	10	7½ × ½ flat	
Badger State	4,000	10,500	7	3,500*	2,800	129 5	256,000	12	1⅞ dia.	
Belmont	4,000	10,500	7	3,400*	2,600	129 5	256,000	12	1⅞ dia.	
Belmont (old)	3,800	8,900	5	3,400	2,250	114	Timber	10	7½ × ½ flat	
Berkely	3,900	8,500	5	3,200	2,600	80	Timber	10	1¼ dia.	
Black Rock No. 1	3,800	10,000	10	2,800	2,600	152	250,000	10	1½ dia.	
Black Rock No. 3	3,800	10,000	10	2,800	2,600	152	250,000	10	1½ dia.	
Diamond	3,850	8,960	5	3,400	2,800	100	318,000	10	6 × ½ flat	
Elm Orlu	4,000	10,000	7	3,300	2,600	94	65,000	10	6 × ½ flat	
High Ore	3,850	8,960	5	3,400	2,600	100	292,000	10	6 × ½ flat	
Leonard	3,800	8,500	5	2,800	3,000	141	346,425	12	1½ dia.	
Mountain Consolidated	4,000	10,500	7	3,600*	2,800	129 5	256,000	12	1⅞ dia.	
Mountain Consolidated (old)	3,850	8,900	5	3,500	1,000	100	Timber	10	1¼ dia.	
Mountain View	4,000	11,000	5	2,600	2,200	80	183,000	7.5	7 × ½ flat	
Never Sweat	3,200	5,700	3.5	2,800	2,600	100	315,000	10	1¼ dia.	
Original	3,900	7,800	4	3,800	2,600	112	318,000	10	1½ dia.	
Orphan Girl	3,050	No skips	....	1,000	1,500	70	79,000	7	1 dia.	
Pennsylvania	3,500	8,500	5	3,400	2,800	100	315,000	12	1½ dia.	
Pittsmont	1,250	3,875	5	1,600	2,600	58 5	41,500	8	6 × ½ flat	
St. Lawrence	3,900	7,000	7	2,100	1,000	97	117,000	10	7 × ½ flat	
Speculator	2,100	11,000	5	2,800	2,200	50	42,200	10	1½ dia.	
Stewart	3,800	8,500	5	3,800	2,800	100	292,000	10	1½ dia.	
Tramway	3,850	8,500	5	2,800	2,800	100	315,000	12	1½ dia.	
West Colusa	3,500 2,200	No skips	. .	2,200	2,800	50	43,000	10	4 × ⅜ flat	
Elkton (Colo.)	.....	.....	7.6†	. ..	.....	55	....	5	3½ × ⅜ flat	
Cia. Minera De Penoles (Mex.)	.....	5,000	5	1,000	.....	90	80,000	7	1⅛ dia.	
Tonopah-Belmont (Nev.)	....	....	.....	1,420	1,000	75	63,000	7	1 dia.	
Copper Queen (Ariz.)		5,990	1 85	1,700	...	60	35,250	7	1¼	2000
Union Shaft (Nev.)	1,200	.....	2	2,000	1,000	50	42,000	7	1 dia.	500
Basin and Bay State (Mont.)	...	.....	.....	. .	.....	70	79,000	10	1¼ dia.	
Quincy Rock House #2 (Mich.)	....	10,000	168 cu. ft.	6,000 (57° incl.)	.....	119′ 3″	839,000	12	1½ dia.	2400

* Hoisting installation designed for 5100 ft.
† This is the working load.

whip to the rope; at the high point in this whipping of the rope the resultant would fall outside the back posts and only the weight of the headframe would prevent overturning. It is better, therefore, to design the frame so that the resultant of the overwind rope falls about one-third the length of the base inside the back posts. As a general rule, the height of the headframe should be determined first; then the hoist is placed such a distance from the shaft that the rope from the drum to the sheave forms an angle of about 45 deg with the horizontal (the back posts make an angle of about 30 deg with the front posts). This will cause only a moderate bending stress in the rope, provided that the sheave diameter is all right. Next the distance between the front and back posts is so chosen that the resultant falls as previously suggested. This ideal location cannot, of course, always be obtained. When there is a considerable length of rope between the drums and the sheaves, idling sheaves are necessary to support the rope. The idling towers should be spaced at unequal distances to prevent the vibration waves from coinciding.

Dimensions of the Headframe. A certain amount of information has already been presented on this topic, especially the height and depth (back post to front post). For a general discussion, the reader is referred to the article by F. L. Burr, previously mentioned. The width (front post to front post) is usually based on a batter of 1 to 2 in. per ft of length of the front post. An examination of a number of the headframes in the Butte district (see reference quoted) shows a ratio between height and width of minimum 1.6; maximum 3.0. Examples of erected headframes given by Ketchum show a minimum of 1.9 and a maximum of 3.3. In four- and six-post headframes the posts are quite often vertical. The width of the top is made sufficiently large to accommodate the sheaves and structural details and sometimes the extension of the guides. If for any reason the width at the bottom must be restricted to a definite amount, which is less than the maximum ratio given, the overturning effect of wind load combined with dead load should be investigated. For this purpose the total wind pressure is assumed as being a concentrated load acting at the center of the figure. The weight of the frame, sheaves, etc., resists this with a moment equal to the weight times one-half the width of the base.[1] If, because of unalterable conditions, the resultant falls outside the base of the headframe, anchor bolts and foundations must be designed to prevent overturning.

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[1] Peele (ed.), *op. cit.*, Sec. 12, p. 63.

The Design of the Headframe Itself. The design will affect the stresses essentially, depending on the type of headframe: A-type; four- or six-post type; or inclined shaft. In some cases exceptionally narrow or wide bases for the frame must be chosen. An unstable condition will result with the narrow base.

EXAMPLE

To illustrate the application of the foregoing discussion a headframe of the A-type will be designed to satisfy the following conditions.

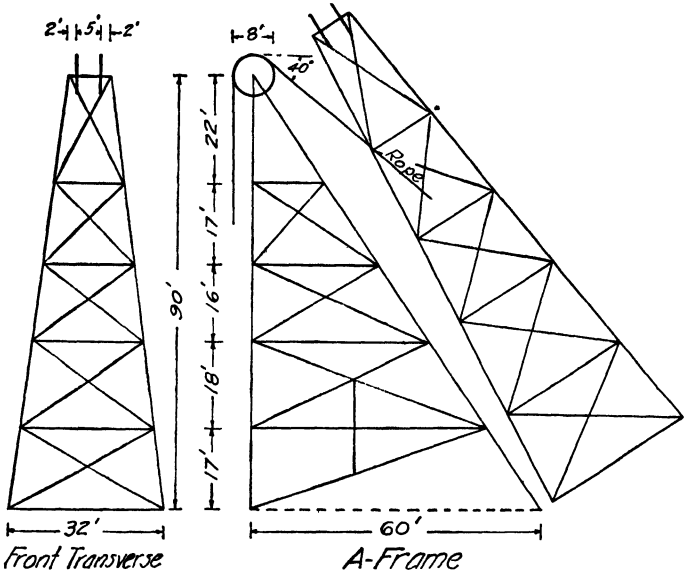

Fig. 45. General layout of headframe.

Depth of shaft = 1000 ft
Weight of skip = 5000 lb
Weight of ore = 10,000 lb
Acceleration = 3.2 ft per sec^2
Rope = 1¼ in. and weighs 2.5 lb per ft
Weight of headframe = 80,000 lb
Weight of one sheave and bearing boxes = 3030 lb
Diameter of sheave = 8 ft
Rope passing over sheave makes an angle of 40 deg with the horizontal.
Wind pressure is 30 lb per sq ft horizontally and the headframe is assumed boxed in
Dimensions and general layout shown in Fig. 45.

The headframe will be investigated for stresses caused by

1. Dead loads
2. Live loads
3. Wind loads

The design will consist of selecting front and back posts, bracing, sheave girders, and foundations.

Dead Load. The dead-load stresses result from the weight of the frame and sheaves. For a graphical solution, only half of the frame need be considered.

$$\text{Total load} = \frac{80{,}000}{2} + 3030 = 43{,}030 \text{ lb}$$

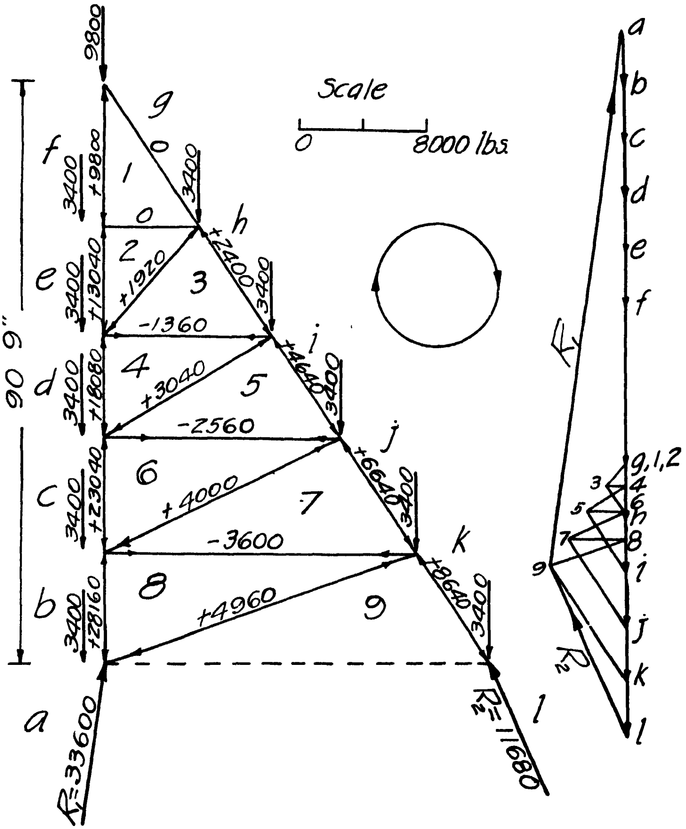

FIG. 46. Dead-load stresses in headframe.

This load is distributed at the panel points as follows: There are 12 panel points (two are at the apex),

$$\frac{40{,}000}{12} = 3333 \text{ lb, call it 3400, at each panel point}$$

At the apex the weight of one sheave wheel and fixtures must be added. We shall take the weight of the sheave, etc., as being 3000 lb.

$$\text{Apex load} = 2 \times 3400 + 3000 = 9800 \text{ lb}$$

Figure 46 shows the distribution of the load. It will be noted that the height of the frame used is the true length (or slant height) of the posts and not the vertical distance of 90 ft. Also it is assumed that only the A frame takes the dead load; actually the transverse frames take a small amount. Only one-half the bracing is shown in Fig. 46; otherwise the solution would be indeterminate. The designation of the forces is by means of letters, with the lettering beginning at the left and proceeding in clockwise order. The panel sections are shown with numbers.

For a graphical solution, a point must be selected at which no more than two unknown forces are acting. Inspection shows this to be the apex. At the apex the force *fg* is held in equilibrium by the two stresses 1-*f* and *g*-1. The load line is laid out as shown in the figure. The stress 1-*f* intersects *g*-1. From *f* in the stress diagram, 1-*f* is drawn parallel to 1-*f* in the figure; from *g*, *g*-1 is drawn to intersect 1-*f*. We find that *g*-1 has a zero stress. We now move to the panel point lying between *e* and *f*. Here the known forces and stresses are *ef* and 1-*f*. The unknown stresses are 1-2 and 2-*e*. The line 1-2 is drawn to intersect 2-*e*. Again it is found that one of the members (1-2) is zero. A moment's consideration will show this to be true. The front post is vertical and consequently is in a stable position. It does not need the upper section of the back post to hold it in equilibrium; so naturally there could be no stress in either *g*-1 or 1-2. Passing next to the panel point between *gh* it is found that the force *gh* and the stresses *g*-1 and 1-2 are known. Holding them in equilibrium are the two stresses *h*-3 and 3-2. The line *h*-3 is drawn to intersect 3-2, thus locating point 3. Panel point *de* is next in order. There are two unknowns, 3-4 and 4-*d*. The process is continued, alternating from one side to the next until point 9 is established. The reactions are 1-9 and 9-*a*. In the stress diagram 9 is connected to 1 and *a*. Their distances when scaled and multiplied by the factor used (8000 lb = 1 in.) give the magnitude of R_1 and R_2. We next determine the sense of the stresses. This is done in exactly the same manner as previously explained (page 10). The sense of the stresses about the panel point *hi* will be determined. Proceeding in a clockwise direction the force *hi* is toward the point; *i*-5 is toward the point (compression); 5-4 is toward the point (compression); 4-3 is away from the point (tension); and 3-*h* is toward the point (compression).

Live Loads. Before beginning the solution of the live loads, the selection of the hoisting rope must be made. For this purpose, the formula $F = W/g\ a$ will be used as discussed on page 112.

$$F = \frac{5000 + 10{,}000}{32.2} \times 3.2 = 1500 \text{ lb}$$

for accelerating the load.

Total pull due to skip, load, and acceleration is

$$15{,}000 + 1500 = 16{,}500 \text{ lb}$$

Since the shaft is 1000 ft deep, a factor of safety of 7 will be used (Table 17). A rope having an ultimate breaking strength of

$$7 \times 16{,}500 = 115{,}500 \text{ lb}$$

is indicated. A rope having a greater strength than this must be selected as the weight of the rope is still to be considered. From Table 19, a 1¼-in. rope is indicated. It has an ultimate strength of 130,000 lb, and weighs 2.5 lb per ft.

$$\text{Weight of rope} = (1000 + 90) \times 2.5 = 2725 \text{ lb}$$

$$F = \frac{2725}{32.2} \times 3.2 = 273 \text{ lb}$$

Total pull due to rope = 2725 + 273 = 2998 lb

The total pull on the rope is 16,500 + 2998 = 19,498 lb, which requires a rope with an ultimate strength of 7 × 19,498 = 136,486 lb.

The 1¼-in. rope gives a safety factor of

$$\frac{130{,}000}{19{,}498} = 6.67$$

which is all right.

The working load as found above is 19,498 lb. To simplify the calculations to follow, 19,500 lb will be used.

The live loads will be determined on the basis of three conditions of loading and breaking rope.[1] They are

Case 1. Both skips loaded

Case 2. One skip empty, one skip loaded, and one rope breaking

Case 3. Both ropes breaking

Case 2 is the condition under which the headframe would be most likely to fail from actual operating conditions (jamming of the ascending skip in the shaft). Many designers consider only case 3, which really is sufficient. The present problem will be solved for the three cases.

It is first necessary to draw the stress diagram involving the rope and to determine the value and direction of the resultant. Figure 47 shows the determination of the resultant, and Table 23 the values of the resultant for the various conditions.

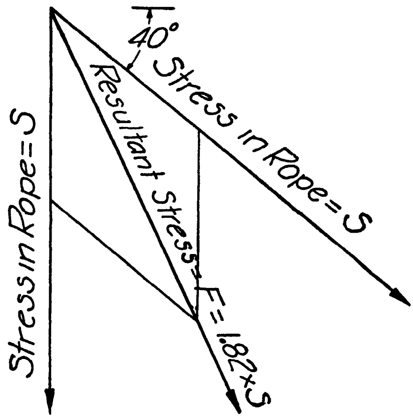

FIG. 47. Determination of resultant from stresses acting on rope.

TABLE 23. RESULTANT FORCES

Loading	S, lb.	F, lb.
Empty skip	5,000	9,100
Working load	19,500	35,490
Breaking load	130,000	236,600

The values of F are the forces acting at the sheaves. In order that we may determine the live-load stresses in the A frame and the transverse frames, F must be transferred to the ends of the sheave girder as reactions. This is done by taking moments about the ends of the girder. (The width of the top of the frame is assumed as being equal to the length of the sheave girder. No serious error results from such an assumption.) For this purpose the values of F are combined as suggested by the three cases. Figure 48 shows the beam under consideration.

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[1] KETCHUM, M. S., "The Design of Mine Structures," p. 49.

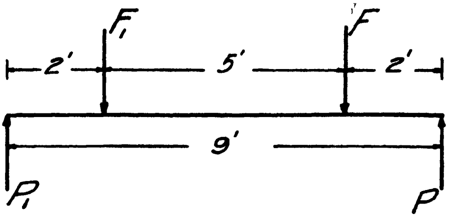

FIG. 48.

The forces are denoted by F_1 and F and the reactions by P_1 and P. Table 24 gives the values.

TABLE 24. FORCES ACTING ON A-FRAME
POUNDS

Case	F_1	F	P_1	P	E_1	E
1	35,490	35,490	35,490	35,490	35,770	35,770
2	9,100	236,600	59,655	186,044	60,150	187,550
3	236,600	236,600	236,600	236,600	238,510	238,510

P_1 and P are found by taking moments about one end of the beam on which the forces F_1 and F are acting. In case 1 the load is uniformly distributed, so that $P_1 = P = F_1 = F$.

Case 2. Taking moments about P_1,

$$\begin{aligned} 9100 \times 2 &= 18{,}200 \text{ ft-lb} \\ 236{,}600 \times 7 &= 1{,}656{,}200 \text{ ft-lb} \\ P \times 9 &= 1{,}674{,}400 \text{ ft-lb} \\ P &= 186{,}044 \text{ lb} \end{aligned}$$

Taking moments about P,

$$\begin{aligned} 236{,}600 \times 2 &= 473{,}200 \text{ ft-lb} \\ 9{,}100 \times 7 &= 63{,}700 \text{ ft-lb} \\ P_1 \times 9 &= 536{,}900 \text{ ft-lb} \\ P_1 &= 59{,}655 \text{ lb} \end{aligned}$$

Case 3. The loading is uniformly distributed so that $P_1 = P = F_1 = F$.

The reactions P_1 and P are vertical. They must be resolved into the plane of the A-frame thus giving the values for E_1 and E. Figure 49 shows how this is done.

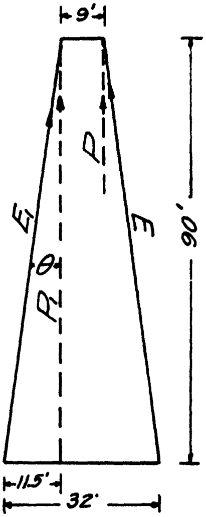

FIG. 49.

$$E_1 = \frac{P_1}{\cos\theta}$$

$$E = \frac{P}{\cos\theta}$$

$$\theta = \tan\frac{11.5}{90} = 7°16'$$

$$\cos 7°16' = 0.992$$

Case 1

$$E_1 = E = \frac{35{,}490}{0.992} = 35{,}770$$

Case 2

$$E_1 = \frac{59{,}655}{0.992} = 60{,}150$$

$$E = \frac{186{,}044}{0.992} = 187{,}500$$

Case 3

$$E_1 = E = \frac{236{,}600}{0.992} = 238{,}510$$

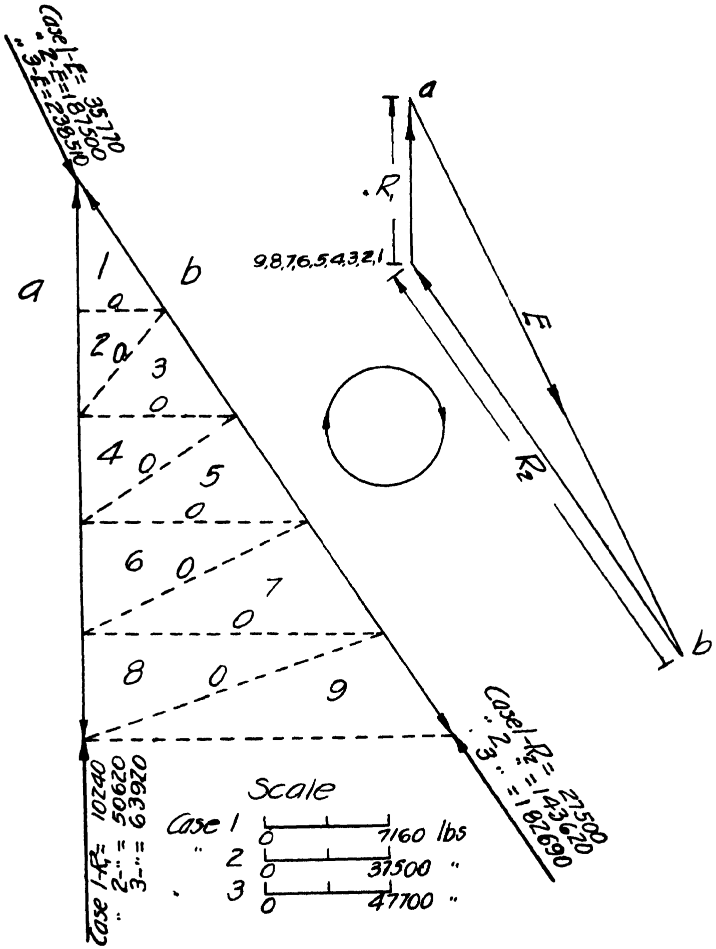

FIG. 50. Live-load stresses in A-frame.

Live-load Stresses in A-frame. Figure 50 shows the determination of the live-load stresses in the A-frame. Only the maximum values in each case are used. The direction of E is the same as F in Fig. 47. The load line ab is laid out to a convenient scale and all three cases determined by means of one stress diagram. It is found that the bracing takes none of the stresses.

Live-load Stresses in Transverse Frames. Before beginning the solution of the live-load stresses in the front and rear transverse frames, P_1 and P for case 2 must be resolved into the planes of the two transverse frames. This may be done trigonometrically (law of sines) or graphically as shown in Fig. 51. In the figure, H_1 and H are front-transverse-frame components and G_1 and G the rear-transverse-frame components. It is suggested that in the solution of the transverse frames the maximum components (H and G) be assumed as acting on the left-hand side of the frame, as shown in Figs. 52 and 53. Otherwise the stresses in the bracing will have their signs reversed from that shown. If this is not done, later calculations are made more complicated. It is not necessary to use all three cases in the above solution; case 2 involving the minimum and maximum loading is sufficient. It is found from Fig. 51 that

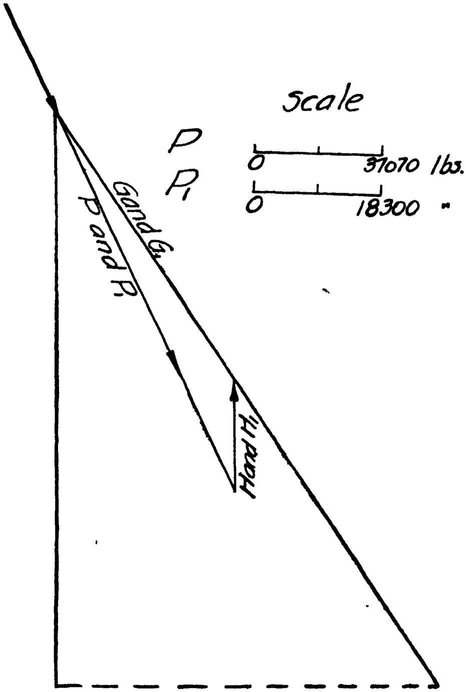

FIG. 51.

$$H_1 = 15{,}920 \text{ lb}$$
$$H = 48{,}950 \text{ lb}$$
$$G_1 = 45{,}750 \text{ lb}$$
$$G = 142{,}670 \text{ lb}$$

The solution of the front transverse frame (Fig. 52) will be briefly discussed; the procedure for the back frame is identical with the front. The load line *abc* is laid out to a suitable scale. By inspection we find that two unknown stresses, *b*-10 and 10-*a* hold the force *ab* in equilibrium. This is the point then at which the problem starts. From *a*, 10-*a* is drawn intersecting *b*-10. This establishes the point 10. Next the stress *b*-10 and the force *bc* are held in equilibrium by the two stresses *c*-11 and 11-10. The line 11-10 intersects *c*-11, thus giving point 11. The next step is to return to the other side of the frame where we find that the known stresses *a*-10 and 10-11 are held in equilibrium by the stresses 11-12 and 12-*a*, and thus the solution continues until the intersection at 19 is located. It is now found that the stress *c*-19 is held in equilibrium by the reaction *cd* and the stress *d*-19. The point *d* falls on the load line, dividing it into *cd* and *da* which gives the reactions R_1 and R_2, respectively. The sense of the stresses is determined as has been done before. In Fig. 53 is shown the back transverse.

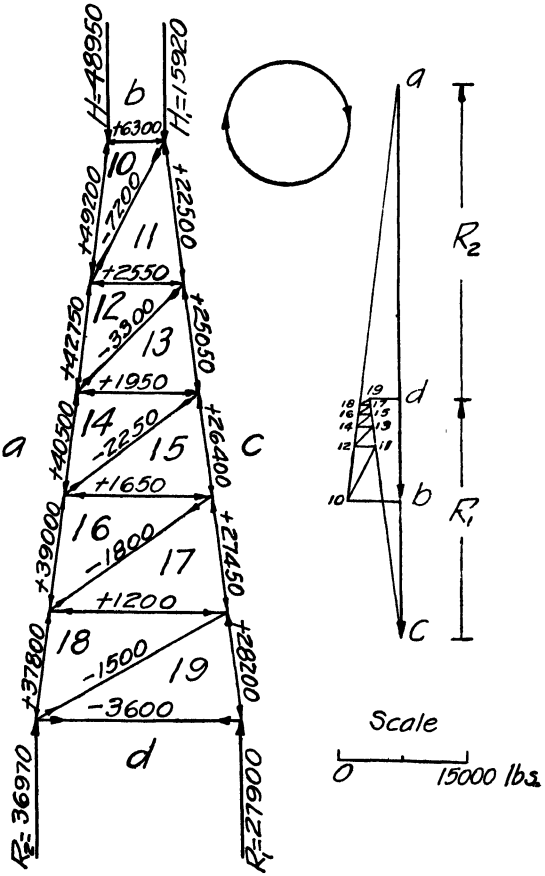

FIG. 52. Live-load stresses in front transverse frame.

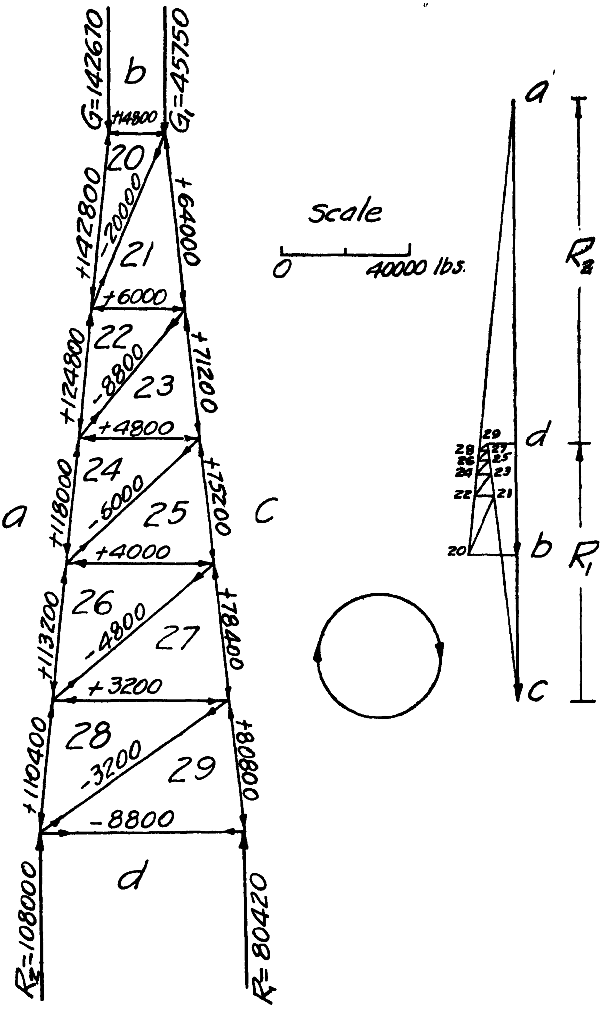

FIG. 53. Live-load stresses in back transverse frame.

Wind-load Stresses. The headframe is assumed to be boxed in for the purpose of determining the wind stresses. The boxing does not extend beyond the frame. A wind pressure of 30 lb per sq ft of vertical surface will be used. There are the following four cases:

Case 1. The wind strikes the A-frame, and stresses in the front transverse frame are determined.

Case 2. Same as case 1, except that stresses in the back transverse frame are found.

Case 3. The wind acts on the back transverse frame and the resulting stresses in the A-frame are determined.

Case 4. Same as case 3, but with the wind on the front transverse frame. The action of the wind is assumed to be acting at the panel points. It is first necessary to determine the areas of the various surfaces resisting the wind and the respective pressures thereon. We need consider but half the areas of the two transverse frames for determining the stresses in the A-frame (cases 3 and 4). A quick and sufficiently accurate way to do this is to draw the three views of the headframe to scale (Fig. 45) and scale the lengths of the members making up the trapezoids (panels). The same thing is done with the A-frame area, except that the entire area is used. The wind pressure on the back transverse is the same as on the front transverse because the vertically projected area of the two frames is equal. With respect to the transverse frames, one-half of the pressure on each panel is assumed as acting at the panel point. The top-panel point will then have a force acting on it equal to one-half the pressure on the panel immediately below it. The next panel point will have a force equal to half of the upper panel plus half of the next panel; the remainder of the forces are determined in a like manner.

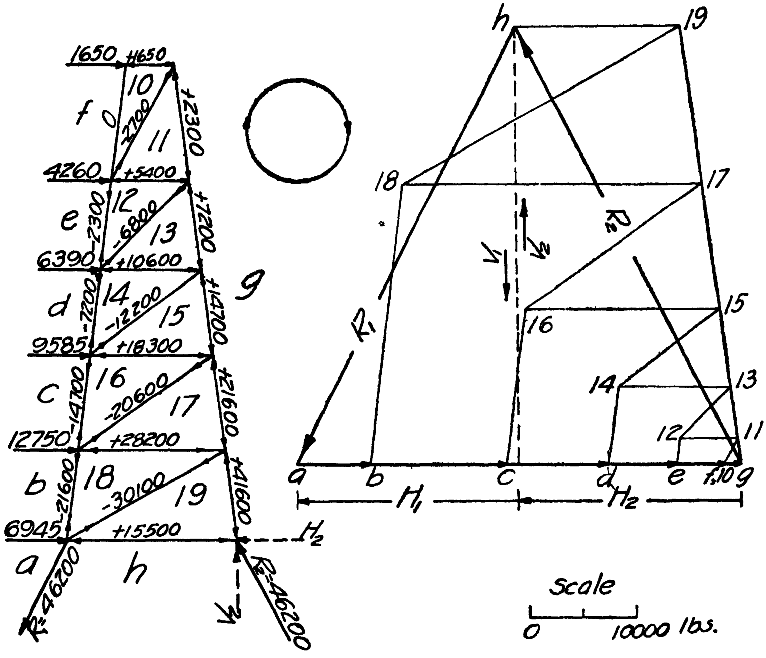

FIG. 54. Wind-load stresses in front transverse, case 1.

In the case of the A-frame, the upper panel is triangular and the force at the apex is one-third of the pressure. The force on the next lower panel point is one-third of the triangular area plus one-quarter of the next panel (there are four panel points supporting this area). In a like manner the remainder of the forces are found. It must be remembered that the forces on any given side of the frame are resisted by a part of the structure at right angles to this side. For example, the pressure acting on the transverse frames is resisted by the members making up the A-frame.

The solution for case 1 is shown in Fig. 54. An assumption must be made for the solution if the member 19-*h* is left in the structure, which is usually done. It will be recalled that in the knee-braced-truss problem the total wind pressure *H* on the side of the building was assumed to be resisted by *H*/2 acting on each column. We do the same thing here. The magnitude of the vertical and horizontal components of each reaction is assumed to be equal. If the member 19-*h* is left out the reactions can be determined without any assumption. The load line *abcdefg* is laid out in clockwise order, beginning with the force *ab* and ending with force *fg*. At the panel point *fg*, there are two unknown stresses, *g*-10 and 10-*f*. The line *g*-10 is drawn parallel to the stress *g*-10 and intersects 10-*f*. The point 10 is found to coincide with *f*, so that the stress in 10-*f* is zero. It is next found that the known stress 10-*g* is held in equilibrium by the stresses *g*-11 and 11-10. The intersection at 11 is readily found. At the panel point *ef* the force *ef* and stresses *f*-10 and 10-11 are known. The stresses 11-12 and 12-*e* are to be found. The solution is thus continued until the intersection at 19 is found. To determine *h*-19 and the reactions R_1 and R_2 the total force acting at the panel points is, according to the

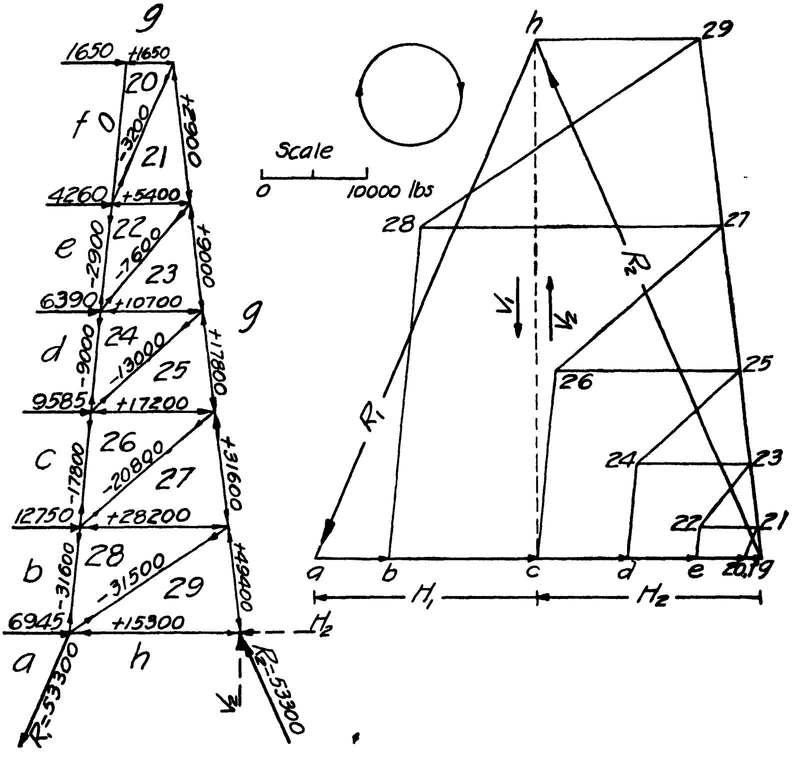

FIG. 55. Wind-load stresses in back transverse, case 2.

assumption previously made, held in equilibrium by half of this force acting at the base of each of the supports. The load line is bisected (thus giving two equal horizontal components) and a perpendicular (vertical components) erected to intersect *h*-19 drawn parallel to the member *h*-19. Point *h* is thus established. The two reactions are $gh = R_1$ and $ha = R_2$, and $R_1 = R_2$ in magnitude.

The stresses in the back transverse frame (case 2) are shown in Fig. 55.

The solution of case 3 now follows. It is illustrated in Fig. 56. The load line is laid out clockwise as is shown in the stress diagram. The determination of the stresses begins at the apex, or panel point *ab*. The known force *ab* is balanced by stresses *b*-1 and 1-*a*. Next we find that *a*-1 is in equilibrium with 1-2 and 2-*a*. The stress in 1-2 is zero. The stresses at panel point *bc* are found next. The solution is readily continued. There is no member connecting the bottom of the two posts, so that the reactions are determinable without any assumptions as were necessary with the transverse frames. The reactions are $g\text{-}9 = R_2$ and $9\text{-}a = R_1$.

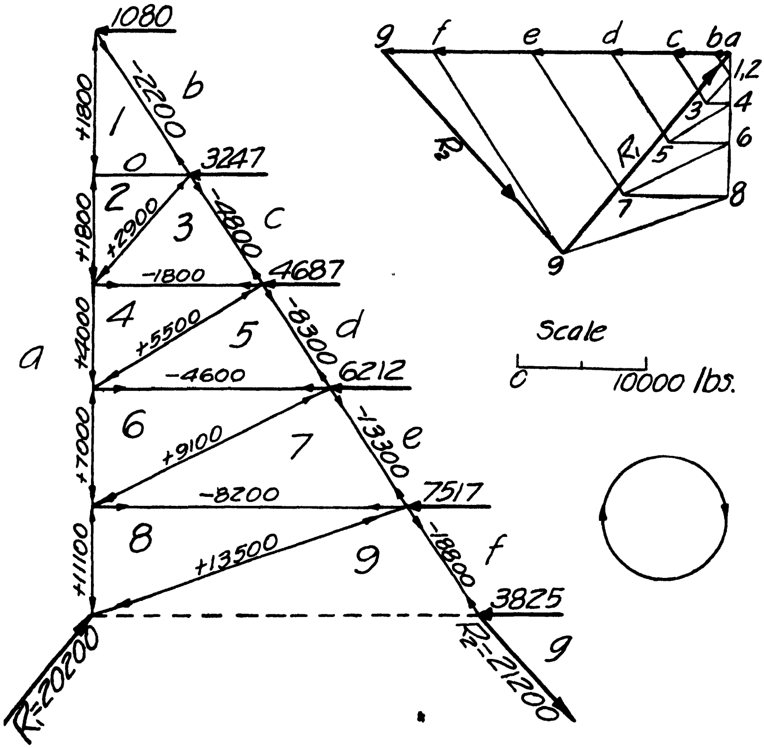

FIG. 56. Wind-load stresses in A-frame, case 3.

Figure 57 shows case 4 with the wind against the front of the A-frame. The same order and procedure are followed as for case 3.

Summary of Stresses. Table 25 gives a tabulation of the dead-load, live-load, and wind-load stresses for the various cases.

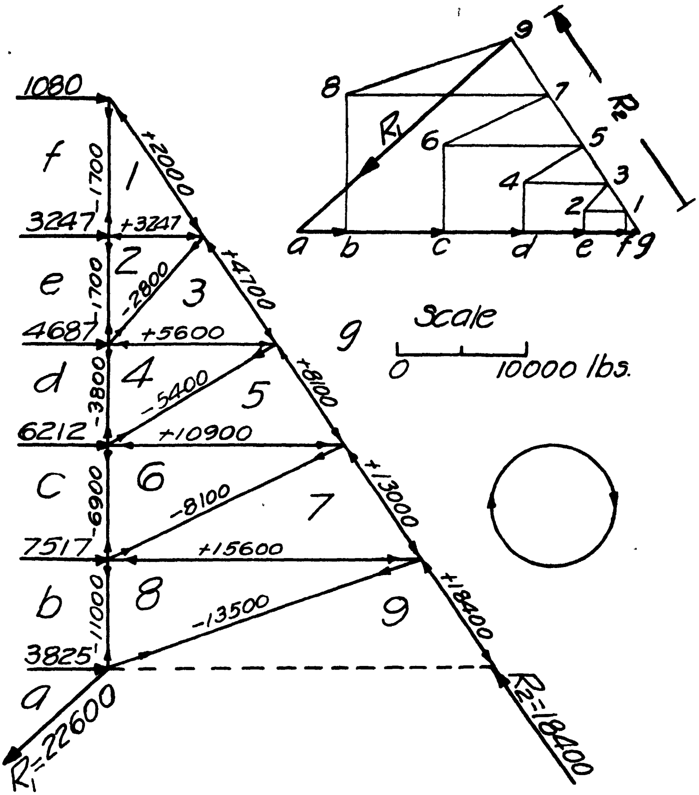

Fig. 57. Wind-load stresses in A-frame, case 4.

SELECTION OF MEMBERS FOR THE HEADFRAME

The allowable stresses for structural steel, from which the headframe is made, are

Tension = 18,000 lb per sq in.

$$\text{Compression} = 18{,}000 - 60\frac{l}{r}$$

The material used should not be less than $\frac{5}{16}$ in. thick.

The slenderness ratio l/r must not be greater than 120 for main members; for secondary members the ratio must not exceed 200.

Bearing stress in plates, shear in rivets, compression and shear in concrete, etc., are the same as for other steel structures.

COMBINATION OF STRESSES[1]

Working Load. The allowable stress is one-half the allowable stress for dead loads.

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[1] Ketchum, M. S., "The Design of Mine Structures," p. 106.
Peele, (ed.), *op. cit.* Sec. 12, p. 69.

TABLE 25. SUMMARY OF STRESSES IN HEADFRAME

		Dead-load stresses	Live-load stresses			Wind-load stresses			
			Case 1	Case 2; (*a* and *b*)	Case 3	Case 1	Case 2	Case 3	Case 4
Front posts		+28,160	+10,240	+ 50,620	+ 63,920	−21,600 +41,600	....	+11,100	−11,000
Back posts		+ 8,640	+27,500	+143,620	+182,690		−31,600 +49,400	−18,800	+18,400
Bracing	2–3	+ 1,920	0	0	0			+ 2,900	− 2,800
	4–5	+ 3,040	0	0	0			+ 5,500	− 5,400
	6–7	+ 4,000	0	0	0			+ 9,100	− 8,100
	8–9	+ 4,960	0	0	0			+13,500	−13,500
Strut	1–2	0	0	0	0			0	+ 3,247
	3–4	− 1,360	0	0	0			− 1,800	+ 5,600
	5–6	− 2,560	0	0	0			− 4,600	+10,900
	7–8	− 3,600	0	0	0			− 8,200	+15,600
Bracing	10–11			− 7,200		− 2,700			
	12–13			− 3,300		− 6,800			
	14–15			− 2,250		−12,200			
	16–17			− 1,800		−20,600			
	18–19			− 1,500		−30,100			
Strut	11–12			+ 2,550		+ 5,400			
	13–14			+ 1,950		+10,600			
	15–16			+ 1,650		+18,300			
	17–18			+ 1,200		+28,200			
	19–*d*			− 3,600		+15,500			
Bracing	20–21			− 20,000			− 3,200		
	22–23			− 8,800			− 7,600		
	24–25			− 6,000			−13,000		
	26–27			− 4,800			−20,800		
	28–29			− 3,200			−31,500		
Strut	21–22			+ 6,000			+ 5,400		
	23–24			+ 4,800			+10,700		
	25–26			+ 4,000			+17,200		
	27–28			+ 3,200			+28,200		
	29–*d*			− 8,800			+15,300		

Breaking Load. The allowable stress for breaking loads is 1½ times the allowable stress for dead loads.

Wind Load. The unit stress for wind-load stress + dead-load stress + 2 times the working-load stress must not exceed 20,000 lb per sq in. When the wind-load stress is greater than 25 per cent of the dead-load stress + 2 times the working-load stress, the area of the member must be increased so that the total unit stress is not more than 25 per cent above the dead-load stress + 2 times the working-load stress. Wind-load stresses are not combined with breaking-load stresses.

The preceding discussion and stress analysis has followed the lines suggested by Ketchum and others. The ultimate result is the effect that the assumption of the simultaneous breaking of both ropes will have on the final selection of the members. A condition whereby this could happen is possible; but it would have to be entirely deliberate and would require the assistance of others in addition to the hoistman. Consideration of such a remote possibility seems unnecessary. Because of the prevalence of the method in the past its application will be illustrated.

Not too far from the realm of possibility is the chance that one rope may be loaded when the other breaks. Also it is doubtful if any mine management would permit regular hoisting procedure if a wind of near hurricane intensity were blowing; hence the doubtfulness of including the wind load in the selection of members when the breaking strength of the rope is used.

Some engineering firms base the design on the breaking strength of the rope without considering the load on the other rope, the dead load, or the wind load. The importance of the wind load would depend on the locality and on whether the frame were actually boxed in. Stresses in struts and bracing (secondary members) are relatively small. If the l/r ratio is observed for compression members, and if for tension members sizes selected which will detail well with the posts, an excess of area beyond that required for wind-load assumptions will invariably result. In other words, if proper detailing is adherred to, there will be sufficient area even if the wind load is not considered.

The selection of members for the foregoing problem will be illustrated by both methods—one using all the stresses, and the other using the dead load and case 2, Table 24.

SELECTION OF MEMBERS

Method Using All the Stresses. *Front Posts.* The length of a section of the posts may be taken as $^{90}/_{5}$ = 18 ft = 216 in. The stresses are, from Table 25,

Dead load = +28,160 lb
Working load = +10,240 lb
Breaking load = 63,920 lb
Wind load = +41,600 lb
Wind load = −21,600 lb

The maximum stresses are taken; for instance, on the front post we have wind stresses resulting from three conditions (Cases 1, 3, and 4). Case 1 shows the greatest stress.

Before proceeding further, the make-up of the section must be decided upon. For the purpose of illustration, standard channels will be used. They will be

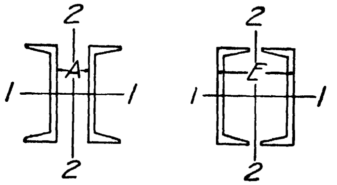

TABLE 26. SPACING OF CHANNELS FOR EQUAL MOMENTS OF INERTIA ABOUT TWO RECTANGULAR AXES 1-1 AND 2-2

Depth of channel, in.	Wt. per ft. of one channel, lb.	Area of section of one channel, sq. in.	*A*, in.	*E*, in.	Depth of channel, in.	Wt. per ft. of one channel, lb.	Area of section of one channel, sq. in.	*A*, in.	*E*, in.
3	4.00	1.19	1.29	3.05	8	18.75	5.51	4 37	6 65
3	5.00	1.47	1 17	2.93	8	21.25	6.25	4 22	6 58
3	6.00	1.76	1.10	2.94					
					9	13.25	3.89	5 62	8 06
4	5.25	1.55	2 08	3.92	9	15 00	4.41	5 48	7 84
4	6.25	1.84	1 96	3 80	9	20 00	5 88	5 14	7 46
4	7.25	2.13	1.88	3.72	9	25.00	7.35	4 83	7 31
5	6.50	1.95	2 79	4 75	10	15.00	4.46	6.33	8 89
5	9.50	2.65	2.57	4.49	10	20 00	5.88	5.96	8 40
5	11.50	3.38	2.35	4.39	10	25.00	7.35	5 66	8.14
					10	30 00	8 82	5.41	8.01
6	8.00	2.38	3.51	5.59	10	35.00	10.29	5.18	7.94
6	10.50	3.09	3.29	5.29					
6	13.00	3.82	3.08	5.16	12	20.50	6.03	7 68	10.48
6	15.50	4.56	2.90	5.10	12	25 00	7.35	7 35	10.07
					12	30.00	8 82	7.06	9 78
7	9.75	2.85	4.21	6.41	12	35.00	10 29	6 83	9.59
7	12.25	3.60	4.00	6.12	12	40.00	11.76	6.60	9.48
7	14.75	4.34	3.82	5 94					
7	17.25	5.07	3.65	5.85	15	33.00	9.90	9.51	12.67
7	19.75	5.81	3.49	5.81	15	35.00	10.29	9 42	12.58
					15	40 00	11.76	9 16	12.28
8	11.25	3.35	4 92	7.24	15	45.00	13 24	8.92	12 08
8	13.75	4.04	4.72	6.96	15	50.00	14.71	8 72	11 92
8	16.25	4.78	4.53	6.77	15	55.00	16.18	8.53	11.81

placed back to back and laced on both sides (see Fig 29a). The minimum radius of gyration allowed is $216/r = 120$; $r = 1.80$ in. On consulting a steel handbook, a number of channels are found that might suit the minimum requirements. As ¾-in.-diameter rivets are to be used, it must be kept in mind that a 2¼-in. flange is the minimum. Also the minimum thickness is $\frac{5}{16}$ in. (0.312 in.). With these conditions in mind a 6-in. channel at 15.5 lb would appear suitable. Further inspection of the tables suggests a 7-in. channel at 14.75 lb. A slight increase in flange width is obtained and ¾ lb per ft is saved. This selection will be tried. The radius of gyration about xx is 2.51 in.; and the area of one channel is 4.32 sq in. From Table 26[1], if two 7-in. channels at 14.75 lb are spaced 3.82 in. apart back to back, the moment of inertia, and consequently the radii of gyration, about both rectangular axes are equal. If they are moved farther apart, the radius of gyration about 1-1 (xx) does not change; that about 2-2 increases. The 7-in. channels selected will be spaced 7 in. apart back to back. The area for both channels is $2 \times 4.32 = 8.64$ sq in. The allowable unit stress in compression is

$$P = 18{,}000 - 60\frac{l}{r} = 18{,}000 - 60 \times \frac{216}{2.51} = 12{,}840 \text{ lb per sq in.}$$

Area for dead load $= 28{,}160 \div 12{,}840 = 2.19$ sq in.
Area for working load $= 10{,}240 \div (12{,}840 \times \frac{1}{2}) = 1.59$ sq in.

The stress for wind-load compression is 41,600 lb, which is far more than 25 per cent of $28{,}160 + 2 \times 10{,}240 = 48{,}640$ lb, and therefore area must be provided for the wind stresses.

Area for wind-load, dead-load, and working-load stresses is

$$(28{,}160 + 2 \times 10{,}240 + 41{,}600) \div 1\tfrac{1}{4} \times 12{,}840 = 5.62 \text{ sq in.}$$

As the wind load in tension is less than the dead load in compression, it need not be considered.

$$\text{Area for breaking load} = 63{,}920 \div 1\tfrac{1}{2} \times 12{,}840 = 3.31 \text{ sq in.}$$

Combination of areas. The areas are combined as follows:

Area for dead load and working load $= 2.19 + 1.59 = 3.78$ sq in.
Area for dead load, working load, and wind load $= 5.62$ sq in.
Area for dead load and breaking load $= 2.19 + 3.31 = 5.50$ sq in.

The area of the section, two 7-in. channels at 14.75 lb, is 8.64 sq in.; therefore the selection is satisfactory.

Two 8-in. channels at 13.75 lb would be satisfactory, except that the web thickness is a little less than the $\frac{5}{16}$-in. minimum.

The front posts will be made of two 7-in. channels at 14.75 lb spaced 7 in. back to back and laced together.

In selecting the back posts, the same procedure is followed. Under certain conditions it might develop that the back-to-back spacing for the purpose of ensuring a least radius of gyration about 1-1 would be larger than that already selected for the front posts. If this happens, the spacing of the front-post channels is simply increased to match the back posts. Two 9-in. channels at 20 lb meet the requirements. They are a little larger than is actually required but will detail better with the back-post sheave girders.

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[1] "Cambria Steel Handbook," 7th ed., p. 219.

Strut 7-8. The struts are secondary members; therefore the l/r ratio may be as great as 200. The length of the member is 48 ft 6 in. = 582 in.; minimum $r = {}^{582}\!/_{200} = 2.96$ in.

Dead load = -3600 lb
Wind load = $+15{,}600$ lb
Wind load = -8200 lb

The wind load in compression is greater than the dead load in tension; the member must be designed for the difference. An 8-in. channel at 13.75 lb has $r = 2.99$ in.; the web thickness is a little under the minimum. Either a 9-in. channel at 20 lb or a 10-in. channel at 20 lb can be used. Because of the length of the member, the 10-in. channel is the better choice. The radius of gyration is 3.66 in.; the area of two channels is 11.72 sq in., and the web thickness is 0.379 in. These channels may be spaced 7 in. without shifting the least radius of gyration to the 2-2 axis.

$$\text{Compressive stress} = 15{,}600 - 3600 = 12{,}000 \text{ lb}$$

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{582}{3.66} = 8450 \text{ lb per sq in.}$$

Area for dead-load tension and wind-load compression is

$$12{,}000 \div 1\tfrac{1}{4} \times 8450 = 1.14 \text{ sq in.}$$

Area for dead-load tension and wind-load tension is

$$(-3600 - 8200) \div 1\tfrac{1}{4} \times 18{,}000 = 0.52 \text{ sq in.}$$

The area of the rivet holes must be deducted from the gross area (11.72 sq in.) Using $\frac{3}{8}$-in gusset plates and $\frac{3}{4}$-in. rivets (field driven),

Rivets in single shear = $\left(\frac{3}{4}\right)^2 \times \frac{\pi}{4} \times 10{,}000 = 4430$ lb
Gusset plate = $\frac{3}{8} \times \frac{3}{4} \times 16{,}000 = 4500$ lb
Front-post channel web = $\frac{3}{4} \times 0.419 \times 16{,}000 = 5040$ lb
Back-post channel web = $\frac{3}{4} \times 0.379 \times 16{,}000 = 4550$ lb
Strut channel web = $\frac{3}{4} \times 0.379 \times 16{,}000 = 4550$ lb

The critical section is rivets in single shear.

Total tension = $-3600 + (-8200) = -11{,}800$ lb
Minimum number of rivets = $11{,}800 \div 4430 = 2.17$, or 3

Two rows of rivet holes will be deducted from each channel.

Area of one hole = $(\frac{3}{4} + \frac{1}{8}) \times 0.379 = 0.332$ sq in.
Area for four holes = $4 \times 0.332 = 1.328$ sq in.
Net area = $11.72 - (1.33 + 1.14) = 9.25$ sq in.

This is excessive, but because of the l/r ratio the channels chosen are used.

The remaining struts and bracing are found in a like manner. When the actual detailing of the structure is undertaken, it will be found that many more than the minimum number of rivets found by calculations will be needed.

Selection of Members Using Dead Load and Case 2. *Front Posts.* The stresses are

Dead load = +28,160 lb
Live load = +50,620 lb

The length is 216 in.; the least $r = {}^{216}/_{120} = 1.80$ in.

For the illustrations to follow, wide flange or *CB* sections will be used.

An 8- by 8-in. at 35 lb has a web thickness of 0.315 in.; the least *r* about *yy* is 2.03 in.; the area is 10.30 sq in. The actual depth of the section is 8.12 in., and the width of the flange is 8.027 in.

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{216}{2.03} = 11{,}620 \text{ lb per sq in.}$$

Dead-load area = 28,160 ÷ 11,620 = 2.42 sq in.
Live-load area = 50,620 ÷ 11,620 = 4.35 sq in.
Total area = 2.42 + 4.35 = 6.77 sq in.

While this section meets the requirements, it will not detail with the struts and bracing as found later. Therefore, a 12- by 8-in. at 45 lb will be used. The area is 13.24 sq in.; $r = 1.94$ in.

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{216}{1.94} = 6670 \text{ lb per sq in.}$$

This selection provides sufficient area.

Back Posts. The stresses are

Dead load = +8,640 lb
Live load = +143,620 lb

The length of the section is 108.5/5 × 12 = 260 in., the least $r = {}^{260}/_{120} = 2.18$ in.

A 10- by 10-in. at 49 lb has an area of 14.40 sq in. and $r = 2.54$ in. But it will not detail with the rest of the structure. A 12- by 10-in. at 53 lb will be used. The area is 15.59 sq in.; $r = 2.48$ in.

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{260}{2.48} = 11{,}710 \text{ lb per sq in.}$$

Area for dead load = 8642 ÷ 11,710 = 0.74 sq in.
Area for live load = 143,620 ÷ 11,710 = 12.27 sq in.
Total area = 0.74 + 12.27 = 13.01 sq in.

Strut 7-8. The length is 48 ft. 6 in. = 582 in.; least $r = {}^{582}/_{200} = 2.96$ in. The member is designed for −3600-lb dead load, and, as a column, with a least radius of gyration as near 2.96 as possible. The most economical selection might prove to be channels, as two 10-in. channels at 20 lb would weigh but 40 lb per ft; but the cost of lacing or battening them must be considered. The nearest wide flange section that will detail with the posts is a 12- by 12-in. at 65 lb.; $r = 3.02$ in.

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{582}{3.02} = 6420 \text{ lb per sq in.}$$

Assuming a maximum of seven rows of rivets after detailing,

$$7 \times 0.332 = 2.32 \text{ sq in.}$$
$$\text{Net area} = 19.11 - 2.32 = 16.79 \text{ sq in.}$$
$$P = 16.79 \times 6420 = 108{,}000 \text{ lb}$$

This far exceeds any wind load that might occur.

Lateral bracing in the horizontal plane, including struts 7-8 on the A-frames and struts 17-18 and 27-28 of the transverse frames, should be provided. A wide flange 12 by 6½-in. at 36 lb will connect struts 7-8 at their mid-points. Diagonal bracing, consisting of two 4- by- 3 by 5⁄16-in. angles with the long legs vertical, will connect the ends of the 12- by 6½-in. section with each post. The same type of bracing should be used in the plane of strut 5-6.

The remaining struts and bracing are found in a similar manner; the fact that they must detail with each other and with the posts must be kept in mind.

SHEAVE GIRDERS

The sheaves are supported by diaphragms, which in turn rest on the sheave girders. The sheave girders are heavy members that run from front post to front post and from back post to back post. A precise determination of the sheave girders can be made only if the details of the sheave supports, spacing, etc., are known. The method that follows is sufficiently accurate, although none of the above details are known. Sheave girders are made of rolled shapes or built-up plate-and-angle girders. If rolled shapes are used, the choice usually lies between I beams and channels. The latter are most suitable as two of them may be used spaced apart; this gives a more stable beam and permits simpler connections to the posts.

The forces F_1 and F, Table 27, act on the sheaves in the direction of the resultant between the ropes. They must be resolved into components acting in the plane of the back posts and in the plane of the front posts. This can be done graphically as shown in Fig. 58. Table 27 gives the values of these components in the front and back posts.

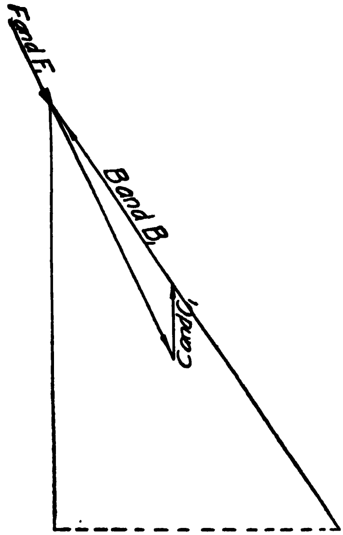

FIG. 58.

The sheave girders perform the function of a beam, so the stress to be calculated is the bending moment. If a built-up section must be used, shear should be determined. Figure 48 shows the loading, except that the components determined in Table 27 replace F_1 and F; and R_1 and R_2 replace P_1 and P. Only the maximum conditions are considered; they are case 1 and case 3, working loads and breaking loads, respectively.

Two solutions will be made: (1), using case 1 and case 3, and (2) using only case 2. As has been suggested, the simultaneous breaking of both ropes is very unlikely.

First Solution. *Front-post Girder.* For the working load, case 1, the bending moment, $M_1 = 10{,}115 \times 2 \times 12 = 242{,}760$ in.-lb.

TABLE 27. SHEAVE-GIRDER LOADS

Case	F_1	F	Front posts		Back posts	
			C_1	C	B_1	B
1	35,490	35,490	10,115	10,115	26,795	26,795
2	9,100	236,600	2,594	67,431	6,871	178,633
3	236,600	236,600	67,431	67,431	178,633	178,633

For case 3, $M_3 = 67{,}431 \times 2 \times 12 = 1{,}618{,}344$ in.-lb.

From strength of materials,

$$M = \frac{SI}{c}$$

where M = bending moment, in. -lb
S = allowable unit stress
c = distance from neutral axis (center of gravity) to extreme outer fiber (edge of surface)
I = moment of inertia of shape used

As a trial, two 15-in. channels at 33.9 lb are selected. $I = 312.6$ in.^{4}· $S = 18{,}000$ lb per sq in.; $c = \frac{15}{2} = 7.5$ in.

$$M = \frac{18{,}000 \times 312.6 \times 2}{7.5} = 1{,}500{,}000 \text{ in.-lb}$$

which is large enough for the working load. Breaking loads are based on 1½ times the allowable unit stress for working loads.

$$\text{Breaking load} = 1\tfrac{1}{2} \times 1{,}500{,}000 = 2{,}250{,}000 \text{ in.-lb}$$

which is more than necessary. It might appear that the excess is too great. An inspection of the handbooks available showed that no 12-in. channel has a sufficiently large moment of inertia. To build up a section would cost more than the one selected. An I beam could be used, but it will not detail as well. Therefore, for the front post girder we shall use two 15-in. channels at 33.9 lb back to back.

Back-post Girder. Case 1, working load,

$$M_1 = 26{,}795 \times 2 \times 12 = 643{,}080 \text{ in.-lb}$$

Case 3, breaking load,

$$M_3 = 178{,}633 \times 2 \times 12 = 4{,}287{,}192 \text{ in.-lb}$$

The largest standard channel shown in the handbooks available is 15 in., which is far too small. It will be necessary to build up a section (plate girder) of angles

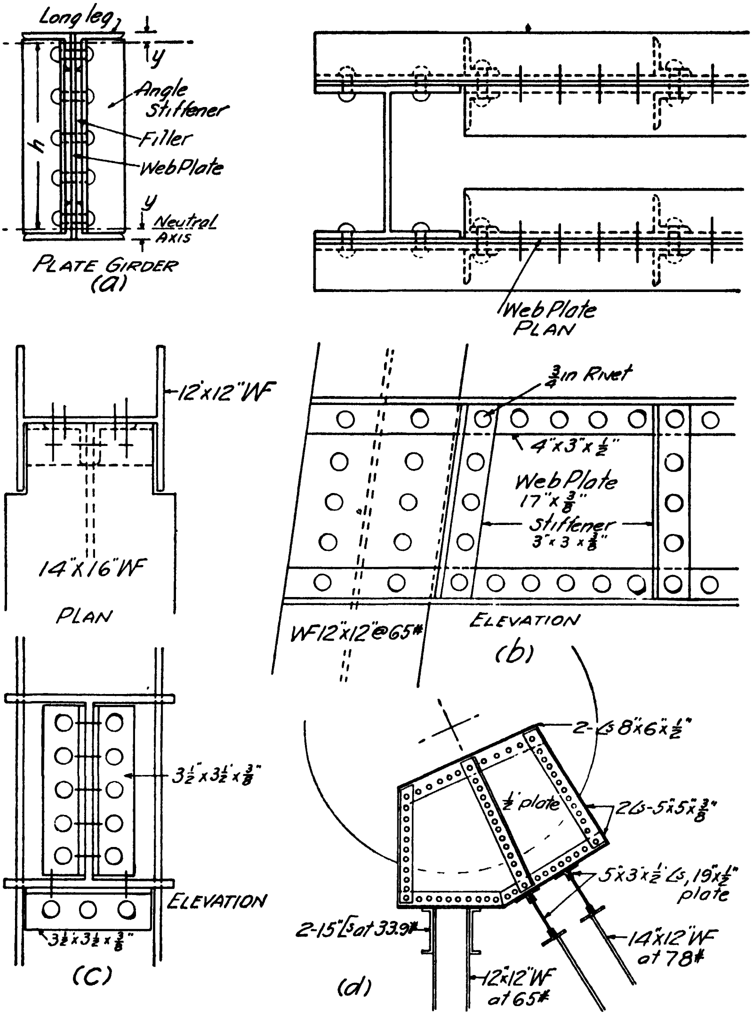

FIG. 59. Headframe details.

and a plate. Two of these girders will be used. Figure 59*a* shows a sketch of a plate-and-angle girder. The bending moment of a plate-and-angle girder (exclusive of filler and stiffeners) is given by

$$M = hfF$$

where M = bending moment, in.-lb
F = area of one flange (two angles)
f = allowable stress in flanges = 18,000 lb per sq in.
h = distance between centers of gravity of angles, in.

This gives the stress in tension; compression is taken with the same value.

For a first trial, a section made up of 4- by 3- by ½-in. angles and a 20- by ⅜-in. web plate will be investigated. The angles are placed with the long legs out. Referring to Fig. 59*a* and consulting a handbook, the area of two 4- by 3- by ½- in. angles is 6.50 sq in., and $y = 0.83$ in.

$$h = 20 - (2 \times 0.83) = 18.34 \text{ in.}$$

For riveting the angles to the web plate, ¾-in. rivets will be used. The area of the section made by the hole must be deducted from each angle.

$$\text{Net area} = 6.50 - 2(\tfrac{1}{2} \times \tfrac{3}{4} + \tfrac{1}{8}) = 5.62 \text{ sq in.}$$

Substituting the values found, in $M = hfF$,

$$M_1 = 18{,}000 \times 5.62 \times 18.34 \times 2 = 3{,}710{,}000 \text{ in.-lb}$$

which is large enough for working loads. For breaking loads,

$$M_3 = 1\tfrac{1}{2} \times 3{,}710{,}000 = 5{,}570{,}000 \text{ in.-lb}$$

It is considerably in excess of that needed. Before changing the size of the member, the shear should be determined. Only the web plate resists shear. The stiffener will be held by one rivet at each pair of angles and by three equally spaced rivets between the end rivets. This gives a total of five rivet holes in the web plate.

$$\text{Gross area of web plate} = 20 \times \tfrac{3}{8} = 7.50 \text{ sq in.}$$

The net area, after deducting for five ¾-in. rivet holes, is

$$7.50\ (5 \times 0.33) = 5.85 \text{ sq in.}$$

The actual shear for breaking loads is

$$178{,}633 \div (5.85 \times 2) = 15{,}250 \text{ lb per sq in.}$$

The allowable shear on the net section of the web plate is

$$1\tfrac{1}{2} \times 13{,}000 = 19{,}500 \text{ lb per sq in.}$$

which shows that the web plate is too large.

The suitability of a 17- by ⅜-in. plate will be checked.

$$\text{Net area} = (178{,}633 \div 2) \div 19{,}500 = 4.57 \text{ sq in.}$$
$$\text{Gross area} = 4.57 + 1.65 = 6.22 \text{ sq in.}$$
$$\text{Width of plate} = 6.22 \div \tfrac{3}{8} = 16.6 \text{ in.}$$

A 17- by ⅜-in. plate is satisfactory.

Under each sheave diaphragm and at each end of the girder, 3½- by 3- by ⅜-in. angles are used for stiffeners.

The back-post sheave girders will consist of two plate girders, each of which is made of four 4- by 3- by ½-in. angles, with the long legs out, with a 17- by ⅜-in. web plate.

Girders Using Case 2. *Front Posts.* The reactions at the ends of the girder are

$$R_1 = \frac{2594 \times 7 + 67{,}431 \times 2}{9} = 17{,}000 \text{ lb}$$

$$R = \frac{2594 \times 2 + 67{,}431 \times 7}{9} = 53{,}025 \text{ lb}$$

The maximum moment is under the 67,431-lb load.

$$M = 53{,}025 \times 2 \times 12 = 1{,}280{,}000 \text{ in.-lb}$$

$$M = \frac{sI}{c} \quad \text{or} \quad \frac{M}{s} = \frac{I}{c}$$

$$\frac{I}{c} = 1{,}280{,}000 \div 18{,}000 = 71.2 \text{ in.}^3$$

For two channels, $I/c = \frac{1}{2} \times 71.2 = 35.6$ in.^{3}

From the handbook, the nearest selection is a 15-in. channel at 33.9 lb. The moment of inertia = 312.6 in.^{4}, and $c = \frac{1}{2} \times 15 = 7.5$ in.

$$M = 18{,}000 \times 312.6 \times 2 \times \frac{1}{7.5} = 1{,}510{,}000 \text{ in.-lb}$$

Two 15-in. channels at 33.9 lb will be used. They provide sufficient area for shear.

Back Posts. The reactions are

$$R_1 = \frac{6871 \times 7 + 178{,}633 \times 2}{9} = 45{,}040 \text{ lb}$$

$$R = \frac{6871 \times 2 + 178{,}633 \times 7}{9} = 140{,}464 \text{ lb}$$

The maximum moment is

$$M = 140{,}464 \times 2 \times 12 = 3{,}380{,}000 \text{ in.-lb}$$

For one girder,

$$\frac{I}{c} = 3{,}380{,}000 \div 18{,}000 \times \frac{1}{2} = 93.8 \text{ in.}^3$$

There are no channels this large. A built-up plate girder must be used. Two girders consisting of four 4- by 3- by ½-in. angles with 19- by ⅜-in. web plates will be tried.

Area of one angle = 3.25 sq in. $y = 0.83$ in.
Area of two angles = 2 × 3.25 = 6.50 sq in.

$$h = 19 - 2 \times 0.83 = 17.34 \text{ in.}$$

$$\text{Net area} = 6.50 - 2[\tfrac{1}{2} \times (\tfrac{3}{4} + \tfrac{1}{8})] = 5.62 \text{ sq in.}$$

$$M = hfF = 17.34 \times 18{,}000 \times 5.62 \times 2 = 3{,}510{,}000 \text{ in.-lb}$$

An 18-in. web plate gives 3,310,000 in.-lb, which is too small.

Gross area of plate = 19 × ⅜ = 7.12 sq in.
Net area = 7.12 − 5 × 0.33 = 5.47 sq in.
Shear = 140,464 ÷ 2 × 5.47 = 12,820 lb per sq in.

The allowable shear is 13,000 lb per sq in. The selection is satisfactory. Figures 59*a* and *b* show a plate girder.

FOOTINGS FOR THE POSTS

In designing the footings for the posts, it is better to use a rather strong concrete. For this purpose a 1:2:4 mix will do. The total load to be sustained will be the dead load plus the breaking load. The front and back posts will be designed to take the sum of these two reactions. The wind-load reaction would only be included in those cases where the maximum wind pressure was a common and more or less continuous occurrence. Usually hoisting would be curtailed under exceptional wind conditions. Before proceeding with the footing, the direction and position of the resultant from various combinations must be determined. The size and number of anchor bolts necessary will depend on this information.

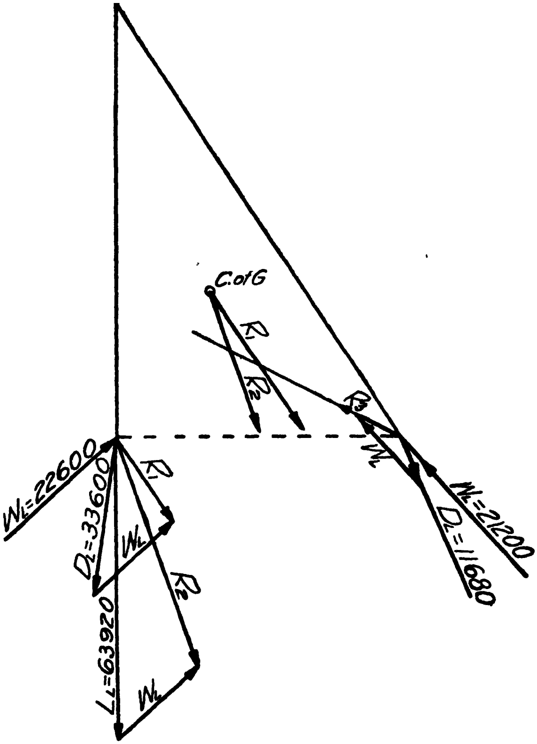

FIG. 60. Resultants of stresses acting on A-frame.

Figure 60 shows the resultants from combining dead load and wind load and wind load and breaking load. It is found that none of the resultants passes outside the frame. We need not bother with designing anchor bolts; good practice, however, requires that not less than two bolts be used for each post. In the case where the forces are acting on the front post, the resultants both fall inside the frame when transferred to the center of gravity. The resultant of wind load and dead load on the back post passes below the center of gravity, so that there is no overturning effect. If a tendency toward overturning is found, the piers must be heavy enough to prevent such action. In the actual design of the footings for our headframe, the wind load will not be combined with the breaking load since hoisting would undoubtedly cease under conditions of maximum wind pressure.

We shall now combine the dead loads (front and back posts) and the breaking load. The breaking load to be used is that which would result if the skip were hoisted into the sheave and the hoist operated until the rope broke. This gives the maximum overturning effort. The force will be equal to the ultimate strength of the rope. Opposed to this is the weight of the headframe and sheaves, acting in the direction of the dead-load resultants. The dead-load reactions must be doubled because the entire headframe is considered here. Figure 61 shows the combination of these forces and the direction and magnitude of the resultant. If *R* had fallen outside the back post, one of two things would be necessary: anchor bolts and piers designed to prevent overturning, or the back post would have to be moved back, necessitating the redesigning of the structure.

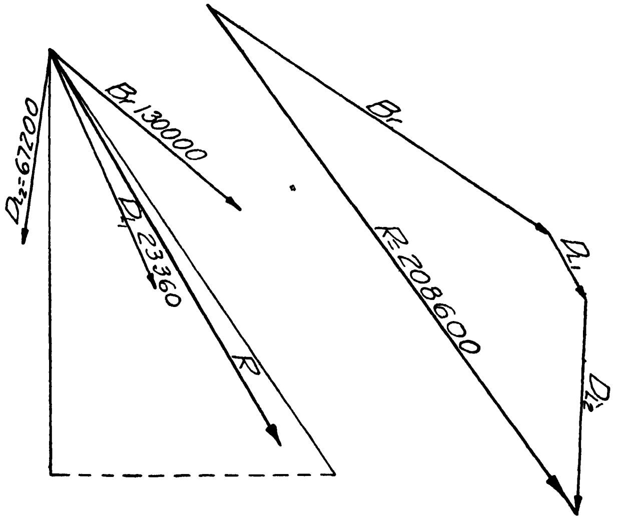

FIG. 61. Resultant of breaking-load and dead-load stresses.

The allowable unit compressive stress with dead loads for 1:2:4 concrete will be taken at 450 lb per sq in.; the allowable shear is 150 lb per sq in. The allowable unit stress for live loads is 1½ times that for dead loads. The bearing pressure on the soil is taken at 2 tons per sq. ft.

Back Posts. The reactions for the back posts are

Dead load = 8640 lb
Live load = 182,690 lb

The dead-load reaction given in Fig. 46 is 11,680 lb. The 8640 lb used above is the component in the plane of the back posts. As the footing to be designed will be quite heavy, its weight must be considered. For this purpose the weight will be estimated at 30,000 lb. The bearing area for the dead load is

$$(8640 + 30{,}000) \div 4000 = 9.6 \text{ sq ft}$$

For the live load,

$$182{,}690 \div 1\tfrac{1}{2} \times 4000 = 30.45 \text{ sq ft}$$

Total area required $= 9.66 + 30.45 = 40.11$, or 40 sq ft

The size of the base plate on the back post is determined as follows: The member is made of two 9-in. channels at 25 lb, 8 in. back to back. The width of the flange on one of the channels is 2.81 in. The width of the plate is $2 \times 2.81 + 8 = 13.62$, or 14 in. A 5- by 5- by ⅝-in. angle will be riveted to the flanges in order to connect the column to the base plate. The length of the base plate is $(2 \times 5) + 9 = 19$, or 20 in. The 20-in. dimension is measured in the plane of the A-frame. The dimension of the bottom of the footing may now be found (see page 100). The area of the base plate is 1.94 sq ft.

$$\begin{aligned} y:1.66&::\sqrt{40}:\sqrt{1.94} \\ y &= 7.54 \text{ ft} \\ x:1.17&::\sqrt{40}:\sqrt{1.94} \\ x &= 5.31 \text{ ft} \end{aligned}$$

The bottom of the footing will be 7.5 by 5.5 ft. As has been discussed in a previous chapter, the area of the top of the pier should be at least twice the area of the base plate. Before proceeding farther, it would be well to check the unit stress on the top of the pier. The area of the top is

$$14 \times 20 = 280 \text{ sq in.}$$

The load is $8640 + (182{,}690 \div 1\tfrac{1}{2}) = 130{,}433$ lb. The unit compressive stress $= 130{,}433 \div 280 = 466$ lb per sq in., which is slightly greater than 450 lb per sq in. but which will be perfectly safe. Some authorities consider 600 lb per sq in. as safe for bearing.

The area of the top of the footing $= 2 \times 1.94 = 3.88$ sq ft.

$$\begin{aligned} y:1.66&::\sqrt{3.88}:\sqrt{1.94} \\ y &= 2.34 \text{ ft} \\ x:1.17&::\sqrt{3.88}:\sqrt{1.94} \\ x &= 1.65 \text{ ft} \end{aligned}$$

The depth of the footing will be calculated by two methods: one will be similar to that given on page 100; the other will be based on punching shear and reinforcing rods. By the first method the depth is

$$\frac{(7.54 - 1.66)}{2} \times \tan 60^\circ = 5.09 \text{ ft}$$

When the depth is based on punching shear, we get the following:

$$\frac{40 - 1.94}{40} \times 130{,}433 = 123{,}911 \text{ lb}$$

$$\frac{123{,}911}{[(2 \times 14) + (2 \times 20)] \times 120} = 15 \text{ in.}$$

to the reinforcing steel, which is insufficient for the anchor bolts. It would not be desirable to use anchor bolts less than $1\frac{1}{4}$ in. in diameter. To develop their full strength such a bolt should be embedded the following depth (page 103):

$$x = \frac{10{,}000 \times 0.893}{\pi \times 1\frac{1}{4} \times 80} = 28.4, \text{ or } 30 \text{ in.}$$

In order that the bending moment be eliminated from the calculations, the piers will be made to conform to the dimensions indicated by the first method. The dimensions are

Bottom:
  Width = 5 ft 6 in.
  Length = 7 ft 6 in.
Top:
  Width = 1 ft 8 in.
  Length = 2 ft 7 in.
Depth = 5 ft

Length of anchor bolts in concrete: 5 ft.

A recalculation of the weight of the pier as determined gives

$$\tfrac{5}{3}(40 + 3.88 + \sqrt{40 \times 3.88}) \times 140 = 13{,}154 \text{ lb}$$

This is less than half that assumed. No change will be made in their size.

For the purpose of illustrating the procedure for determining the size and amount of reinforcing steel, the above footing will be so designed. When reinforcing steel is used, the depth to the steel is determined by the depth for punching shear. This was found to be 15 in. The thickness of concrete below the reinforcing will be 5 in. On page 97 a formula for bending moment in the rods was given. It applies to square footings. For rectangular construction this formula must be modified.[1] If the length of the footing is not greater than 50 per cent more than the width, we have

$$M_{1\text{-}1} = (\tfrac{1}{2} a_1c^2 + 0.6\, c^2b)w \text{ in.-lb}$$
$$M_{2\text{-}2} = (\tfrac{1}{2} ab^2 + 0.6\, cb^2)w \text{ in.-lb}$$

When the above ratio is exceeded, the moment becomes

$$M_{3\text{-}3} = \left(\frac{wb^2}{2}\right) g \text{ in.-lb}$$
$$M_{4\text{-}4} = \left(\frac{a_1c^2}{2} + \frac{bc^2}{2}\right) w \text{ in.-lb}$$

where $M_{1\text{-}1}$ = moment with reference to edge of column parallel to length of footing

$M_{2\text{-}2}$ = moment with reference to edge of column parallel to width of footing

$M_{3\text{-}3}$ = moment with reference to edge of column parallel to width of footing

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[1] Hool and Kinne, "Foundations, Abutments and Footings," 2d ed., p. 235, McGraw-Hill Book Company, Inc., New York.

$M_{4\text{-}4}$ = moment with reference to edge of column parallel to length of footing
a_1 = width of column base
a = length of column base
c = offset normal to length of the column base
g = width of footing
w = allowable bearing pressure, lb per sq ft on soil

Before continuing with the determination of the reinforcing, the depth to the rods would be checked for shear[1] across the width and the length of the footing. The plane through which the shear is to be determined is located a distance equal to the depth of the rods measured from the face of the pier. The shearing stress should not exceed 40 lb per sq in. The distance to the rods has been found to be 15 in. In the plane through the length of the footing the shear will be very small; therefore it will not be determined. The length of the footing is 7.5 ft; the width is 5.5 ft. The length of the column base is 1.66 ft. The plane across the width of the footing is

$$\frac{7.5}{2} - \frac{1.66}{2} - \frac{15}{12} = 1.67 \text{ ft}$$

from the end. The bearing pressure on the footing is

$$130{,}433 \div 40 = 3261 \text{ lb per sq ft}$$

The total shear at the section across the footing is equal to the upward pressure of the area outside the section, which equals the width or length of the footing times the distance of the section from the edge of the footing times the allowable bearing pressure, and is represented by

$$V = 5.5 \times 1.67 \times 3261 = 29{,}952 \text{ lb}$$

The shear as a measurement of diagonal tension is

$$v = \frac{V}{bjd}$$

where b = length of section
d = depth to reinforcing steel
j = 0.875 (see page 98).

$$v = \frac{29{,}952}{5.5 \times 12 \times 0.875 \times 15} = 34.5 \text{ lb per sq in.}$$

which is safe, since 40 lb per sq in. is permissible. If v is greater than 40 lb, the depth to the reinforcing steel is increased. We shall now continue with the steel determination. The length of the footing under inspection does not exceed the width by more than 50 per cent. The moments in the steel are

$$\begin{aligned} M_{1-1} &= (\tfrac{1}{2} \times 1.16 \times 2.17^2 + 0.6 \times 2.17^2 \times 2.92) \times 3261 \times 12 \\ &= 429{,}670 \text{ in.-lb} \\ M_{2-2} &= (\tfrac{1}{2} \times 1.66 \times 2.92^2 + 0.6 \times 2.17 \times 2.92^2) \times 3261 \times 12 \\ &= 711{,}030 \text{ in.-lb} \end{aligned}$$

---

[1] Actually this is diagonal tension. It is usually spoken of as shear.

The area of the steel is found from

$$A_s = \frac{M}{f_s jd}$$

where M = bending moment, as determined above
f_s = allowable unit stress in steel = 16,000 lb per sq in.
j = 0.875
d = depth to steel

$$A_{s(1-1)} = \frac{429{,}670}{16{,}000 \times 0.875 \times 15} = 2.04 \text{ sq in.}$$

Seven ⅝-in. diameter rods will give the required area. The bond stress is (page 98)

$$u = \frac{7.5 \times 2.17 \times 3261}{7 \times 1.96 \times 0.875 \times 15} = 295 \text{ lb per sq in.}$$

which is altogether too high; the allowable stress is 80 lb per sq in. If 21 bars are used, the bond stress is 98 lb. By using deformed bars, the design will be safe.[1]

$$A_{s(2-2)} = \frac{711{,}030}{16{,}000 \times 0.875 \times 15} = 3.4 \text{ sq in.}$$

Twelve ⅝-in.-diameter bars will provide sufficient area.

$$u = \frac{5.5 \times 2.93 \times 3261}{12 \times 1.96 \times 0.875 \times 15} = 169 \text{ lb per sq in.}$$

By using 20 bars the bond stress is reduced to a point of safety for deformed bars.

The foregoing illustration was carried out mainly to indicate the procedure for selecting the reinforcing steel. Actually, the depth of the footing should be increased, thus reducing the number of reinforcing bars necessary. It would be advisable to double the depth. On this basis the steel required would be much less. The point is not so much the actual cost per pound of steel, as the labor involved in preparing and placing the bars. On the basis of 30 in. to the steel, the amount required for M_{2-2} will be calculated.

$$v = \frac{29{,}952}{5.5 \times 12 \times 0.875 \times 30} = 17.25 \text{ lb per sq in.}$$

$$M_{2-2} = 711{,}030 \text{ in.-lb}$$

$$A_{s(2-2)} = \frac{711{,}030}{16{,}000 \times 0.875 \times 30} = 1.7 \text{ sq in.}$$

If only six ⅝-in. bars are used (they provide area enough), we shall get exactly the same bending stress as before. If 12 bars are used,

$$u = \frac{5.5 \times 2.92 \times 3261}{12 \times 1.96 \times 0.875 \times 30} = 84.5 \text{ lb per sq in.}$$

---

[1] Allowable stress for deformed bars is 100 lb per sq in. Such bars are used almost entirely at the present day.

While the bond stress is slightly higher than the 80 lb allowed, 12 bars will do. Of, if deformed bars are used, 12 bars are more than sufficient.

Front-post Footings. The reactions for the front posts are

Dead load = 28,160 lb
Live load = 63,920 lb

If the weight of the footing is estimated at 15,000 lb, the dead-load area required is

$$(28{,}160 + 15{,}000) \div 4000 = 10.8 \text{ sq ft}$$

Area for live load = 63,920 ÷ 6000 = 10.7 sq ft
Total area = 10.8 + 10.7 = 21.5, or 22 sq ft

The size of the base plates on the front posts is 13 by 18 in. The area of the base plate is 1.62 sq ft. The dimensions of the footing are

Bottom:
  Width = 4 ft
  Length = 5.6 ft
Top:
  Width = 1.5 ft
  Length = 2.1 ft

$$\text{The depth} = \frac{5.6 - 1.5}{2}\tan 60° = 3.5 \text{ ft}$$

HEADFRAME FOR AN INCLINED SHAFT

The front posts have an inclination equal to that of the shaft. The back posts are placed so that they fall just outside the resultant. In other respects the design does not differ from the vertical-post type.

Figure 62 shows the determination of the dead-load stresses in a headframe for an inclined shaft. Exactly the same procedure is followed as for the headframe previously discussed. It should be noted that the sense of the stresses in the diagonal and horizontal members is the reverse of that found in the previous example. The live-load-stress diagram for the A frame does not differ from the one for a vertical-post headframe. The remaining live-load and wind-load stresses are determined as before.

HEADFRAME FOR A FOUR-COMPARTMENT SHAFT

The headframe to accommodate a four-compartment shaft in which the compartments are not all on the same center line will usually be of the four-post type. Such an arrangement becomes necessary at many metal mines and is commonly used at coal mines. The shaft arrangement necessitating such a layout might be two skip compartments (accessible by means of one pair of sheaves) and one large cage compartment and manway adjoining the cage compartment. There may or may not be a counterweight for the cage. The sheave for the cage would be to the rear of the skip sheaves (more or less in tandem).

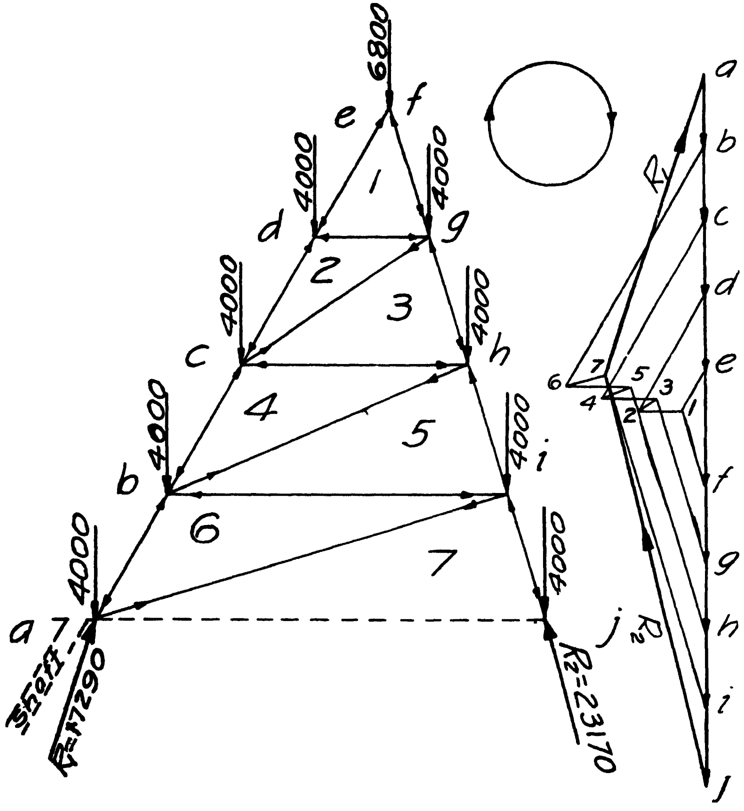

FIG. 62. Dead-load stresses in the A-frame for an inclined shaft.

Four-post Headframe. The four-post headframe is statically indeterminate. By assuming, in turn, certain members as being redundant (*i.e.*, for the purpose of making the solution, temporarily inactive) an approximation that is perfectly safe may be made. There are a number of solutions for such problems. The one to be explained here is known as "the method of least work."[1] It is based on one redundant member. For a treatise on statically indeterminate structures the reader is referred to Ketchum's book and to "Statically

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[1] KETCHUM, M. S., "The Design of Mine Structures," p. 93.

Indeterminate Stresses," by Parcel and Maney. The discussion to follow should prove sufficient for the usual four-post headframe.

It is necessary that the approximate area of the various members of the structure be known before the solution is started. An arbitrary assumption can, of course, be made; however, a considerable amount of tedious calculations and drafting is avoided if a close approximation of the areas can be used. An inspection of data available on erected headframes suggests the procedure that will follow. In a great many cases (in fact most), the four posts and the back braces are made of the same size, weight, and shape material. When selecting the horizontal members, they are usually chosen the same size, as is also the diagonal bracing. If the members are designed for dead-load and wind-load stresses, it is customary to assume that only the tower (four posts) takes these loads. Ordinarily if the frame is designed for the breaking strength of the rope, no consideration need be given to the dead and wind loads. This does not apply, of course, if the headframe supports ore bins, etc. We shall base the design on the breaking load. As was stated above, an inspection of available data suggests a method that reduces the amount of drafting and calculations involved. It was found that the area of the horizontal members is approximately one-half of the area of the back braces and that the area of the diagonal bracing is one-quarter of the area of the back braces. These ratios are not exact but are very close, and if followed, the labor necessary for determining the stresses in the statically indeterminate headframe is very materially reduced.

The first step in the solution of our problem is the selection of the rope. This has been covered in an earlier part of the book. The resultant of the breaking stress in the rope is found as previously explained. From this resultant the horizontal and vertical components are determined. They act at the center of the sheave which, because of the construction, is located at some point between the two vertical posts. By means of moments, the vertical and horizontal forces at the center of the sheave are transferred to the two posts. For simplicity, only one set of wheels is used in the example to follow; if more than one set are present, the forces are transferred in the same manner as used for more than one concentrated load on a beam. The magnitude of the horizontal forces remains unchanged throughout these operations.

The vertical force determined as acting on the end of the middle post is combined with the horizontal force. Their resultant is then resolved into the plane of the back brace and the front posts. This operation approximately determines the force acting on the back posts. This approximation is necessary to estimate the areas of the members. To determine the area of the back braces, we take a section whose length is the over-all length of the brace divided by the number of panels. It is also necessary to assume (purely as an aid toward reducing the number of trials) that the l/r ratio here is not greater than 70 instead of the customary 100. Also the unit stress for the breaking load is, as before, 1½ times that for dead loads.

The selection of the cross-sectional shape of the member involves the same consideration as for the simple A-frame. Channels for the posts and angles for the bracing are commonly used. When angles are selected for the horizontal members, at least two must be used as they resist compression. For tension one is usually sufficient. With this general discussion in mind the design of a four-post frame will be taken up.

EXAMPLE

Figure 63 shows the dimensions, etc., of the frame. The resultant of the breaking stress in a 1¼-in. rope at 45 deg with the horizontal is 240,500 lb. The vertical and horizontal components of the resultant are 221,500 and 95,000 lb, respectively. The forces acting on the two posts are (letting F_1 be the force on the outside post and F_2 the force on the middle post), by moments,

$$F_1 = \frac{221{,}500 \times 2\frac{1}{2} - 95{,}000 \times 1}{13} = 35{,}290, \text{ or } 35{,}300 \text{ lb}$$

$$F_2 = \frac{221{,}500 \times 10\frac{1}{2} + 95{,}000 \times 1}{13} = 186{,}210, \text{ or } 186{,}200 \text{ lb}$$

Combining F_2 and H, we get E. The resultant E is resolved into the plane of the back brace. The force on the back brace is found to be 203,000 lb. The length of the section is 58.5/3 = 19.5 ft, or 234 in. If l/r is taken equal to 70, the approximate radius of gyration for a section of 234 in. is $^{234}/_{70} = 3.34$ in. From the handbook, a 10-in. channel at 20 lb has $r = 3.66$ in. Two of these, 8 in. back to back and laced together, will be used.

$$\frac{P}{A} = 1\frac{1}{2}\left(16{,}000 - 70 \times \frac{234}{3.66}\right) = 17{,}295 \text{ lb per sq in.}$$

$$P = 11.72 \times 17{,}295 = 202{,}697 \text{ lb}$$

which is satisfactory. On the basis of a back-brace area of 11.72 sq in., the areas of the remaining members should be

Horizontal member $= \frac{1}{2} \times 11.72 = 5.86$ sq in.
Diagonal member $= \frac{1}{4} \times 11.72 = 2.93$ sq in.

Consulting the handbook, two 4½- by 3- by $\frac{7}{16}$-in. angles provide sufficient area (6.18 sq in.) for the compression members; and the area (3.31 sq in.) of one 4- by 4- by $\frac{7}{16}$-in. angle satisfies the tension members. These areas will be used in the solution to follow.

The stresses are determined in three steps. It is first considered that the back brace takes none of the load. On this assumption Fig. 64 shows the graphical determination of the stresses in the tower. The forces ab = 95,000 lb, bc = 35,300 lb, and cd = 186,200 lb are laid out to a suitable scale, as shown in the stress diagram. The stresses are readily determined by starting at the panel point ab. The horizontal force H = 95,000 lb is assumed to be resisted equally by the two posts.

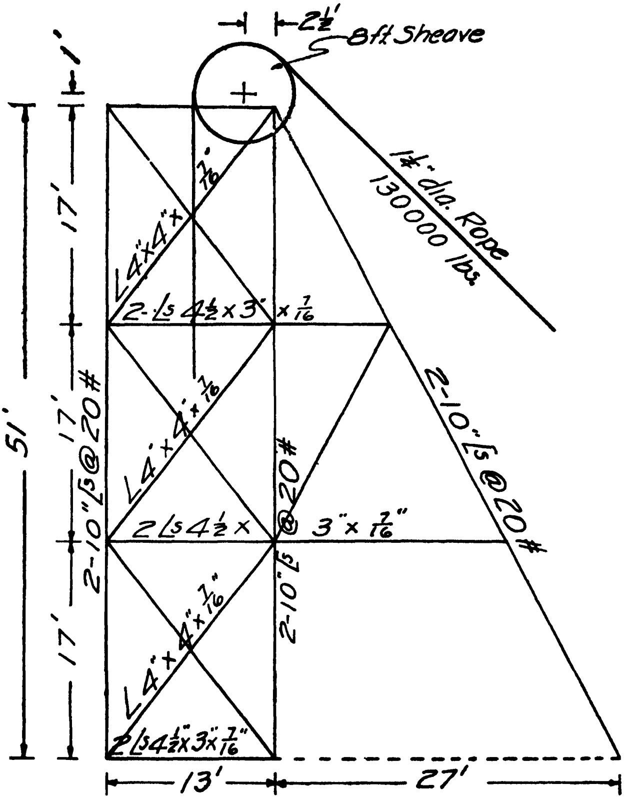

FIG. 63. Layout for a four-post headframe.

It is next assumed (Fig. 65) that the back brace is acted upon by a force S_{12} = 1 lb, and that all of the bracing between the tower and the brace is redundant. On this basis we determine graphically the stresses in the member in terms of S_{12} = 1 lb. There are now available sufficient data to calculate an approximate value for S_{12}. If S_{12} is determined, the remaining members of the headframe, with the exception of the bracing between the tower and the back brace (they are considered redundant), can be found. The tabulation of the data is given in Table 28.[1]

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[1] KETCHUM, M. S., "The Design of Mine Structures," p. 95. (Same system followed as given in reference.)

TABLE 28. CALCULATION OF S_{12} FOR FOUR-POST HEADFRAME, FIRST TRIAL

Member	Area A, sq. in.	Length L, in.	Stress S, in terms of S_{12}	$\frac{dS}{dS_{12}} = U$	$\frac{L}{AE} = B$	$BS\left(\frac{dS}{dS_{12}}\right)$
1-2 = S_1	3.31	258	$-152000 + 0.75S_{12}$	+0.75	0.000002598	$-0.2962 + 0.000001463S_{12}$
2-f = S_2	11.72	204	$+305000 - 1.48S_{12}$	−1.48	0.0000005802	$-0.2620 + 0.000001272S_{12}$
2-3 = S_3	6.18	156	$+95000 - 0.46S_{12}$	−0.46	0.0000008414	$-0.0368 + 0.000000178S_{12}$
a-3 = S_4	11.72	204	$-84000 + 0.60S_{12}$	+0.60	0.0000005802	$-0.0293 + 0.000000209S_{12}$
3-4 = S_5	3.31	258	$-152000 + 0.75S_{12}$	+0.75	0.000002598	$-0.2962 + 0.000001463S_{12}$
4-g = S_6	11.72	204	$+430000 - 2.09S_{12}$	−2.09	0.0000005802	$-0.5220 + 0.000002535S_{12}$
4-5 = S_7	6.18	156	$+95000 - 0.46S_{12}$	−0.46	0.0000008414	$-0.0368 + 0.000000178S_{12}$
a-5 = S_8	11.72	204	$-208000 + 1.21S_{12}$	+1.21	0.0000005802	$-0.1461 + 0.000000850S_{12}$
5-6 = S_9	3.31	258	$-152000 + 0.75S_{12}$	+0.75	0.000002598	$-0.2962 + 0.000001463S_{12}$
6-d = S_{10}	11.72	204	$+552000 - 2.69S_{12}$	−2.69	0.0000005802	$-0.8620 + 0.000004200S_{12}$
6-e = S_{11}	6.18	156	$+47500 - 0.23S_{12}$	−0.23	0.0000008414	$-0.0092 + 0.000000044S_{12}$
				$\frac{dW}{dS_{12}} =$	$\sum BS\left(\frac{dS}{dS_{12}}\right) = y_{12} =$	$-2.7928 + 0.000013855S_{12}$
fd = S_{12}	11.72	696	S_{12}	+1.00	0.00000198	$-y_{12} = 0.00000198S_{12}$

Column (1) gives the member; the area is shown in column (2); the length in column (3); the stress S, in terms of S_{12}, is given in column (4); column (5) gives the stresses in the member on the basis of $S_{12} = 1$ lb; values of $B = L/AE$* are found in column (6); the product of columns (4), (5), and (6) are given in column (7).

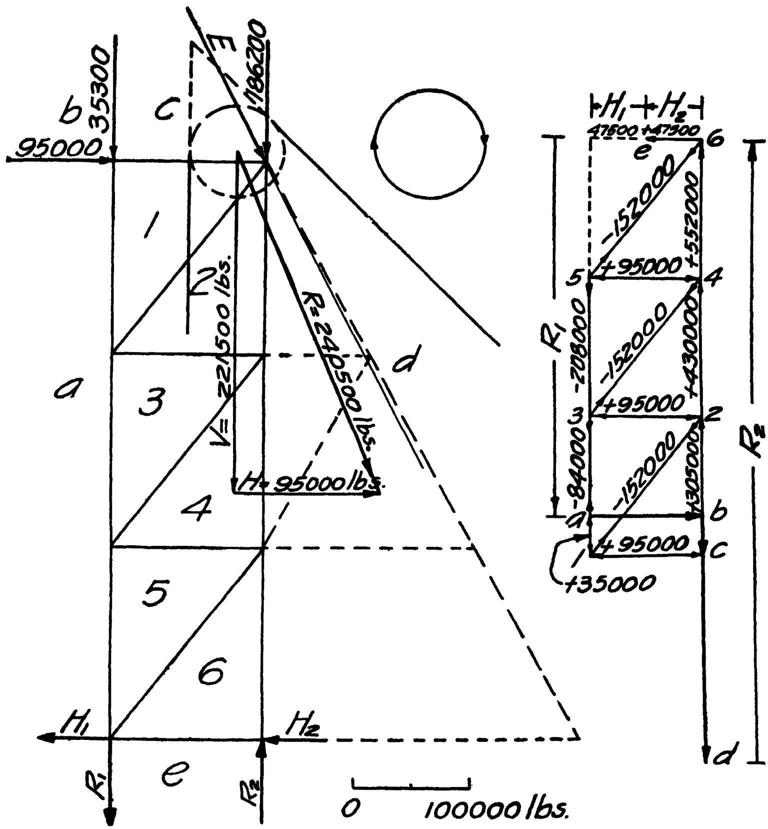

FIG. 64. Stresses in the tower, back brace redundant.

The sum of the values in column (7), from S_1 and S_{11}, is

$$\frac{dW}{dS_{12}} = \sum BS\left(\frac{dS}{dS_{12}}\right) = y_{12}$$

$$= -2.7928 + 0.000013855S_{12}$$

The deformation of the member S_{12} is

$$B\frac{L}{AE}\frac{dS}{dS_{12}} - y_{12} = 0.00000198 \times S_{12} \times 1$$

$$y_{12} = 0.00000198S_{12}$$

---

* Modulus of elasticity for steel. Taken equal to 29,000,000.

The two values of y_{12} are equated, thus giving a value for S_{12}.

$$
\begin{aligned}
y_{12} &= -2.7928 + 0.000013855S_{12} \\
-y_{12} &= 0.000001980S_{12} \\
S_{12} &= 176{,}500 \text{ lb}
\end{aligned}
$$

Figure 66 shows the final determination of the stresses. We now have available the forces *ab* = 95,000 lb, *bc* = 35,300 lb, *cd* = 186,200 lb, and *df* = 176,500 lb = S_{12}. If the bracing between the back brace and the tower is considered redundant, the stresses in the members, with the above forces acting, may be determined. The load line *abcdf* is laid out as shown. The solution is started at panel point *abc*1. The forces *ab* and *bc* are held in equilibrium by the stresses *c*-1 and 1-*a*. Solving next the stresses at the opposite panel point, the known forces and stresses are *bc*, *df*, and 1-*c*. The two stresses *f*-2 and 2-1 are to be determined. The remainder of the solution is readily carried out. From Fig. 66 the maximum stress in the front posts is +80,000 lb; in the horizontal members it is +13,000 lb; and in the diagonal members it is −21,000 lb. The member *c*-1 must be designed also as a beam or girder to support the sheave girder; therefore it is considered separately from the other members. Its design does not present any particular

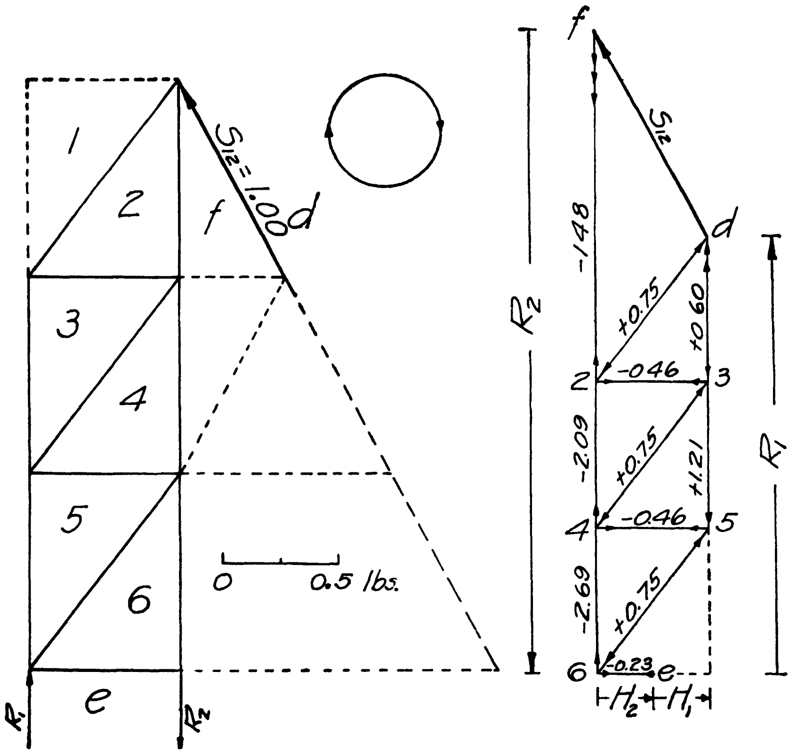

FIG. 65 Relative stresses in members, when S_{12} = 1 lb.

difficulty; therefore it will not be discussed further. It is a case of combined bending and direct stress.

On the basis of the maximum stresses found, new members are selected. It is found that two 9-in. channels at 20 lb, two 7-in. channels at 12.25 lb, two angles 3 by 2 by 5⁄16 in., and one angle 2½ by 2½ by 5⁄16 in. will supply the area required for the back braces, posts, horizontal members, and diagonal members, respectively. Using the areas corresponding to the new selection and recalculating (Table 29),

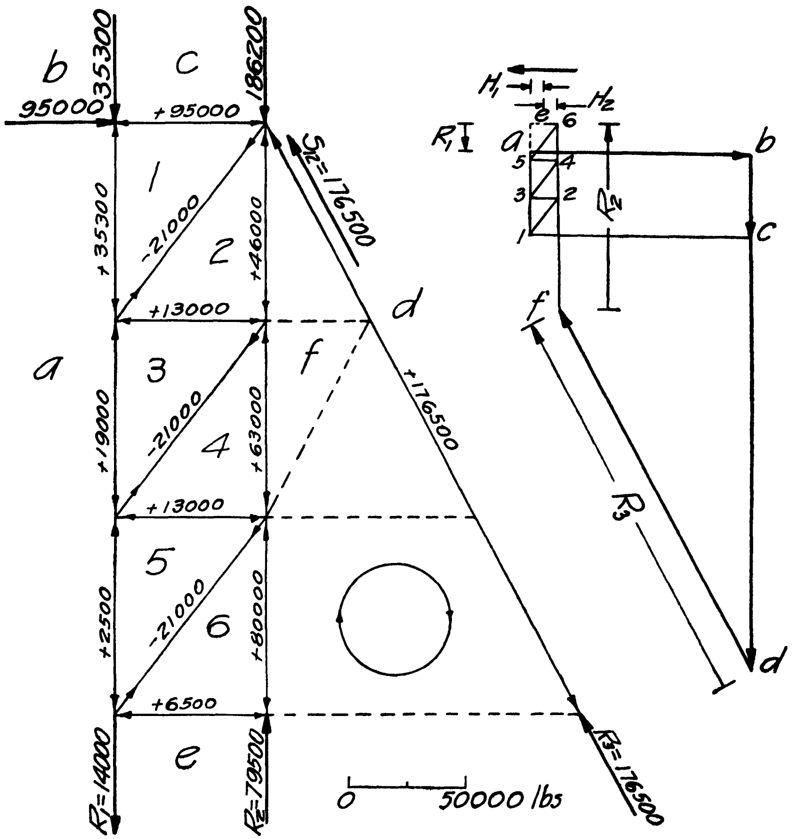

FIG. 66. Final stresses in four-post headframe.

we find that S_{12} becomes equal to 173,200 lb. This differs but little from the first value. It would cause but a slight change in the stresses shown in Fig. 66. If the second choice of members were used in the construction, they would not provide, in most cases, sufficient area after deducting for rivet holes. In addition, as no calculations were made for dead-load or wind-load stresses, they would on this score prove too small. The material shown in Fig. 63 should be used.

It should be noted that the second calculation of S_{12} checks very closely the first determination of this force. The basis on which the solution was started was the

TABLE 29. CALCULATION OF S_{12} FOR FOUR-POST HEADFRAME, FINAL TRIAL

Member	Area A, sq. in.	Length L, in.	Stress S, in terms of S_{12}	$\frac{dS}{dS_{12}} = U$	$\frac{L}{AE} = B$	$BS\left(\frac{dS}{dS_{12}}\right)$
1-2 = S_1	1.47	258	$-152000 + 0.75S_{12}$	+0 75	0 000005850	$-0.6670 + 0.000003295S_{12}$
2-*f* = S_2	7.16	204	$+305000 - 1.48S_{12}$	−1.48	0.000000908	$-0.4100 + 0.000001990S_{12}$
2-3 = S_3	2.94	156	$+\ 95000 - 0.46S_{12}$	−0 46	0.000002653	$-0.1159 + 0.000000562S_{12}$
a-3 = S_4	7.16	204	$-\ 84000 + 0.60S_{12}$	+0.60	0.000000908	$-0.0458 + 0.000000327S_{12}$
3-4 = S_5	1.47	258	$-152000 + 0.75S_{12}$	+0.75	0.000005850	$-0.6670 + 0.000003295S_{12}$
4-*g* = S_6	7.16	204	$+430000 - 2.09S_{12}$	−2 09	0.000000908	$-0.7264 + 0.000004360S_{12}$
4-5 = S_7	2.94	156	$+\ 95000 - 0.46S_{12}$	−0.46	0.000002653	$-0.1159 + 0.000000562S_{12}$
a-5 = S_8	7.16	204	$-208000 + 1.21S_{12}$	+1.21	0.000000908	$-0.2290 + 0.000001331S_{12}$
5-6 = S_9	1.47	258	$-152000 + 0.75S_{12}$	+0 75	0.000005850	$-0.6670 + 0.000003295S_{12}$
6-*d* = S_{10}	7.16	204	$+552000 - 2.69S_{12}$	−2 69	0.000000908	$-1.3490 + 0.000006580S_{12}$
6-*e* = S_{11}	2.94	156	$+\ 47500 - 0.23S_{12}$	−0.23	0.000002653	$-0.0208 + 0.000000108S_{12}$
				$\frac{dW}{dS_{12}} = BS\left(\frac{dS}{dS_{12}}\right) = y_{12} =$		$-5.0138 + 0.000025705S_{12}$
fd = S_{12}	7.16	696	S_{12}	+1 00	0.000003240	$-y_{12} = 0.00000324S_{12};$

assumption made with regard to the ratio of the areas. If some such method had not been used, many more calculations would have been necessary, at least for those who were not experienced in this type of work.

The four-post type, and thus an indeterminate solution, may be avoided by setting the center line of the front posts on the center line of the divider separating the compartments. Thus the skips come up on the outside of the front transverse frame and the cage and counterweight, or cages, on the inside.

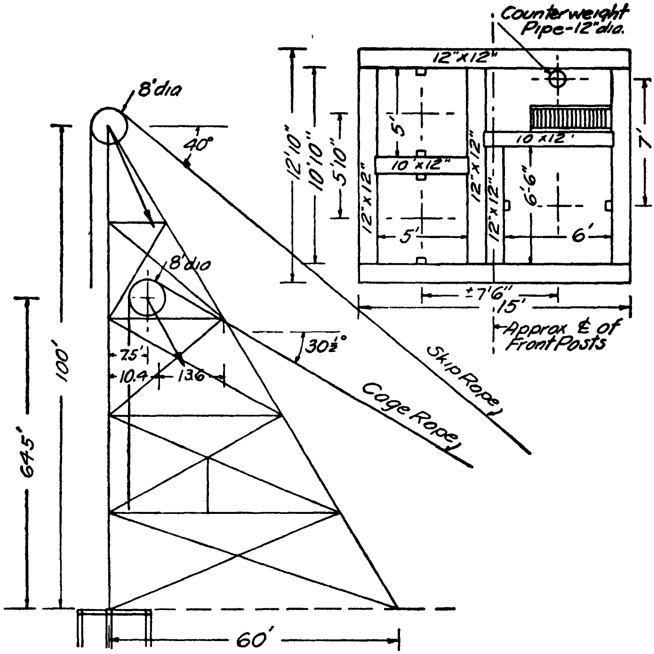

Fig. 67. Four-compartment shaft headframe.

Many shafts might conveniently be designed as shown in Fig. 67. When this type is used, it is customary to use a four-post headframe. The solution to follow is suggested for designing a headframe to suit the shaft shown in Fig. 67.

The skip and contents weigh 5000 lb and 10,000 lb and require a $1\frac{1}{4}$-in. rope with a breaking strength of 130,000 lb. For the cage, a weight of 5000 lb will be taken. The load capacity will be equivalent to 10,000 lb; the acceleration is 3.2 ft per sec^2; the depth of shaft is 1000 ft; the weight of sheave, etc., is 3030 lb; the weight of headframe is 80,000 lb. The width of the transverse frame is 10 ft at the top and 37 ft at the bottom.

The headframe will be designed for case 2, as already discussed. The weight of the counterweight (page 280) is equal to the cage plus one-half the load.

$$\text{Counterweight} = 5000 + \tfrac{1}{2} \times 10{,}000 = 10{,}000 \text{ lb}$$

The dead load is

$$\text{One-half of the frame} = \tfrac{1}{2} \times 80{,}000 = 40{,}000 \text{ lb}$$

There are 12 panels.

$$\text{Panel load} = 40{,}000 \div 12 = 3330 \text{ lb}$$

At the apex, the load is

$$2 \times 3330 + 3030 = 9690 \text{ or } 9700 \text{ lb}$$

The length of the member supporting the cage sheave is approximately 24 ft and the sheave is about 7½ ft from the front-post center line (see Fig. 67).

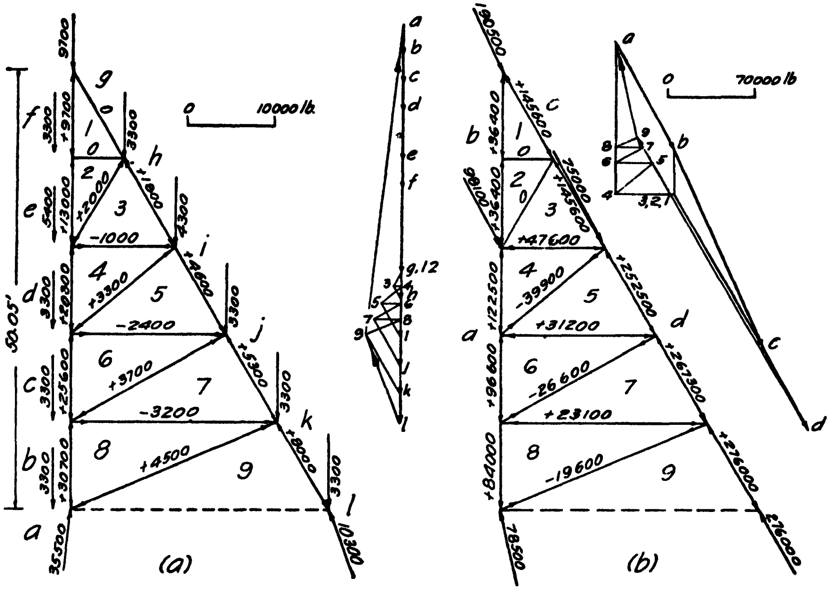

FIG. 68. Stresses in four-compartment shaft headframe.

Transferring the weight of the sheave to its equivalent reactions at the ends of the member,

$$R = \frac{16.5 \times 3030}{24} = 2080 \text{ lb}$$

$$R_1 = \frac{7.5 \times 3030}{34} = 950 \text{ lb}$$

Panel *d-e* = 2080 + 3330 = 5410, or 5400 lb
Panel *h-i* = 950 + 3330 = 4280, or 4300 lb
Rest of panel loads = 3300 lb each

The dead-load stress diagram is shown in Fig. 68*a*.

Live loads. To select a rope for the cage,

$$F = \frac{5000 + 10{,}000}{32.2} \times 3.2 = 1500 \text{ lb} \qquad \text{accelerating force}$$

$$\text{Total pull} = 15{,}000 + 1500 = 16{,}500 \text{ lb}$$

A factor of safety of about 8 will be used. As the rope is still unknown, $8\frac{1}{2}$ will be used in the calculations.

$$16{,}500 \times 8\tfrac{1}{2} = 140{,}250 \text{ lb} = 70.12 \text{ tons}$$

A $1\frac{3}{8}$-in. rope with an ultimate strength of 78.5 tons is required. It weighs 3.03 lb per ft.

Weight of rope = $(1000 + 65) \times 3.03 = 3230$ lb
$F = 3230/32.2 \times 3.2 = 323$ lb
Total pull due to rope = $3230 + 323 = 3553$ lb
Total pull on rope = $16{,}500 + 3553 = 20{,}053$ lb
Ultimate strength of rope needed = $20{,}053 \times 8 = 160{,}424$ lb = 80.21 tons

If the $1\frac{3}{8}$-in. rope is used, the factor of safety is

$$157{,}000 \div 20{,}053 = 7.83$$

On consulting Table 17, this is found satisfactory.

The resultant forces on the headframe are shown in Table 30.

TABLE 30.—RESULTANT FORCES ON FOUR-COMPARTMENT SHAFT HEADFRAME

Loading	S, lb	F,* lb
Skip side:		
Empty skip	5,000	9,100
Breaking rope	130,000	236,000
Cage side:		
Counterweight	10,000	17,450
Breaking rope	157,000	274,000

* The conversion factor is $F = 1.745S$.

Following the method illustrated by Figs. 48 and 49, Table 31 is obtained for case 2.

TABLE 31. LIVE-LOAD STRESSES

Loading	F_1	F	P_1	P	E_1	E
Skip side	9,100	236,000	56,300	188,800	56,800	190,500
Cage side	17,450	274,000	120,050	171,400	....	98,100 75,000

For the skip side,

$$P = \frac{2.08 \times 9100 + 7.92 \times 236{,}000}{10} \qquad 188{,}800 \text{ lb}$$

$$P_1 = \frac{7.92 \times 9100 + 2.08 \times 236{,}000}{10} = 56{,}300 \text{ lb}$$

For the cage side,

$$P = \frac{5.65 \times 17{,}450 + 12.65 \times 274{,}000}{20.8} = 171{,}400 \text{ lb}$$

$$P_1 = \frac{15.15 \times 17{,}450 + 8.15 \times 274{,}000}{20.8} = 120{,}050 \text{ lb}$$

$$E_1 = \frac{P_1}{\cos \theta}$$

$$= 56{,}400 \div \cos 7°45' = 56{,}800 \text{ lb}$$

$$E = 189{,}300 \div \cos 7°45' = 190{,}500 \text{ lb}$$

where $\theta = 7°45'$.

The values given for P_1, P, E_1, and E in Table 31, cage side, are those at the end of the sheave axis on the A-frame girders. They must be transferred to the panel points at the ends of the girder. Figure 69 shows a sketch to accompany the following calculations.

For load P_1,

$$R = \frac{120{,}050 \times 13.6}{24} = 68{,}000 \text{ lb}$$

$$E = 68{,}700 \text{ lb (front post)}$$

$$R_1 = \frac{120{,}050 \times 10.4}{24} = 52{,}000 \text{ lb}$$

$$E_1 = 52{,}600 \text{ lb (back post)}$$

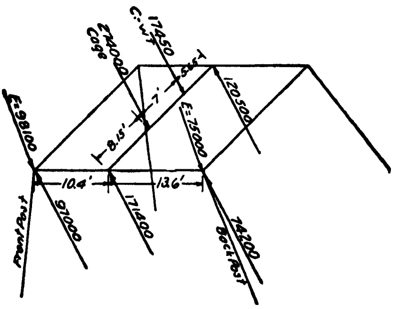

Fig. 69.

For load P,

$$R_2 = \frac{171{,}400 \times 13.6}{24} = 97{,}000 \text{ lb} \qquad E_2 = 98{,}100 \text{ lb (front post)}$$

$$R_3 = \frac{171{,}400 \times 10.4}{24} = 74{,}200 \text{ lb} \qquad E_3 = 75{,}000 \text{ lb (back post)}$$

R and R_2 are the loads on the front posts acting along the rope resultant. The vertical component of R and R_2 acts on the front transverse frame.

R_1 and R_3 act in a similar way on the back posts; the components parallel to the posts are the back transverse loads.

Figure 68*b* shows live loads on an A-frame.

By the method shown in Fig. 51 the transverse-frame loads are found.

Loading	H_1	H	G_1	G
Skip side.	11,700	39,400	45,900	154,500
Cage side.	44,100	64,200	65,500	94,600

Figure 70 shows the stress diagrams; *a* is the front transverse frame; *b* is the back transverse frame. Table 32 summarizes the stresses.

Selection of Members. The headframe will be constructed of wide flange-beam sections.

Back Posts. The stresses are

Dead load = +8000 lb
Live load = +276,000 lb

The length of the section is 118 ÷ 5 = 23.6 ft = 258 in.; the least

$$r = {}^{258}\!/_{120} = 2.33 \text{ in.}$$

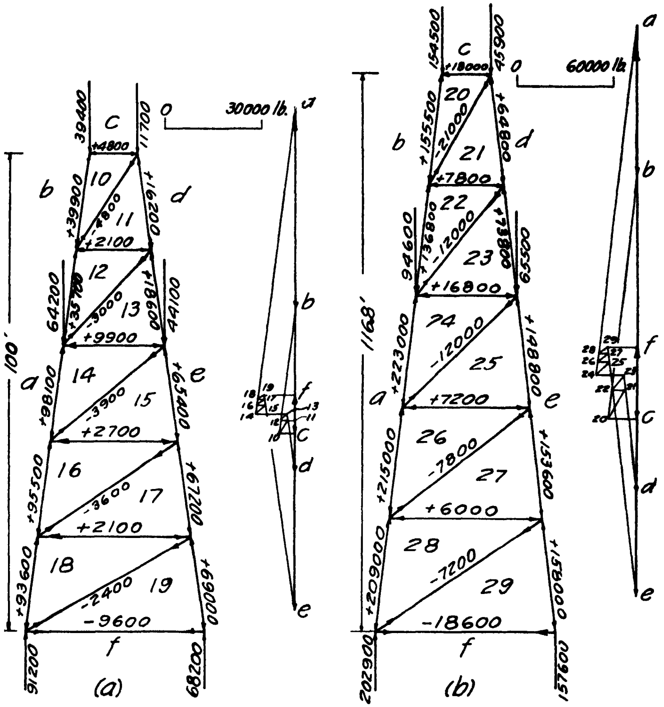

Fig. 70. Stresses in four-compartment shaft transverse frames.

Trying a 14- by 12-in. wide flange at 78 lb, the area is 22.94 sq in.; $r = 3.00$ in.

$$\frac{P}{A} = 18{,}000 - 60 \times \frac{258}{3.00} = 12{,}840 \text{ lb per sq in.}$$

Area for dead load = 8000 ÷ 12,840 = 0.62 sq in.
Area for live load = 276,000 ÷ 12,840 = 21.50 sq in.
Total area = 0.62 + 21.50 = 22.12 sq in.

The selection is just large enough.

The post will set with the web of the section parallel to the front-post—back-post plane. This makes possible a better arrangement for connecting the sheave girders to the posts, as will be illustrated later.

TABLE 32. STRESSES IN FOUR-COMPARTMENT SHAFT HEADFRAME

Member	Dead-load stress	Live-load stresses			Approx. length of members ft
		A-frame	Front transverse	Back transverse	
Front posts	+30,700	+122,500	+98,100	...	101
Back posts	+8,000	+276,000	...	+223,000	118
1-2	0	0	...	...	12
3-4	−1,000	+47,600	...	...	24
5-6	−2,400	+31,200	...	...	36
7-8	−3,200	+23,100	...	...	48
2-3	+2,000	0	...	...	24
4-5	+3,300	+39,900	...	...	32
6-7	+3,700	+26,600	...	...	41
8-9	+4,500	−19,600	...	...	52
c-10	...	...	+4,800	...	10
11-12	...	...	+2,100	...	16
13-14	...	...	+9,900		20.8
15-16	...		+2,700		26
17-18	...		+2,100		32
19-*f*	...		−9,600	...	37
10-11			−4,800	...	24
12-13			−3,000	...	27
14-15			−3,900	...	31
16-17		...	−3,600	...	35
18-19	...		−2,400	...	40
c-20			...	+18,000	10
21-22			...	+7,800	16
23-24	...		...	+16,800	20 8
25-26		...	...	+7,200	26
27-28	...	...	...	+6,000	32
29-*f*	...	...	...	−18,000	37
20-21	...	...	...	−21,000	27
22-23	...	...	...	−12,000	29
24-25			...	−12,000	31
26-27	...	...	...	−7,800	37
28-29	...	...	...	−7,200	42

Front Posts. The length of the section is ${}^{101}\!/_{5} = 20.2$ ft $= 242$ in.; the least $r = {}^{242}\!/_{120} = 2.01$ in.

Dead load $= +30{,}700$ lb
Live load $= +122{,}500$ lb

One dimension of the member should be 12 in. in order to detail with the back post. A 12- by 12-in. wide flange at 65 lb (area = 19.11 sq in.; $r = 3.02$ in.) is the best selection. A 12- by 10-in. wide flange at 53 lb would provide plenty of area but would not detail well with the back post or the secondary members.

Member 8-9. The length of this member is 52 ft = 624 in.; least

$$r = {}^{624}\!/_{200} = 3.12 \text{ in.}$$

Dead load $= +4500$ lb
Live load $= -19{,}600$ lb

A 12- by 12-in. wide flange at 106 lb, an area of 31.19 sq in., and $r = 3.11$ in. are necessary.

Member 28-29. The length is 42 ft and = 504 in.; least $r = 2.52$ in. A 12- by 8-in. wide flange at 45 lb, an area of 13.24 sq in., and $r = 1.94$ in. are satisfactory. It provides sufficient area for any wind load that might occur. The member will always be in tension, so that the radius of gyration does not enter into the selection; in addition it is connected at its mid-point by a similar cross member.

Member 27-28. This member, and others up to 21-22, may be called upon to take a wind-load compression. The l/r ratio will be considered. The length is 32 ft = 384 in.; least $r = 1.94$ in. A 12- by 8-in. wide flange at 45 lb may be used.

The members of the front transverse frame are all 10- by 8-in. sections at 41 lb, which detail well with the front posts. Back transverse-frame members are 12- by 8-in. sections at 45 lb.

SHEAVE GIRDERS

For Skips. The components of the forces F_1 and F are found as in Fig. 58 and Table 27. Table 33 gives the data.

TABLE 33. SHEAVE-GIRDER LOADS

Loading	F_1	F	Front posts		Back posts	
			C_1	C	B_1	B
Skip	9100	236,600	1890	49,200	7,450	193,000
Cage	17,450	274,000	720	11,300	16,800	264,300

For the skip, the front-post reactions are

$$R_1 = \frac{7.92 \times 1890 + 2.08 \times 49{,}200}{10} = 11{,}730 \text{ lb}$$

$$R = \frac{2.08 \times 1890 + 7.92 \times 49{,}200}{10} = 39{,}360 \text{ lb}$$

The back-post reactions are

$$R_1 = \frac{7.92 \times 7450 + 2.08 \times 193{,}000}{10} = 46{,}100 \text{ lb}$$

$$R = \frac{2.08 \times 7450 + 7.92 \times 193{,}000}{10} = 154{,}300 \text{ lb}$$

Sheave Girder, Front Posts. The maximum moment will be at the 49,200-lb load.

$$M = 2.08 \times 39{,}360 \times 12 = 983{,}000 \text{ in.-lb}$$

Two channels will be considered. For one channel,

$$\frac{I}{c} = 983{,}000 \div 18{,}000 \times \tfrac{1}{2} = 27.3 \text{ in.}^3$$

A 15-in. channel at 33.9 lb is the lightest section with sufficient I/c value. For this channel, $I/c = 41.7$ in.^{3} Two such channels are satisfactory.

Back Posts. The maximum moment is under the 193,000-lb load.

$$M = 2.08 \times 154{,}350 \times 12 = 3{,}850{,}000 \text{ in.-lb}$$

A built-up plate girder may be used as there are no channels of sufficient size. A wide flange beam could also be used. For a wide flange beam,

$$\frac{I}{c} = 3{,}850{,}000 \div 18{,}000 = 214 \text{ in.}^3$$

A 14- by $14\frac{1}{2}$-in. at 136 lb has $I/c = 216$. It will not detail as well with the back posts as a built-up section.

Two plate girders will be used. Using $\frac{3}{4}$-in. rivets and assuming six holes in the web plate, the depth of a $\frac{1}{2}$-in. plate for shear is found

$$6 \times \tfrac{1}{2} \times (\tfrac{3}{4} + \tfrac{1}{8}) = 2.63 \text{ sq in.}$$
$$\text{Net area for shear} = 154{,}350 \div 13{,}000 = 11.86 \text{ sq in.}$$

There are two girders; therefore the area for each is $11.86 \times \frac{1}{2} = 5.98$ sq in.

Gross area $= 2.63 + 5.98 = 8.61$ sq in.
Depth between center of gravity axis $= 8.61 \div \frac{1}{2} = 17.22$ in.

The angles will be 5 by 3 by $\frac{1}{2}$ in.; $y = 0.75$ in.

$$\text{Actual depth of plate} = 17.22 + 2 \times 0.75 = 18.72 \text{ in.}$$

A 19- by $\frac{1}{2}$-in. plate will be tried.

Area of two angles $= 2 \times 3.75 = 7.50$ sq in.
$h = 19 - 2 \times 0.75 = 19 - 1.50 = 17.50$ in.
Area of two rivets holes $= 2 \times \frac{1}{2} \times (\frac{3}{4} + \frac{1}{8}) = 0.88$ sq in.
Net area $= 7.5 - 0.88 = 6.62$ sq in.
$M = hfF = 17.50 \times 18{,}000 \times 6.62 \times 2 = 4{,}170{,}000$ in.-lb

This is larger than necessary. If smaller angles are used, they do not provide sufficient moment. The plate selected just provides for the shear. For the back-post sheave girders, two plate girders, each consisting of four 5- by 3- by ½-in. angles with 19- by ½-in. web plates, will be used.

Girders for the Cage. Member 3-4 on both A frames will be designed as a girder to support the ends of the sheave girders. Figure 69 shows a sketch of a suitable arrangement with the loading. Until a detail drawing is made, the exact point of attachment for the sheave girders to member 3-4 is difficult to determine. The length of the diaphragm supporting the sheaves must be known. The approximate location of the sheave girder is (distance from front-post plane center line)

Vertical component distance = 7.5 ft
Component parallel to back post = 10.4 ft

The resultant of the cage rope is almost parallel to the slope of the back post. Consequently the vertical component is almost negligible compared with the other component.

From Fig. 69, the reaction on the front-post end of member 3-4 is 97,000 lb.

$$M = 97{,}000 \times 10.4 \times 12 = 12{,}100{,}000 \text{ in.-lb}$$

It is found that 7- by 4- by 1-in angles are necessary. The area is 20.0 sq in.; $y = 1.10$ in.

Net area $= 20.0 - 2 \times 1 \times (\frac{3}{4} + \frac{1}{8}) = 18.25$ sq in.
$h = 12{,}100{,}000 \div (18{,}000 \times 18.25) = 36.6$ in.
Depth $= 36.6 + 2 \times 1.10 = 38.8$ in.

Checking for shear, with a 39- by ¾-in. plate, and deducting for 11 rivet holes,

Net area $= (39 \times \frac{3}{4}) - 11 \times \frac{3}{4} \times (\frac{3}{4} + \frac{1}{8}) = 17.02$ sq in.
Actual shear $= (98{,}100 \times \cos 29°45') \div 17.02 = 5020$ lb per sq in.

As 13,000 lb per sq in. is permitted, the selection is satisfactory.

For member 3-4 on each A-frame, a plate girder consisting of four 7- by 4- by 1-in. angles with a 39- by ¾-in. web plate will be used. A wide flange beam, 33 by 15¾ in. at 210 lb could have been used also. It weighs a little less than the plate girder.

Sheave Girder. Vertical Component. The reactions at member 3-4 are

$$R_1 = \frac{8.15 \times 11{,}300 + 15.15 \times 720}{20.8} = 4970 \text{ lb}$$

$$R = \frac{12.65 \times 11{,}300 + 5.65 \times 720}{20.8} = 7050 \text{ lb}$$

The greatest moment will be at the cage support.

$$M = 7050 \times 8.15 \times 12 = 690{,}000 \text{ in.-lb}$$

$$\frac{I}{c} = 690{,}000 \div 18{,}000 = 38.3 \text{ in.}^3$$

Because of the 5⁄16-in. minimum thickness, a wide flange beam, 10 by 8 in. at 41 lb with $I/c = 44.5$ in.^{3}, is used. The web thickness is 0.328 in.

Back-post Component. The reactions are

$$R_1 = \frac{8.15 \times 264{,}300 + 15.15 \times 16{,}800}{20.8} = 115{,}600 \text{ lb}$$

$$R = \frac{12.65 \times 264{,}300 + 5.65 \times 16{,}800}{20.8} = 165{,}500 \text{ lb}$$

$$\text{Maximum moment} = 165{,}500 \times 8.15 \times 12 = 16{,}200{,}000 \text{ in.-lb}$$

$$\frac{I}{c} = 16{,}200{,}000 \div 18{,}000 = 900 \text{ in.}^3$$

If a wide flange beam is used, one that is 36 by 16½ in. at 250 lb would do. This would detail with the 39-in. plate girder making up member 3-4, but would not prove satisfactory if the 33- by 15¾-in. beam was used. The web of the member chosen will be placed parallel to the resultant of the rope. The wide flange beam is a little too large to fit well without cutting away some of the flange. Two plate girders will be selected.

$$\text{Moment for each girder} = \tfrac{1}{2} \times 16{,}200{,}000 = 8{,}100{,}000 \text{ in.-lb}$$

Trying 6- by 4- by 1-in. angles, the area of two is 18.00 sq. in.; $y = 1.17$ in.; there are six rivet holes in the web and two in angles.

$$\begin{aligned} \text{Net area} &= 18.00 - 2 \times 1 \times (\tfrac{3}{4} + \tfrac{1}{8}) = 16.25 \text{ sq in.} \\ h &= 8{,}100{,}000 \div (18{,}000 \times 16.25) = 27.7 \text{ in.} \\ \text{Depth of plate} &= 27.7 + 2 \times 1.17 = 30.04, \text{ or } 30 \text{ in.} \end{aligned}$$

Two plate girders of four 6- by 4- by 1-in. angles with a 30- by ½-in. web plate are suitable.

Checking for shear,

$$\begin{aligned} \text{Area of plate} &= 30 \times \tfrac{1}{2} - 6 \times \tfrac{1}{2} \times (\tfrac{3}{4} + \tfrac{1}{8}) \\ &= 12.37 \text{ sq in.} \\ \text{Shear} &= (\tfrac{1}{2} \times 264{,}300) \div 12.37 = 10{,}700 \text{ lb per sq in.} \end{aligned}$$

From the preceding calculations, we find that rather massive members are needed for the cage sheave supports. The calculations should be repeated, using member 13-14 for the sheave-girder supports. This arrangement would require four sheave girders, two for each wheel. The following members are found to be necessary:

For front post to front post (member 13-14), a 33- by 11½-in. wide flange beam at 152 lb is needed.

For back post to back post (member 23-24), a 33- by 11½-in. wide flange beam at 125 lb is needed.

Sheave Girders. A 30- by 11½-in. wide flange beam at 152 lb is needed. Four of these are necessary.

Comparing the two possibilities, on a weight basis, it is found that the first requires 20,445 lb of steel, whereas the second requires 20,366 lb.

The difference in weight is insignificant. However, there will be a greater labor charge on the first method because of the plate girders. On the other hand, they appear to detail better with the posts and each other.

Sheave Diaphragm. One method of constructing a diaphragm is shown in Fig. 59*d*. The upper face is normal to the rope resultant; it is often made horizontal.

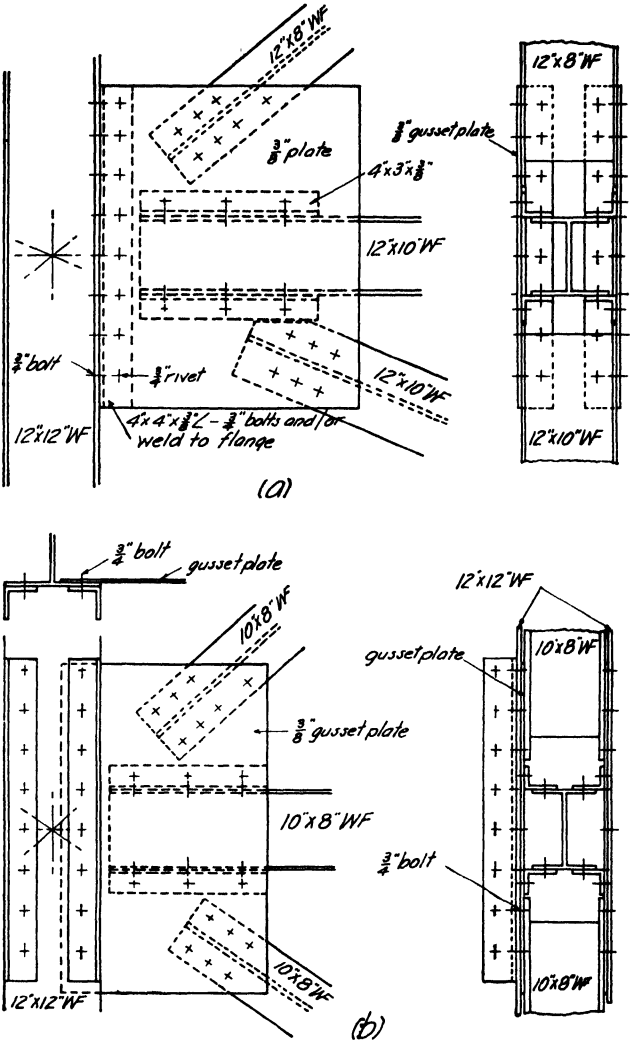

FIG. 71. Headframe details.

(If this is done, the component causing shear must be considered when selecting the bolts that connect the wheel to the diaphragm.)

From Table 31, the maximum skip resultant is 189,300 lb. There are two diaphragms per wheel; therefore each takes ½ × 189,300 = 94,650 lb.

To provide area for attaching the journal boxes and for adjustment, two 8- by 6- by ½-in. angles are used; for the bottom angles, two 5- by 5- by ⅜-in. angles are used. To provide clearance for the wheel, the depth of the plate girder along the resultant is 49 in. A $\frac{1}{2}$-in. web plate is used. Stiffener angles are 5- by 5- by $\frac{3}{8}$-in. angles.

Checking for shear, deducting 14 rivet holes,

$$\text{Area of rivet holes} = 14 \times \tfrac{1}{2} \times (\tfrac{3}{4} + \tfrac{1}{8}) = 6.12 \text{ sq in.}$$
$$\text{Area of plate} = 49 \times \tfrac{1}{2} = 24.50 \text{ sq in.}$$
$$\text{Net area} = 24.50 - 6.12 = 18.38 \text{ sq in.}$$
$$\text{Shear stress} = 94{,}650 \div 18.38 = 5160 \text{ lb per sq in.}$$

As selected, the member is more than ample; but it is necessary in order to provide for the 8-ft wheel.

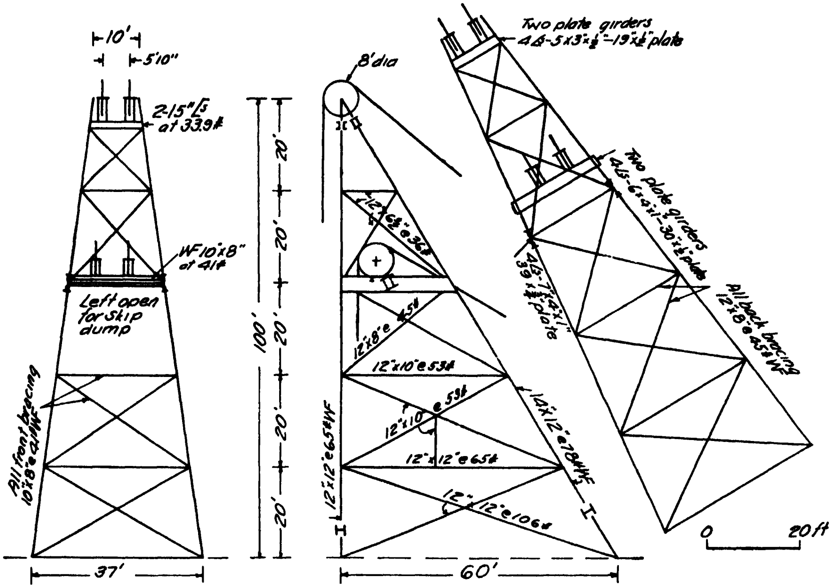

FIG. 72. Assembled four-compartment shaft headframe.

Figure 59*a* gives details of a plate girder; (*b*) shows the connection of plate girders to the posts; *c* shows the connection of a wide flange beam to posts; and *d* shows a common type of diaphragm for supporting the sheaves. The connection of the secondary members to the posts is shown in Fig. 71; *a* is in the plane of the front and back posts (the webs of the posts are parallel to the plane of the posts). To avoid subjecting the rivets that hold the gusset plate to the post to tension, $\frac{3}{4}$-in. bolts are used. The 4- by 4- by $\frac{3}{8}$-in. gusset angles could be welded to the post. Some designers combine both methods. If the rivets are designed for tension, 44.18 per cent of the allowable shearing stress is used.[1] Figure 71*b* shows the connection in the plane of the front posts or the back posts.

Figure 72 shows the headframe. Lateral bracing is placed at the top of the 20-ft and 40-ft sections. It is similar to that provided for the two-post steel headframe already investigated.

Footings for the headframe would be selected as previously explained.

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[1] "Steel Construction Handbook," p. 254.

DESIGN OF WOODEN HEADFRAMES

The design of timber headframes[1] is essentially similar to the design of steel headframes. Connections and joints, however, are more difficult to make. Certain other features occur so that it is well to call attention to these differences. As with trusses, we are immediately confronted with connection difficulties. The most involved of these is the sheave-girder design. In general, there are two types: the A-frame and the four-post types. Each of these have a different type of sheave girder. For various methods that may be used, see footnote. Several ways are suggested in Fig. 73. Timber headframes may be built of solid material with tension members of steel rods or of built-up (laminated) columns for the compression members, or timber connectors may be used. Splicing and joint details are considered exactly as with trusses. Because headframes are usually exposed to the weather and in some instances to upcast air from the mine, the values given in Table 11 should be used, without increasing them. There may, of course, be exceptions, for instance, in the case of an enclosed structure.

Combination of Stresses. Essentially the same procedure is followed as with steel headframes. In the case of the steel frames we made use of the l/r ratio and an assumed r corresponding to a shape listed in the steel handbook. For convenience, this technique is altered somewhat when selecting timber members.

Working Loads. The allowable stress is one-half the allowable stress for dead loads.

Breaking Loads. The allowable stress is 1½ times the allowable stress for dead loads.

Wind Loads. If wind loads are considered, the following applies: When the wind-load stress is greater than 25 per cent of the dead load plus 2 times the working load, then the area provided must include an allowance for wind-load stress as follows:

$$[\text{Wind load} + (2 \times \text{working load}) + \text{dead load}] \div 1\tfrac{1}{2} \times \text{dead load} = \text{area}$$

EXAMPLE

Dead load = +8,100 lb
Working load = +2,900 lb
Breaking load = +22,400 lb
Wind load = +15,100 lb

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[1] Staley, W. W., Design of Small Wooden Headframes, *U. S. Bur. Mines Inf. Cir.* 6943 (1937).

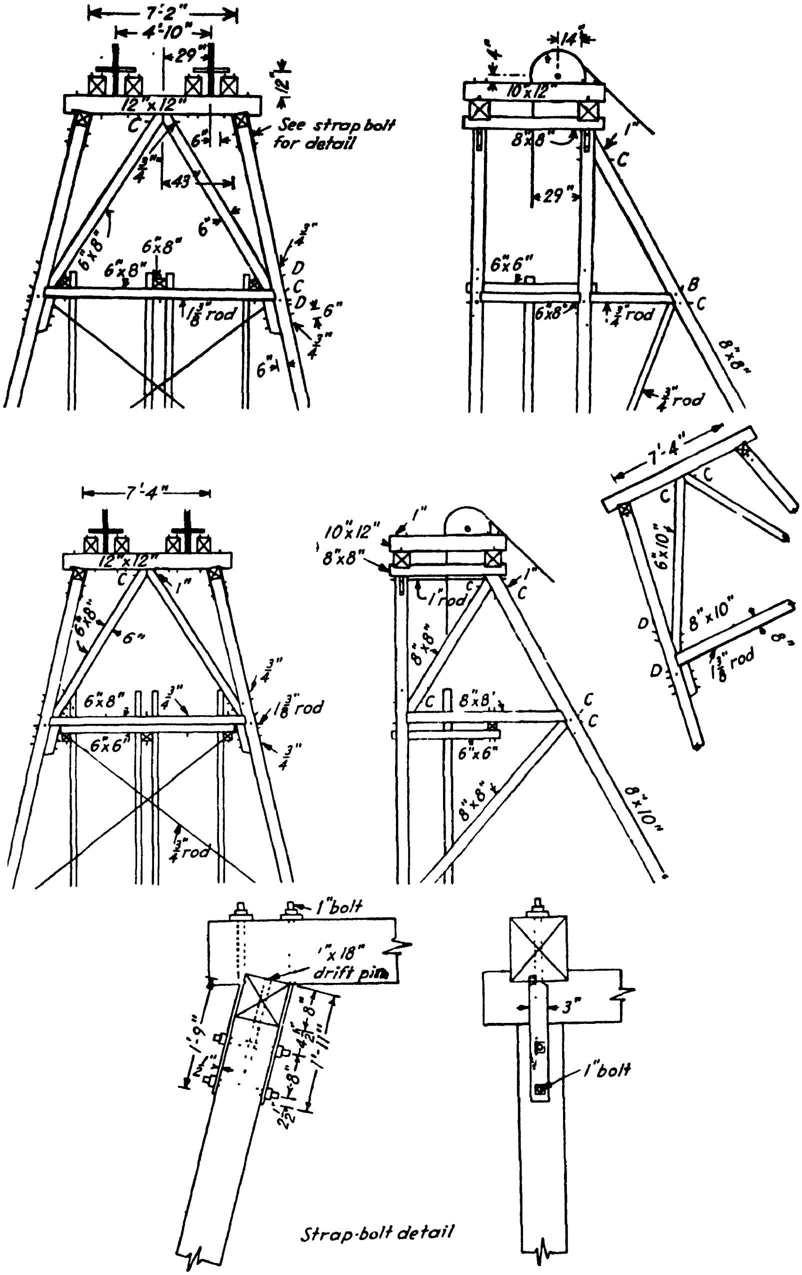

FIG. 73a. Sheave girders for wooden headframes.

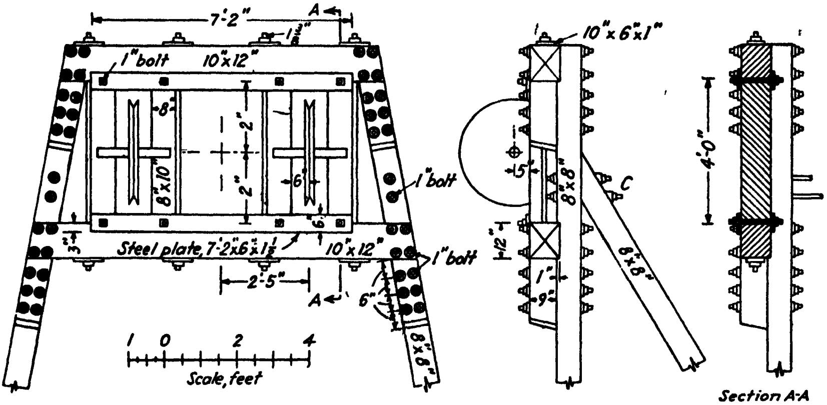

FIG. 73*b*. Sheave girder for wooden headframe.

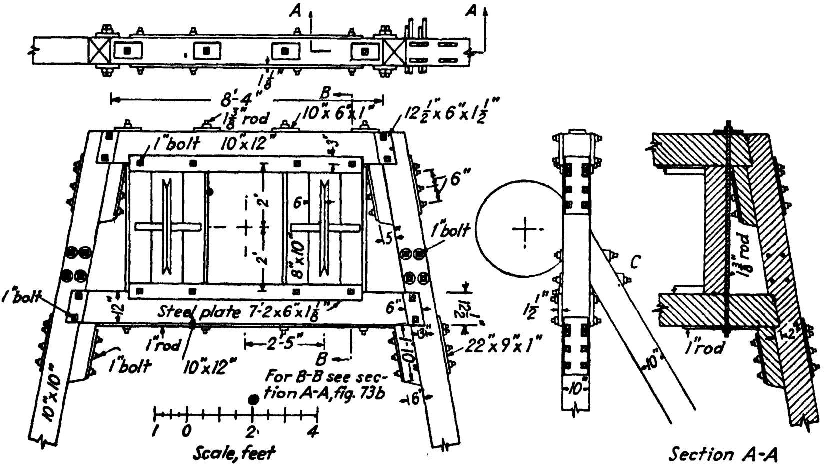

FIG. 73*c*. Sheave girder for wooden headframe.

For dead load and working load,

$$DL + 2 \times WkgL = 8100 + 2 \times 2900 = 13{,}900 \text{ lb}$$

For dead load, working load, and wind load,

$$25\% \text{ of } (8100 + 2 \times 2900) = 3475 \text{ lb}$$

The wind-load (+15,100 lb) is more than 3475 lb and must be considered.

$$(DL + 2 \times WkgL + WL) \div 1\tfrac{1}{4} = (8100 + 2 \times 2900 + 15{,}100) \div 1\tfrac{1}{4} = 23{,}200 \text{ lb}$$

For dead load and breaking load,

$$DL + (BL \div 1\tfrac{1}{2}) = 8100 + (22{,}400 \div 1\tfrac{1}{2}) = 23{,}040 \text{ lb}$$

Of these three combinations, that including the wind load happens, in this example, to be the largest. If the column is assumed to be 15 ft long and made of a 1200 lb per sq in. wood, then from

$$\frac{P}{A} = 1200 - 20\frac{L}{D}$$

and trying a 6- by 8-in. timber,

$$\frac{P}{5.5 \times 7.5} = 1200 - 20 \times \frac{15 \times 12}{5.5} = 545 \text{ lb per sq in.}$$

$$P = 5.5 \times 7.5 \times 545 = 22{,}481 \text{ lb}$$

This is well within 5 per cent of the actual load.

Weight of Timber Headframes. There are insufficient data published to provide any sort of rule for estimating the weight of wooden headframes. In the previous reference to wooden headframes, the weights of three different types of structures were calculated. The average was about 9000 lb. The dimensions were as follows:

Height to center of sheave = 36 ft
Front post to back post (average) = 21 ft
Front post to front post (bottom) = 22 ft
Three panels (12 ft vertically between panels)

For estimating purposes, if the height ratio of the proposed headframe to 36 ft, is taken and is multiplied by 9000 lb, it will give a close guess. The 9000 lb does not include the sheaves.

A headframe to satisfy the following conditions will be selected. The height to center of sheave is 50 ft. The shaft is 500 ft deep and has three compartments, two hoisting and one manway. The hoisting compartments are to take a cage carrying one 20-cu ft (1-ton capacity) car. The shaft timbers are 10 by 10 in. The headframe will be divided into four panels, as shown in Fig. 75 and 78. The timber is Douglas fir with the values given in Table 11.

A headframe of this type would be selected for a mine with relatively small tonnage, for instance, 100 to 150 tons per shift of 7 hours. Forces resulting from accelerating the load need not be considered. Wind-load stresses will not be considered.

From a car manufacturer's catalogue, a 20-cu ft car is found to be 52¾ in. long and 34 in. wide and to weigh 1000 lb. Assuming the cage deck is about 7 in. more than the length of the car, the cage would be 60 in. long. The width to the face of the guides shoes is about 8 in. greater than the car.[1] Hence, the over-all dimensions of the cage would be 42 in. wide by 60 in. long. Figure 74 gives the layout of the shaft. This is necessary to determine the distance from the center of the shaft to the front posts of the headframe, the sheave-girder design, the location of footings, etc.

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[1] Tillson, B. F., "Mine Plant," American Institute of Mining and Metallurgical Engineers, 1938.

Size of Hoisting Rope. From Table 37c, we find the cage will weigh about 2000 lb. The total weight suspended on the rope is

2000 (cage) + 1000 (car) + 2000 (ore) = 5000 lb

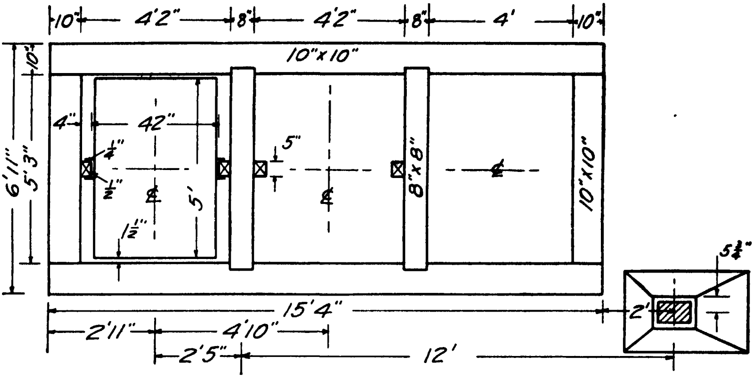

FIG. 74. Shaft collar for wooden headframe.

The factor of safety (both men and ore are handled on the cage), from Table 17, is 8. Because the rope weight is not yet included, 8½ will be used as a trial.

Ultimate strength of rope = 5000 × 8½ = 42,500 lb
= 21¼ tons

From Table 19, a ¾-in. special rope is indicated. It weighs 0.9 lb per ft.

Total weight of rope = (500 + 50) × 0.9 = 495, or 500 lb
Total load supported by rope = 5000 + 500 = 5500 lb
Ultimate strength with factor of safety of 8 = 5500 × 8 = 44,000 lb
= 22 tons

The ¾-in rope selected has a breaking strength of 23.7 tons and is satisfactory.

The working load for determining the stresses in the headframe is 5500 lb.

In designing the headframe, the following loads will be considered.

1. Dead load
2. One cage loaded, and one rope breaking

If boxed in or exposed to exceptionally violent wind storms, the wind load should be included.

Sheaves. If the diameter of the sheave is taken at 70 times the rope diameter (page 111),

70 × ¾ = 52½ in. groove diameter

A bicycle type of sheave with a groove diameter of 48⅜-in. with two-ring oiling boxes and shaft weighs 1040 lb.[1]

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[1] Diameter and weight will vary with the manufacturer's catalogue consulted.

From page 176, the weight of the headframe is estimated to be

$$\tfrac{50}{36} \times 9000 = 12{,}500 \text{ lb}$$

There are two sheaves.

$$\text{Total weight} = 12{,}500 + 2 \times 1040 = 14{,}580 \text{ lb}$$

There are 10 panel points, including the two at the apex.

$$\tfrac{1}{2} \times 12{,}500 \div 10 = 625 \text{ lb at each panel}$$

Apex load $= 2 \times 625 + 1040 = 2290$ lb

Dimensions. The vertical height is 50 ft from the shaft collar to the center of the sheave. The center-to-center distance between hoisting compartments is 4 ft 10 in. For the sheave selected, the length of the shaft is $38\tfrac{1}{2}$ in. The top of the frame should have a minimum width of

$$4 \text{ ft } 10 \text{ in.} + \tfrac{1}{2} \times 38.5 \text{ in.} + \tfrac{1}{2} \times 38.5 \text{ in.} = 8 \text{ ft } \tfrac{1}{2} \text{ in.}$$

For design purposes, it will be taken as 10 ft. The width of the frame at the bottom is found from Fig. 74. It is a $2 \times 12 = 24$ ft. The batter of the front posts in the plane of the A-frame is

$$(24 - 10) \div 2 = 7 \text{ ft in } 50 \text{ ft}$$

This is a batter of 1.68 in. per ft, which is within the usual limits.

The rope will make an angle of 45 deg with the horizontal. Then the resultant will be 22°30′ from the vertical and will intersect a horizontal line along the depth, at the elevation of the shaft collar, of

$$50 \times \tan 22°30' = 20.7 \text{ ft}$$

If this is considered as being two-thirds of the depth,

$$\text{Depth} = 20.7 \times \tfrac{3}{2} = 31.05, \text{ or } 30 \text{ ft}$$

The dimensions are

Width.
  Top = 10 ft
  Bottom = 24 ft
Depth = 30 ft
Height to center line, sheave = 50 ft

The actual length of a front post equals $\sqrt{50^2 + 7^2} = 50.5$ ft; of the back posts, $\sqrt{50.5^2 + 30^2} = 58.7$ ft. Figure 75*a* shows the solution for dead-load stresses.

Live loads. From a solution similar to Fig. 47, the factor for the resultant is found to be 1.86, so that $F = 1.86S$

RESULTANT FORCES

Loading	S, lb	F, lb
Cage, car, ore, and rope.	5,500	10,220
Breaking load.	47,400	88,200

According to the procedure used for Fig. 48, the following values are obtained.

Loading	F_1	F	P_1	P	E_1	E
One cage loaded and one rope breaking. . .	10,220	88,200	30,320	68,100	30,600	68,800

Figure 75*b* shows the live-load stresses in the A-frame.

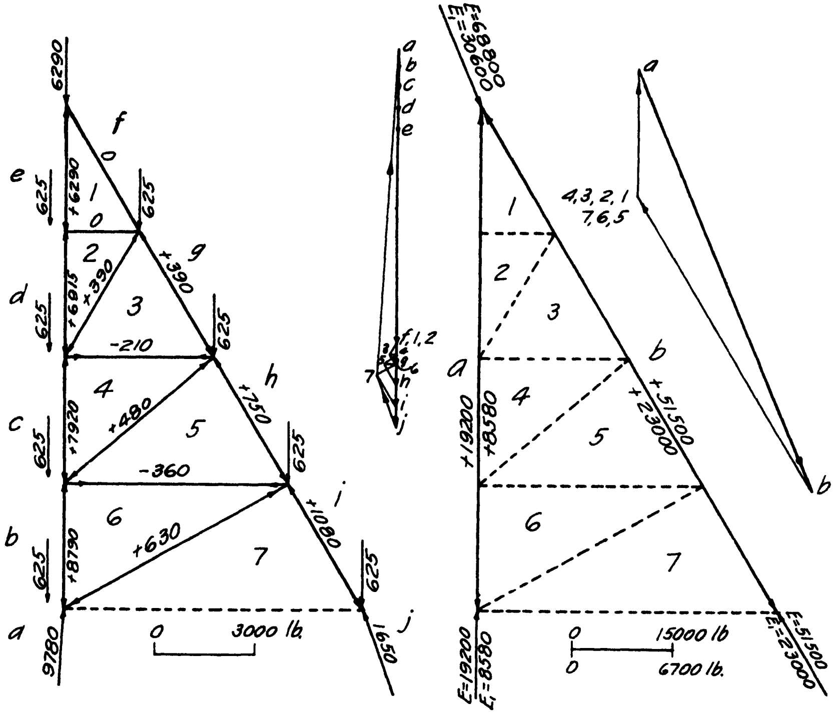

FIG. 75. Dead-load and live-load stresses for wooden headframe.

The angle at the apex between the front post and the back post is

$$\tan^{-1} = {}^{30}\!/_{50} = 31 \text{ deg.}$$

The angle between the resultant and the back post is

$$31 \text{ deg} - 22\tfrac{1}{2} \text{ deg} = 8\tfrac{1}{2} \text{ deg.}$$

P and P_1 must be resolved into components parallel to the back post and parallel to the front posts. These components and the forces P and P_1 make a triangle with side P or P_1 and angles 8½ deg, 22½ deg, and 149 deg opposite the known side. Then, by the law of sines, For P_1,

Back-post component = 22,600 lb
Front-post component = 7700 lb

For P,

Back-post component = 50,700 lb
Front-post component = 17,400 lb

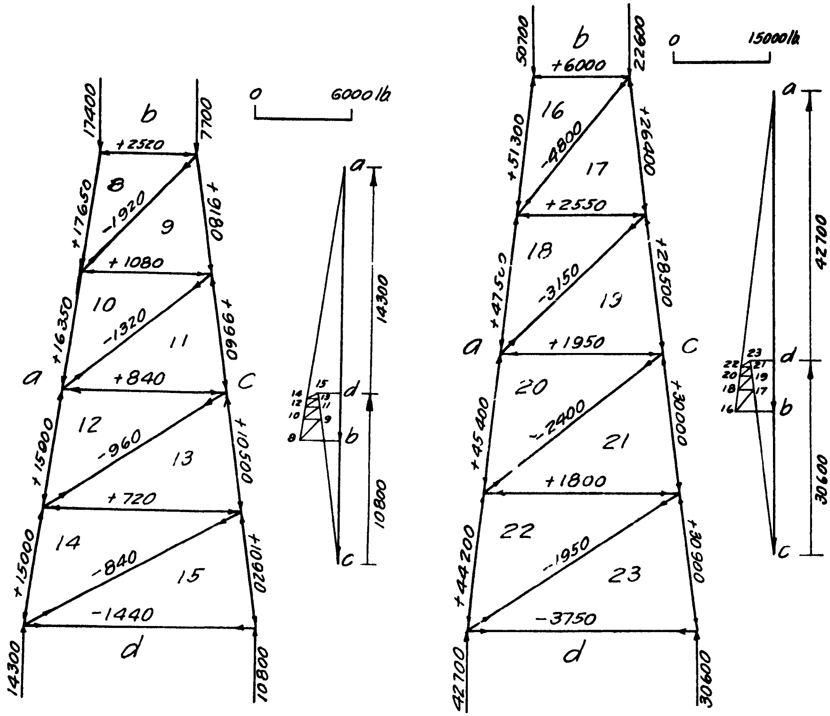

FIG. 76. Transverse-frame stresses, wooden headframe.

Figure 76 shows the front and back transverse-frame stress diagrams. The stresses are tabulated in Table 34.

Because of the detail and the stresses involved, it is well to start the design of a wooden headframe at the sheave girders.

Design of Sheave Girders. The selection of the sheave girders will be based on the forces F and F_1. Force F is the resultant of the breaking strength of the rope. These forces must be resolved into their vertical and horizontal components.

For F_1,

Horizontal component = 10,220 × sin 22°30′ = 3890 lb
Vertical component = 10,220 × cos 22°30′ = 9450 lb

For F,

Horizontal component = 88,200 × sin 22°30′ = 33,700 lb
Vertical component = 88,200 × cos 22°30′ = 81,400 lb

Sheave Girders. There will be an upper and lower sheave girder, and each will be assumed to take one-half the vertical component (the girder will be tied together with the sheave-girder posts so that this assumption is justified).

The loading is ½ × 9450 = 4725 lb, and ½ × 81,400 = 40,700 lb. (They may be taken at 4700 lb and 40,700 lb.)

TABLE 34. MAXIMUM STRESSES IN WOODEN HEADFRAME

Member	Dead load	Live load, A-frame	Transverse frame		Approximate length,
			Front	Back	
Front posts	+8790	+19,200	+17,650	. .	50′6″
Back posts	+1080	+51,500	... .	+51,300	58′8½″
1-2	0	0	... ..	. . .	7′6″
2-3	+390	0	....	.....	14′7″
3-4	−210	0	....	.	15′0″
4-5	+480	0	....		19′7″
5-6	−360	0	. .	...	22′6″
6-7	+630	0	...	.....	25′9″
b-8	.....		+2520	. .	10′0″
8-9	......		−1920	.	17′4″
9-10	....	. .	+1080	. . ..	13′7″
10-11	...	.	−1320	.	19′9″
11-12	.		+840	. . .	17′0″
12-13	... ..	. .	−960		22′8″
13-14	..		+720	. .	20′6″
14-15	. .		−840	....	25′7″
15-d	....	. .	−1440	......	24′0″
b-16	... ..	. .	....	+6000	10′0″
16-17	......	. . .	....	−4800	18′10″
17-18	... ..	....	....	+2550	13′7″
18-19	......	....	....	−3150	21′2″
19-20	......	....	. ...	+1950	17′0″
20-21	......	....	....	−2400	24′0″
21-22	......	....	. ...	+1800	20′6″
22-23	......	... .	... ...	−1950	26′7″
23-d	.... .	... ...	. . .	−3750	24′0″
Reactions:					
Front posts.....	9780	19,200	14,300		
Back posts......	1650	51,500	....	42,700	

The distance between the two loads is (Fig. 74) 4 ft 10 in.; the length of the girder was taken at 10 ft. It is a simple beam with two concentrated loads, each 2 ft 7 in. from the end. By moments the reactions are

$$R = \frac{4700 \times 2.583 + 40{,}700 \times 7.416}{10} = 31{,}400 \text{ lb}$$

$$R_1 = \frac{40{,}700 \times 2.583 + 4700 \times 7.416}{10} = 14{,}000 \text{ lb}$$

The maximum bending moment is at the 40,700-lb load.

$$M = 31{,}400 \times 2.583 \times 12 = 975{,}000 \text{ in.-lb}$$

For the unit stresses to be used, the face values of Douglas fir (see Table 11) were selected. Members such as the sheave girders and the sheave posts should be made of select structural-grade timber. This will materially reduce their size and prevent a top-heavy, bulky appearance for the headframe in addition to providing a safer design. For select structural-grade Douglas fir beams, the unit stress for bending may be taken at 2000 lb per sq in. This would be the value used for dead loads. On page 173, it was stated that breaking loads could be designed on the basis of 1½ times the dead-load unit stress. Then for the breaking load 1½ × 2000 = 3000 lb per sq in. may be used.

Assuming that $b = 12$ in.,

$$d = \sqrt{\frac{6M}{sb}} = \sqrt{\frac{6 \times 975{,}000}{3000 \times 12}} \times 12.8 \text{ in.}$$

To provide area for designing the connections at the posts, etc., a timber with $b = 10$ in. would be more suitable. Using this value, $d = 14.3$. A 10- by 16-in. member is required.

The headframe will be designed with timber connectors. For the conventional method, using solid members and tie rods, the principles followed for designing wooden trusses would be used. Because of the involved bracing the posts will be of solid material. This selection will permit easier connections to be made.

Sheave Posts. Two posts for each wheel are necessary. Each pair will be designed to take the horizontal component of the breaking load.

$$\text{Load per post} = 33{,}700 \times \tfrac{1}{2} = 16{,}850 \text{ lb}$$

To allow clearance for the wheels, the distance between the top and bottom girder is 60 in. Adding one-half the depth of each girder, 60 + (2 × 15.5/2) = 75.5 in., the center-to-center length of the posts is obtained. Assuming the post to act as a simple beam with a concentrated load at the center, each reaction is ½ × 16,850 = 8425 lb.

$$M = 8425 \times 75.5/2 = 317{,}000 \text{ in.-lb}$$

Taking $b = 11.5$ in., to allow width for the journal boxes,

$$d = \sqrt{\frac{6 \times 317{,}000}{3000 \times 11.5}} = 7.4 \text{ in.}$$

The posts will be 8- by 12-in. select structural grade.

At each post the vertical component is 81,400 × ½ = 40,700 lb; and at each end the connection must be designed for ½ × 40,700 = 20,350 lb. This should be adjusted with the live-load factor and becomes 20,350 ÷ 1½ = 13,600 lb. In selecting the split rings for attaching the posts to the girders, group A values are used; and for other connections, group B values are used.

At the sheave posts, the minimum value results from rings at 90 deg to the grain of the sheave girder.

Number of rings = 13,600 ÷ 4660 = 2.9, or 4
Per cent of full load = 2.9/4 = 68.5

The reaction at the end of each sheave girder is 31,400 lb and when corrected becomes 31,400 ÷ 1½ = 20,700 lb. The allowable load perpendicular to the grain for one ring in the sheave girder is 4660 lb for group A wood. For the front posts, group B, parallel to the grain, stands 5735 lb per ring.

Number of rings = 20,700 ÷ 4660 = 4.5, or 6
Per cent of full load = 4.5/6 = 75

Six rings, three rows of two per row, will be used.

Front Posts. The stresses are

Dead load = +8790 lb
Live load = +19,200 lb

$$DL + (BL \div 1\tfrac{1}{2}) = 8790 + (19{,}200 \div 1\tfrac{1}{2}) = 21{,}590 \text{ lb}$$

The unsupported length is 12½ ft = 150 in.

$$\frac{P}{5.5 \times 11.5} = 1200 - (20 \times 150/5.5) = 654 \text{ lb per sq in.}$$
$$P = 5.5 \times 11.5 \times 654 = 41{,}300 \text{ lb}$$

This is more than sufficient, but an 8- by 12-in. timber will be used. This will make up the area removed by the numerous boltholes and in addition will improve the appearance of the structure.

Back Posts. The stresses are

Dead load = +1080 lb
Live load = +51,500 lb

An 8- by 12-in. timber is found suitable for the back posts.

Connection of Back Posts to Front Posts. The adjusted load on the back post is 35,414 lb and it makes an angle with the front post of about 31 deg (this is explained later).

Number of rings = 35,414 ÷ 5000 = 7.1, or 8

Figure 77 shows the details; for other suggestions, see Fig. 73.

The remaining members—bracing and struts—are selected according to principles already used. By using split rings, no steel tie rods are necessary. Compression members consist of two members with spacer blocks; tension members are of single pieces. Because of the exposed condition of a headframe, the minimum thickness of the secondary members is 3 in. All braces and struts will be 3- by 6-in.

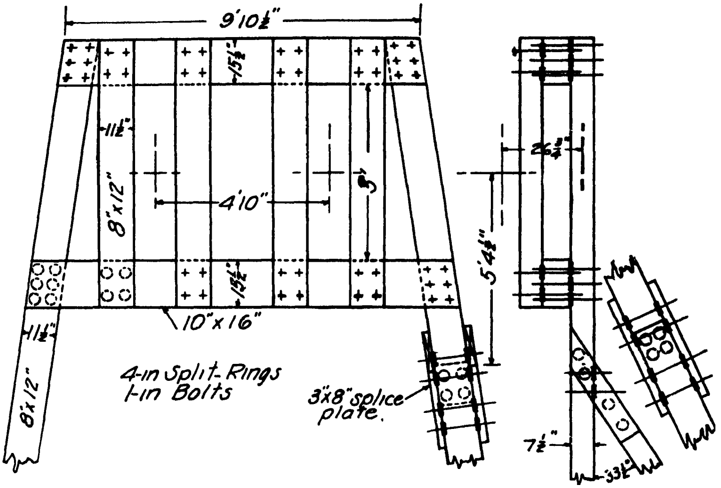

FIG. 77. Sheave-girder details, wooden headframe.

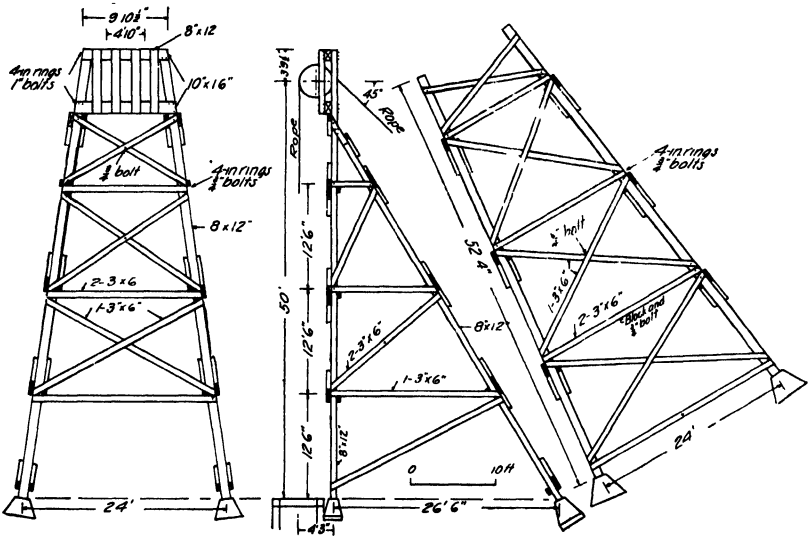

FIG. 78. Wooden headframe.

pieces with 4-in. split rings and ¾-in. bolts. The sheave girders and posts are fastened with 1-in. bolts.

The assembled headframe is shown in Fig. 78. For further details, the reference on wooden headframes should be consulted.

On making a detailed drawing of the apex part of the headframe it is discovered that a minor adjustment of the angle between the back posts and the front post would be necessary. The center of the sheave is found to be about 26¾ in. in front of the vertical center line through the front posts. This is the moment arm of the vertical component. Balancing this moment is the horizontal component times the distance from the center of the sheave to the intersection of the front- and back-post center lines.

This distance is

$$x = \frac{26\tfrac{3}{4} \times 81{,}400}{33{,}700} = 64\tfrac{1}{2}\text{ in.} = 5.38\text{ ft}$$

$$\text{Apex angle} = \tan^{-1} = 30 \div (50.7 - 5.4) = 33°30'$$

The stress diagrams were drawn on the basis of an angle of 30°40′. This change in angle will have no effect on the practical solution of the problem. Redrawing the live load diagram for the A-frame with the adjusted setup (height to apex 45.3 ft, depth 30 ft, as before) indicates a lowering of the stress in the back post of about 8 per cent and an increase in the front post of about 25 per cent. If the depth of the headframe is changed to 26½ ft, the stresses remain as before, or the angle of the rope with the horizontal can be changed to 45° − 3° = 42 deg. The first alternative will be used.

Footings. By following the method suggested on page 99, the size of the footings may be found. They are plain concrete. The bearing on the ground is taken at 4000 lb per sq ft and on the concrete at 600 lb per sq in. With 1200 lb per sq in. for the posts, no base plates are needed.

The load on the front post is

$$9780 + (19{,}200 \div 1\tfrac{1}{2}) = 22{,}630\text{ lb}$$

The load on the back posts is

$$1650 + (51{,}500 \div 1\tfrac{1}{2}) = 36{,}150\text{ lb}$$

The size of the footings are

	Front post	Back post
Top	10½″ × 1′4¼″	10½″ × 1′4¼″
Bottom	2′2¾″ × 3′4¾″	2′9¾″ × 4′3¾″
Depth	2′2″	3′0″

To help hold the posts on the footings, ¼- by 6- by 7½-in. and ¼- by 6- by 11½-in. steel strips will be set 4 in. into the concrete. This will leave a 2-in. projection to enclose the posts.

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CHAPTER V

ORE BINS

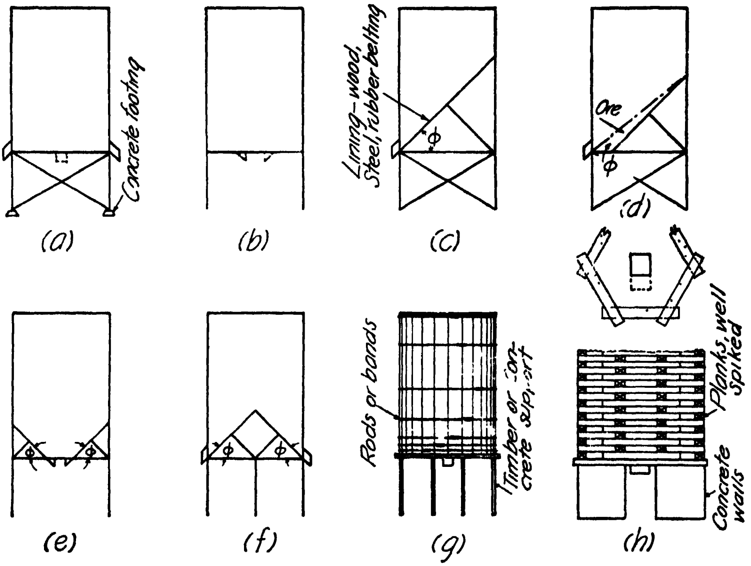
There are exceedingly few prospects or mining operations where an ore bin does not soon become necessary. The capacity of the bins ranges from a few tons to several thousand tons. The shape varies from square to round. The material from which these bins are constructed, depends, to quite some extent, on the capacity—the small ones usually being of wood, and the larger ones of steel or concrete or a combination of these two. When the capacity exceeds a thousand tons or so, suspension bunkers are common.

Considerable discussion has been waged regarding the size of the bin for a given mine output. Within limits, it is generally regarded as good practice to provide a bin at the mine that will store an amount that is from one-half the mine output to the whole mine output per 24 hr. The mining method in use must be considered. Some types of mining provide plenty of storage space underground (stopes, transfer raises, skip pockets). For the concentrator the same reasoning is applied, *i.e.*, to provide sufficient storage space to keep the mill operating for 24 hr if the mine is down or to prevent shutting the mine down if the mill ceases operating.

In times past, too much attention has not been required to keep down the cross-sectional dimensions of timbers. Now large dimensions are almost impossible to obtain. When designing with timber, this must be kept in mind and dimensions exceeding 16 in. avoided when possible. Within the limits of acceptable designing practice there would be no limitations on the use of concrete or steel.

The final design of a wooden ore bin presents much the same difficulties as the design of wooden trusses; joints and connections are troublesome. Many members are oversized because of bearing stresses perpendicular to the grain. A saving of material and the elimination of practically all the long tie rods can be had if timber connectors are used. Massive timbers are replaced by planks and all but one or two of the tension members will be of timber. The posts can run continuous from top to footing; in the conventional type of construction they are interrupted by a horizontal sill. Serious consideration should be given to the use of timber connectors when designing a wooden ore bin.

Bins of a great many geometrical shapes have been erected. The following forms are common; inclined bottoms; partly inclined bottoms; flat bottom; chutes on the outside wall; chutes near the center line of the bottom; round; square; rectangular; hexagonal. Circular bins of steel, of wood staves with surrounding steel rods or flat bands, or of reinforced concrete are rather common. This type of bin may rest on wood sills supported by posts on footings, on concrete walls, or on steel beams.



(a) (b) (c) (d)

(e) (f) (g) (h)

FIG. 79. Common types of ore bins.

Figure 79 shows the more common types of bins. In the figure, types *a* and *b* are flat-bottomed; *a* shows chutes on the outside or, alternatively, a single row under the bin may be used. In *b* the chutes alternate, left and right. The bin shown in *c* is a very common type. The slope of the floor, to ensure self-discharging, should be about 5 deg greater than the angle of repose ϕ. In *d* is a type recommended by the Southwestern Engineering Co.[1] The ore builds up to the angle of repose and forms the floor. This saves wear and tear on the bin and does not tie up too much ore as does the flat-bottomed type.

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[1] MAAG, E. A., Ore Bin Design, *Eng. Mining J.* **139**, No. 1, 66, (1938).

Figure 79*g*, shows a round bin. Instead of the design shown, concrete or steel plates may be used. The base may be of wood sills, steel beams, or reinforced concrete slabs. A hexagonal bin is shown in *h*. This is a very simple and economical bin to construct, especially when planks are readily available. The structure is held together with spikes that penetrate well into each third plank. The planks alternatively overlap at the ends, and are shimed up between the layers with short members. The hexagonal or octagonal bin has also been built of square timbers mortised at the ends like raise cribbing.

Bins may be loaded in one of two ways: level full or surcharged. Only the level-full loading will be considered. Hardly ever are conditions such that mining operations permit or require a surcharged bin.

The principles involved in the design, so far as the simple bin (other than suspension bunkers or surcharged bins) is concerned, will depend on whether the sloping bottom (self-discharging) or flat-bottom bin is selected. Of the bins shown in Fig. 79, *a*, *b*, *g*, and *h* would use the same fundamental principles; and *c*, *d*, *e*, and *f* would use a modification of these principles.

It is necessary to find the pressure acting on the walls and bottom of the bin. If the contents were a true liquid, the problem resolves itself into the expression

$$P = wh \qquad \text{lb per sq ft}$$

where w = weight of 1 cu ft of the liquid, lb
h = depth from surface to point where pressure is to be found (usually the bottom), ft

Broken material like ore does not flow like water. Therefore, the angle of repose ϕ and the friction angle θ enter into the problem. Values for θ depend on the bin lining: wood, discarded rubber belting, steel, or concrete. It would appear to vary from about 40 deg to 45 deg. The angle of repose varies from about 38 deg to about 45 deg for mine-run ore, the former being the more common.

Figure 80 shows the graphical solution of the stresses in a flat-bottom bin, regardless of periphereal outline. A method commonly used[1] is to calculate N_1 from

$$N_1 = \frac{1}{2} wh^2 \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

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[1] KETCHUM, M. S., "The Design of Mine Structures," p. 260, McGraw-Hill Book Company, Inc., New York.

where N_1 = total normal pressure, lb
w = weight per cu ft, lb
h = depth of bin, ft
ϕ = angle of repose

The results thus obtained are considerably greater then when the friction angle enters the calculations. As ϕ and θ increase in value, the disagreement also increases. As shock and wear and tear are difficult to allow for, the formula probably should be used instead of the more exact solution involving friction.

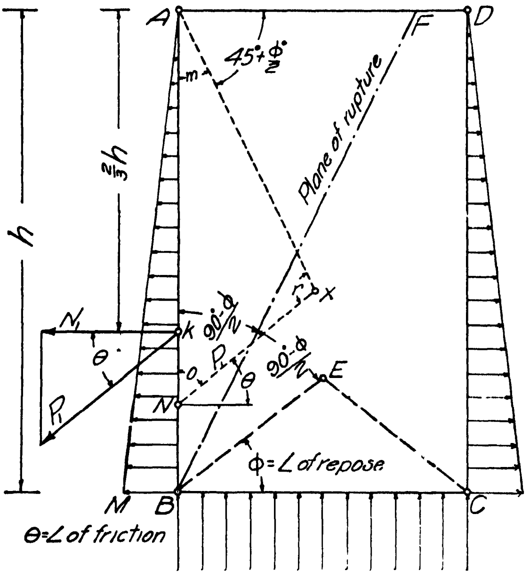

FIG. 80. Stresses in flat-bottomed bin.

In Fig. 80, the essential features will be discussed. Triangle *BEC* represents the material at the angle of repose ϕ. The plane of rupture bisects the angle $90° - \phi$, so that angle *ABF* equals $(90° - \phi)/2$. The plane of rupture is important because it should cut the top of the bin and not fall below *D*. By geometry it can be demonstrated that angle *DAx* equals 45 deg + $\phi/2$, when the effect of the resultant of the mass to the right of *BF* on *ABF* acts in a direction inclined ϕ below the normal to *BF*.

Point *N* is found by calculating the weight of triangle *ABF*; *i.e.*, the area of the figure multiplied by the weight of 1 cu ft of the ore equals *ABF*. To some suitable scale this weight is laid off on *AB*, establishing *N*. Line *Ax* is intersected by *Nx*, *Nx* making an angle with the horizontal of θ deg, the friction angle. Then $P_1 = Nx$ = the total pressure. The point of application of P_1 is at k, $\frac{2}{3}h$ below the top of the bin. By resolving P_1 through θ, the horizontal component N_1, normal to the bin wall, is found. This is the total pressure acting normal to the wall. The pressure at $CL = BM$ is found from

$$N_1 = \tfrac{1}{2}CL \times h$$

or

$$CL = BM = \frac{2N_1}{h}$$

Triangles ABM and DCL may then be drawn. Pressure at any point on DC or AB may be scaled off. The pressure on the flat bottom is directly proportional to the depth of the ore.

In designing bins it is convenient to assume a slice of material 1 ft. thick and to carry out stress calculations on this basis. The above explanation of Fig. 80 was made with data chosen in this way.

When designing cylindrical bins or tanks, the thickness of the steel shell, or the concrete, or the diameter of the circular rods, or the reinforcing steel is found from (see Fig. 83a)

$$PD = 2st$$

where P = internal pressure, lb per sq in.
= $w/144h$.
= graphical solution shown in Fig. 80, or from

$$N_1 = \frac{1}{2}wh^2\left(\frac{1 - \sin\phi}{1 + \sin\phi}\right)$$

w = weight per cu ft of material, lb
h = height of ore or liquid column, ft
D = diameter of bin or tank, in.
t = thickness of steel shell, or bands, or reinforcing steel, in., or for round bars, $t \times$ 1-in. $= a = \pi d^2/4$, where $d = \sqrt{4a/\pi}$ = diameter of rod, in.
s = allowable unit stress in steel, rods, etc., lb per sq in.

The above formula holds when d is greater than 30 t.[1]

Dimensions of Ore Bins. In many instances one or more of the dimensions will be limited by the location of the bin. Its height may be controlled by the difference in elevation between the mine portal or

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[1] See any treatise on hydraulics.

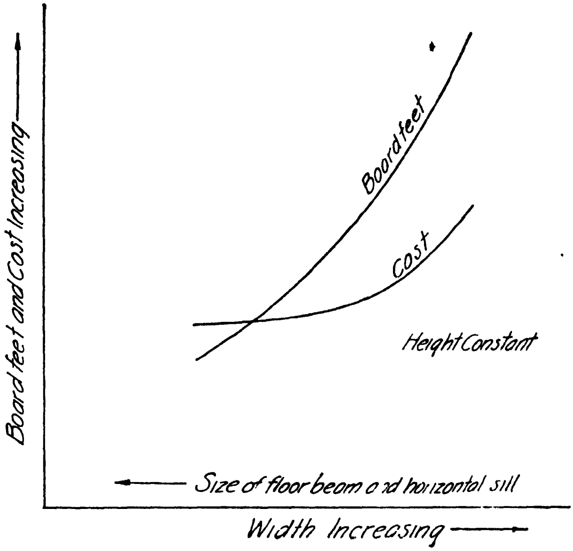

FIG. 81. Cost of ore bins.

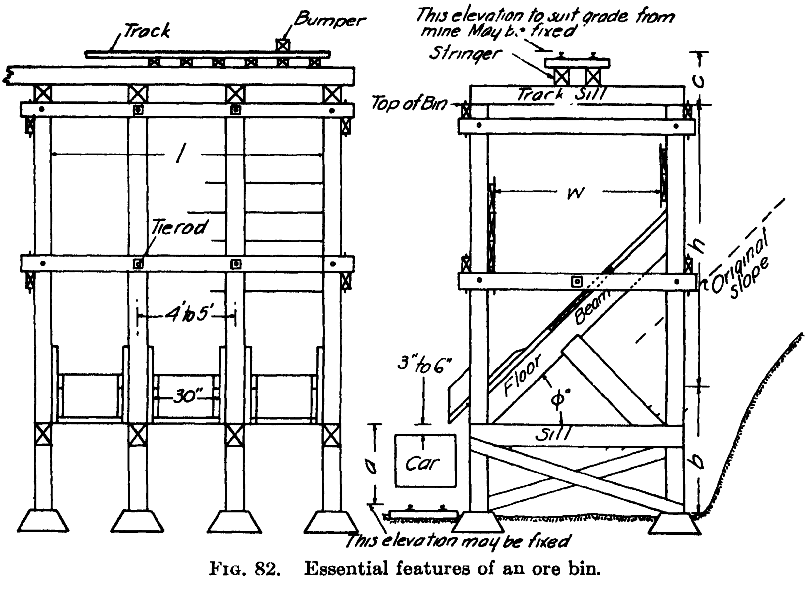

FIG. 82. Essential features of an ore bin.

shaft collar and the road or track or conveyer belt for discharging the bin. The width may be limited by the amount of excavating required to set the structure on a hillside. Ordinarily no restrictions are imposed on the length, which is fortunate. From the standpoint of cost and cross-sectional dimensions of the timber, a low, narrow, long bin is best; *i.e.*, it is best to have a number of sections rather than fewer sections of great height and width. Preliminary calculations[1] gave the information shown in Fig. 81.

The spacing between the floor beams should not be excessive. A distance just sufficient to build in the chute (4 to 5 ft) is desirable. This may require an extra chute for a given bin, but the size of the beams will be materially reduced because the load is distributed over more of them. No great change in the amount of timber required is experienced.

After the preliminary dimensions have been selected, the plane of rupture should be located in order to make sure that the bin is not too narrow. The preceding remarks are meant to convey the idea that capacity should be gained through length and not by height or width for the most economical results.

Figure 82 is a sketch of a bin listing various dimensions. The overall height is $c + h + b$; h is the effective height so far as the capacity is concerned.

DESIGN OF CYLINDRICAL BINS

A bin to suit the following requirements will be designed.

Height = 15 ft = h
Ore = 100 lb per cu ft = w
Capacity = 100 tons
Car = 20 cu ft (53 in. long × 34 in. wide × 42 in. high)
Angle of repose = 38 deg = ϕ
Friction angle = 45 deg = θ
Gate slope = 45 deg
Gate opening = greater than 30 in.
Material of construction—wood with round steel rods and concrete footings
  Wood species—Douglas fir (s = 1200 lb per sq in. for planks and staves; 300 lb per sq in. for bearing; 1800 lb per sq in. for beams and columns)
  Steel rods—10,000 lb per sq in.
  Concrete—400 lb per sq in.
  Ground bearing—4 tons per sq ft

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[1] Williams, R. T., Determining the Most Economical Dimensions of Small Wooden Ore Bins, unpublished thesis, School of Mines, University of Idaho, 1936.

Solution:

$$\begin{aligned} 100 \text{ tons} &= 2000 \times 100 = 200{,}000 \text{ lb} \\ \text{Volume} = v &= 200{,}000 \div 100 = 2000 \text{ cu ft} \\ d &= \sqrt{4v \div \pi h} \\ &= \sqrt{\frac{4 \times 2000}{15\pi}} = 13 \text{ ft} \end{aligned}$$

Total normal pressure on wall $= N_1$.

$$\begin{aligned} N_1 &= \frac{1}{2} wh^2 \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \\ &= \frac{1}{2} \times 100 \times 15^2 \left(\frac{1 - \sin 38^\circ}{1 + \sin 38^\circ}\right) = 2670 \text{ lb on a 1-ft vertical strip of the wall} \end{aligned}$$

From the area of a triangle, $a = \frac{1}{2}bh$, the base is

$$b = \frac{2a}{h} = \frac{2 \times 2670}{15} = 356 \text{ lb total pressure at bottom on 1-sq ft of wall}$$

The plane of rupture cuts the surface of the ore (top of bin) at

$$h \tan \frac{90^\circ - \phi}{2} = 15 \tan \frac{90^\circ - 38^\circ}{2} = 7.32 \text{ ft from the wall}$$

This is inside the diameter of the bin; therefore the size of the bin selected is satisfactory. With Fig. 80 as a guide, the weight of block AFB, where $AF = 7.32$ ft, for a 1-ft strip of ore is

$$\begin{aligned} a &= \tfrac{1}{2}bh = \tfrac{1}{2} \times 15 \times 7.32 = 55 \text{ sq ft} \\ \text{Weight of block } AFB &= 100 \times 55 = 5500 \text{ lb} \end{aligned}$$

From Fig. 80,

$$\begin{aligned} \angle m &= 90^\circ - (45^\circ + 38^\circ/2) = 26^\circ \\ \angle y &= 90^\circ - \theta = 90^\circ - 45^\circ = 45^\circ \\ \angle r &= 180^\circ - (m + o) = 180^\circ - 26^\circ - 45^\circ = 109^\circ \end{aligned}$$

In the triangle ANx, side AN is equal to the weight of block AFB laid out to some suitable scale. Side Nx is the pressure exerted by the block ANx. It is necessary to find this side.

$$\begin{aligned} \frac{\sin m}{\sin r} &= \frac{Nx}{AN} \\ \frac{\sin 26^\circ}{\sin 109^\circ} &= \frac{Nx}{5500} \\ \text{Side } Nx &= 2550 \text{ lb} \\ N_1 &= Nx \times \cos \theta = 2550 \times \cos 45^\circ = 1800 \text{ lb} \end{aligned}$$

This is exactly the same as if N_1 had been found graphically by the method explained by Fig. 80.

It will be noted that the first solution gave $N_1 = 2670$ lb, if the friction angle is not used. The first method is about 48 per cent higher than when friction is considered.

Allowance for shock and wear on the bin walls is difficult to estimate. For this reason, the higher value of N_1 should probably be used. Then $N_1 = 2670$ lb and the horizontal pressure at the bottom of the bin is 356 lb.

The decrease in pressure per foot of bin height is

$$356 \div 15 = 23.7 \text{ lb per sq ft}$$

Total projected pressure on one-half of the bin at the bottom is

$$P = 356 \div 144 = 2.47 \text{ lb per sq in. at the bottom of the bin}$$

From $PD = 2st$

$$2.47 \times 13 \times 12 = 2 \times 10{,}000 \times t$$

$$t = \frac{2.47 \times 13 \times 12}{2 \times 10{,}000}$$

But $t \times 1$ in. = area of rod to resist stress $= \pi d^2/4$.

Therefore

$$\frac{\pi d^2}{4} = \frac{2.47 \times 13 \times 12}{2 \times 10{,}000}$$

$$d = \sqrt{\frac{4 \times 2.47 \times 13 \times 12}{\pi \times 2 \times 10{,}000}} = 0.16 \text{ in., or } \tfrac{1}{4} \text{ in.}$$

A $\frac{1}{4}$-in. rod would be unsatisfactory for several reasons: (1) it is rather small for the exposed condition; (2) a large nunber of rings would be necessary, thus running up the labor cost of building the bin. When making a choice of the rod size, the total weight of the steel and the effect of the spacing between rods on the bending of the wooden staves must be kept in mind. If the spacing becomes too great (use of large diameter rods), then the bin walls will be excessively thick. By trial and error $\frac{3}{4}$-in.-diameter rods seem to be the most suitable. Using this diameter rod, the spacing is found.

Net area of $\frac{3}{4}$-in. rod = 0.302 sq in.

$$PD = 2st = 2s\left(\frac{\pi d^2}{4}\right)$$

$$PD = 2 \times 10{,}000 \times 0.302 = 6040 \text{ lb}$$

$$P = 356 \text{ lb per sq ft}$$

$$D = 13 \text{ ft}$$

Each rod stands 6040 lb. From

$$PD = 2s\left(\frac{\pi d^2}{4}\right)$$

a rod placed near the bottom will resist a pressure on the wall equal to the diameter times a height to be determined. This is

$$hPD = 2s\frac{\pi d^2}{4}$$

$$h = \frac{2 \times 10{,}000 \times 0.302}{P \times 13}$$

The pressure P changes at the rate of 23.7 lb per sq ft as the top of the bin is approached. Each successive area supported by the rod is found as the top is approached.

At the bottom

$$h = \frac{6040}{356 \times 13} = 1.32 \text{ ft}$$

This leaves 15 − 1.32 = 13.68 ft of bin. The pressure at this depth (13.68 ft) is

$$P = 13.68 \times 23.7 = 324 \text{ lb}$$
$$h = \frac{6040}{324 \times 13} = 1.43 \text{ ft}$$

For the next segment the depth is 13.68 − 1.43 = 12.25 ft.

$$P = 12.25 \times 23.7 = 291 \text{ lb}$$
$$h = \frac{6040}{291 \times 13} = 1.6 \text{ ft}$$

In this manner the spacing for each successive rod is found. The rods are placed at these various distances above each other. Table 35 shows the data.

TABLE 35. SPACING OF RODS FOR CYLINDRICAL BIN

Depth at which pressure is taken, H, ft	Pressure, lb per sq ft $H \times P$	h, ft, $\frac{2st}{HP}$	Distance from top of bin to rod
15	356	1.32	15' − 4" = 14'8"
15 − 1.32 = 13.68	324	1.43	14'8" − 1'4" = 13'4"
13.68 − 1.43 = 12.25	291	1.60	13'4" − 1'5" = 11'11"
12.25 − 1.60 = 10.65	253	1.84	11'11" − 1'7" = 10'4"
10.65 − 1.84 = 8.81	209	2.22	10'4" − 1'10" = 8'6"
8.81 − 2.22 = 6.59	156	2.98	8'6" − 2'3" = 6'3"
6.59 − 2.98 = 3.61	86	5.40	6'3" − 3'0" = 3'3"
			3'3" − 5'4"; top rod at 6 in.*

* The last value for h (5.4 ft) extends beyond the top of the bin. The distance to the top of the bin is 3.61 ft.

Side Planks or Staves. Figure 83*b* shows a sketch of the loading. Loads are taken as acting at the points where the rods surround the bin. The maximum moment, assuming the action as a simple beam, is found to be at the 209-lb load.

$$M_{209} = 32{,}300 \text{ in.-lb}$$

This is on a strip 12 in. wide. Assuming that the staves have a width of 6 in., the moment will be ½ × 32,300 = 16,150 in.-lb. From

$$M = \tfrac{1}{6}sbd^2$$
$$d = \sqrt{\frac{6 \times 16{,}150}{1200 \times 6}} = 3.7, \text{ or } 4 \text{ in.}$$

The sides will be made of 4- by 6-in. staves, beveled on the edges. The total bevel angle is

$$\text{Circumference of bin} = \pi D = \pi \times 13 = 40.84 \text{ ft}$$

$$\text{Bevel angle} = \frac{0.5 \times 360°}{40.84} = 4.42°, \text{ or } 4°30'$$

Each edge is beveled at $\frac{1}{2} \times 4°30' = 2°15'$.

Bearing of Rods on Staves

$$\text{Load} = P = 10{,}000 \times 0.302 = 3020 \text{ lb}$$

$$\text{Length of rod acting} = \tfrac{1}{2}\pi d = \tfrac{1}{2} \times \pi \times 13 = 20.4 \text{ ft}$$

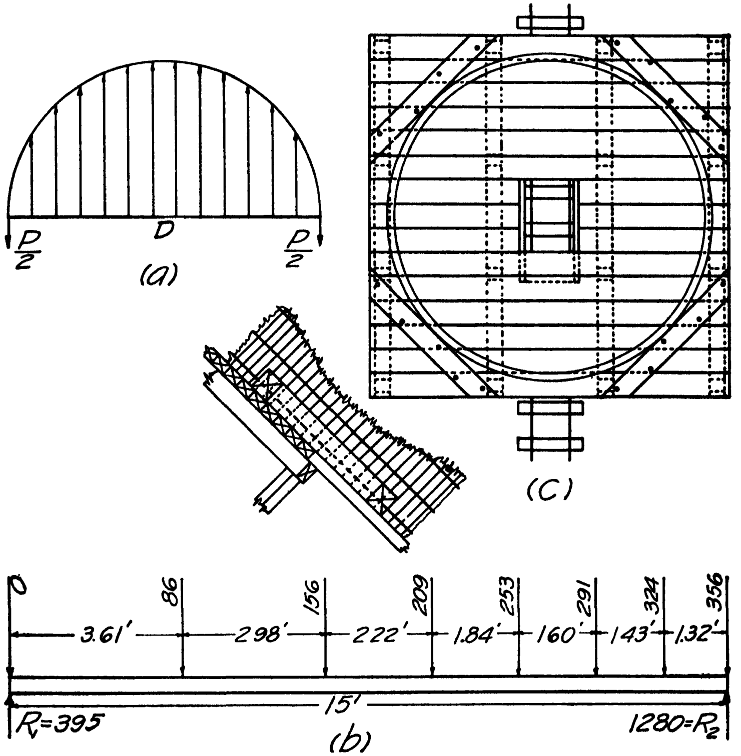

FIG. 83. Design of a cylindrical bin.

The bearing area of a ½-in.-diameter rod, using one-third of diameter as effective, is

$$\tfrac{1}{3} \times \tfrac{1}{2} \times 20.4 \times 12 = 41 \text{ sq in.}$$

$$\text{Bearing pressure} = 3020 \div 41 = 74 \text{ lb per sq in.}$$

The allowable pressure for Douglas fir is 300 lb per sq in.

Floor. The pressure is 100 lb per cu ft × 15 ft = 1500 lb per sq ft as a uniformly distributed load.

Assuming that the beams supporting the floor are 14 ft center to center of the end posts, four posts, with three spaces of 4.7 ft (4 ft 8 in.), will be necessary. This allows ample room for passage of a 20-cu ft car. The unsupported length is 4.7 ft.

$$\begin{aligned} M &= \tfrac{1}{8}wl^2 \times 12 \\ &= \tfrac{1}{8} \times 1500 \times 4.7^2 \times 12 = 49{,}700, \text{ or } 50{,}000 \text{ in.-lb} \\ d &= \sqrt{\frac{6 \times 50{,}000}{1200 \times 12}} = 4\tfrac{1}{2}, \text{ or } 6 \text{ in.} \end{aligned}$$

The floor will be of 6- by 12-in. planks.

Floor Beams. Each will support the floor load halfway to the next beam (Fig. 83*c*). There are three spans or four beams. Distance between beams, center to center, is 4 ft 8 in.

$$\text{Load} = 1500 \times (4.7/2 + 4.7/2) = 7050, \text{ or } 7000 \text{ lb per ft}$$

The maximum moment for a beam with four supports (see Fig. 14), is

$$M = -\tfrac{1}{10}wl^2 \times 12 \qquad \text{in.-lb}$$

where $l = 4.7$ ft.

$$M = -\tfrac{1}{10} \times 7000 \times 4.7^2 \times 12 = -185{,}000 \text{ in.-lb}$$

Trying $b = 8$ in., with $s = 1800$ lb per sq in.,

$$d = \sqrt{\frac{6 \times 185{,}000}{1800 \times 8}} = 8.8 \text{ in.} = 10 \text{ in.}$$

An 8- by 10-in. timber is needed.

Posts. The floor-beam arrangement calls for 16 posts.

Total load on bin = 100 tons = 200,000 lb
Load per post = 200,000 ÷ 16 = 12,500 lb

The posts will be about 5½ ft long. Trying a 4- by 8-in. post,

$$\begin{aligned} \frac{P}{A} &= 1800\left(1 - \frac{L}{60d}\right) = 1800 - 30\frac{L}{d} \\ \frac{P}{4 \times 8} &= 1800 - 30 \times \frac{5.5 \times 12}{4} = 1305 \text{ lb per sq in.} \\ P &= 4 \times 8 \times 1305 = 41{,}760 \text{ lb} \end{aligned}$$

This is far greater than needed. Before trying any other size, the 4 by 8 should be checked for bearing against the beam. Perpendicular to the grain the beam will stand 300 lb per sq in.

$$\text{Area needed} = 12{,}500 \div 300 = 41.6 \text{ sq in.}$$

The 4- by 8-in. post has only $4 \times 8 = 32$ sq in. Therefore, a 6- by 8-in. post with 48 sq in. is necessary. The smaller post with bearing plates might have been used.

Diagonal bracing, top of post to bottom of neighboring post, is used. The weight of the bin supported on the posts is about 10,000 lb. The material selected is ample to take this additional load.

Footings. Footings are selected as explained on page 99.

Summary

Wall planks or staves are 4 by 6 in. by 15 ft beveled on the edges at 2°15′
Floor planks are 6 by 12 in. by 14 ft

Floor beams are 8 by 10 in. by 14 ft
Posts are 6 by 8 in. by 5½ ft
Rods are ¾ in. diameter by 41 ft long.
Bin is 21 ft 10 in. high; 13 ft 8 in. outside diameter; 100 tons capacity

The assembled drawing of the circular bin is shown in Fig. 84. If a track is to rest on top of the bin, the effect of its weight with a loaded car must be considered.

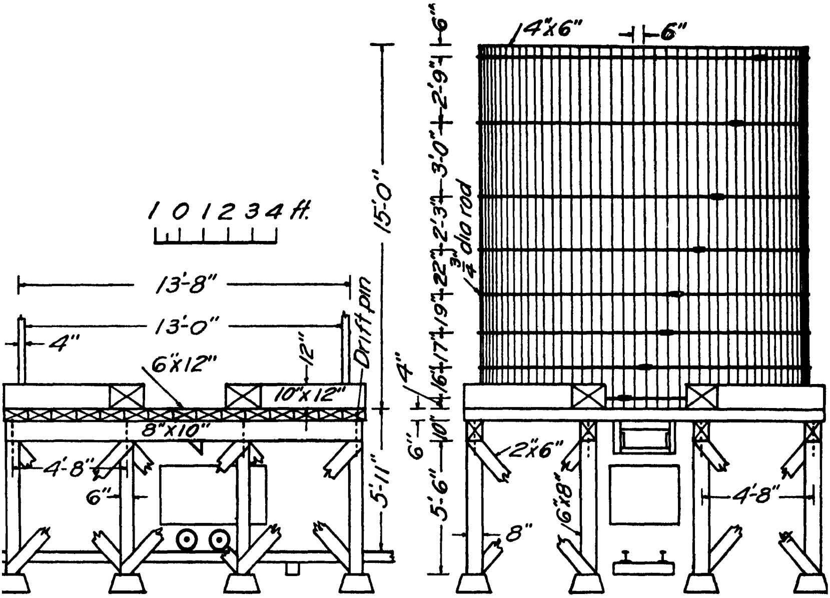

FIG. 84. Assembled cylindrical bin.

SIMPLE RECTANGULAR-TYPE BIN

A bin similar to the one shown in Fig. 79c will now be designed.[1] The bin will be designed by the conventional method and also with Teco timber connectors.

Height = 16.5 ft
Width = 12 ft
Capacity = 100 tons
Slope of bottom (from horizontal) 42 deg
Weight of ore, mine run = 100 lb per cu ft
Angle of repose = 38 deg
Size of car = 20 cu ft
Material of construction is Douglas fir
Footings are concrete

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[1] STALEY, W. W., How to Design Small Wooden Ore Bins, *Eng. Mining J.*, **138**, No. 10, 45, (1937), **139**, No. 1, 66 (1938).

MAAG *loc. cit.*

Figure 85*a* shows the dimensions of the cross section.

Dimensions of Bin. The length of the bin to hold 100 tons is first found. A section 1 ft. thick contains

$$1 \times 12 \times \tfrac{1}{2}(5.7 + 16.5) = 13{,}320 \text{ lb}$$

$$\text{Length} = (100 \times 2000) \div 13{,}320 = 15 \text{ ft}$$

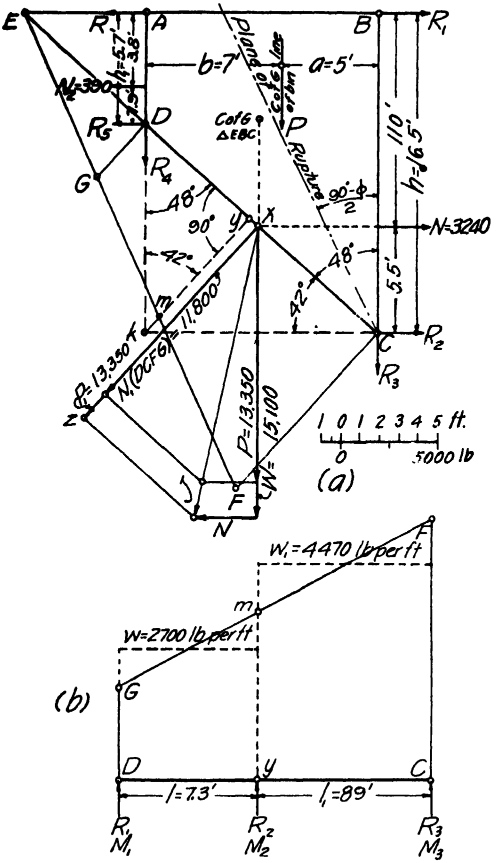

FIG. 85. Stress diagram, rectangular bin. (*Courtesy of Engineering and Mining Journal.*)

The stresses in the bin are found by taking a "slice" of ore 1 ft thick with the dimensions shown in Fig. 85*a*. The stresses resulting from this block of material must be resolved into components which will give the stresses acting in the front and back posts; the beams supporting the floor; the diagonal posts helping to support the floor beams; the sills connecting the front and back posts and supporting the diagonal post and lower end of the floor beam; the tension rods (tie rods); the floor planking; the concrete footings; and various bolts and bearing plates which tie the structure together.

The forces N and N_2 are found from

$$N = \frac{1}{2} wh^2 \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \quad \text{lb}$$

They act at a distance $\frac{2}{3}h$ and $\frac{2}{3}h_1$ below the top of the bin.

$$N = \frac{1}{2} \times 100 \times 16.5^2 \times \left(\frac{1 - \sin 38^\circ}{1 + \sin 38^\circ}\right) = 3240 \text{ lb}$$

$$N_2 = \frac{1}{2} \times 100 \times 5.7^2 \times \left(\frac{1 - \sin 38^\circ}{1 + \sin 38^\circ}\right) = 386, \text{ or } 390 \text{ lb}$$

The stress diagram is shown in Fig. 85*a*. The line of action of N is extended until it intersects the bottom at x. Point x is on a line through the center of gravity of the triangle EBC. Using a suitable scale, the weight W of EBC is laid off as shown.

$$W = \frac{BC}{2} \times EB \times 100 = \frac{16.5}{2} \times 18.3 \times 100 = 15{,}100 \text{ lb}$$

By combining N and W, the resultant J is obtained. This force is resolved into two components, one normal to CD (the bin bottom) and the other parallel to CD. That normal to the bottom is $P_1 = 13{,}350$ lb. It is equal to the weight represented by triangle ECF. To find CF,

$$\text{Area of } ECF = 13{,}350 \div 100 = 133.5 \text{ sq ft}$$

and

$$EC = 24.62 \text{ ft} = 16.5 \div \cos 48^\circ$$

or scaled from the drawing

$$CF = 133.5 \times 2 \div 24.62 = 10.85 \text{ ft}$$

The trapezoid $DCFG$ represents the load on the bottom of the bin.

$$CF = 10.85 \text{ ft} \qquad DG = 3.73 \text{ ft} \qquad DC = 16.16 \text{ ft}$$

$$\text{Weight of } DCFG = DC \frac{CF + DG}{2} \times 100$$

$$= 16.16 \times \frac{10.85 + 3.73}{2} \times 100 = 11{,}800 \text{ lb}$$

Lay off the 11,800 lb on xz, and resolve back to the vertical acting through x. This gives $P = 13{,}350$ lb. The force P is assumed as acting parallel to BC through the center of gravity of $ABCD$, the bin section.

Letting a = the distance, the center of gravity is from BC; $d = a + b$ is the width of the bin; and h and h_1 are the depth of the front and back walls, respectively.

$$a = \frac{d}{3}\left(\frac{h + 2h_1}{h + h_1}\right) \quad \text{ft}$$

$$a = \frac{12}{3}\left(\frac{16.5 + 2 \times 5.7}{16.5 + 5.7}\right) = 5.03, \text{ or } 5 \text{ ft}$$

$$b = 12 - 5 = 7.0 \text{ ft}$$

With the values of N, N_2, and P known, the reactions R, R_1, R_2, R_3, R_4, and R_5 may now be found. They are

$$
\begin{aligned}
R &= \tfrac{1}{3}N_2 \\
R_1 &= \tfrac{1}{3}N \\
R_2 &= \tfrac{2}{3}N \\
R_5 &= \tfrac{2}{3}N_2 \\
R_3 &= \frac{Pb}{a + b} \\
R_4 &= \frac{Pa}{a + b}
\end{aligned}
$$

The bin will be divided into three sections. This will require four sets of posts. The distance, center line to center line of each section, then is 5 ft for the purpose of making the calculations. The maximum load on any section will result from an area 5 ft wide ($2\tfrac{1}{2}$ ft on each side of the center line). This is not true for the two end walls. They have only one-half the load. However, it is customary to assume that they are loaded the same as the others.

Taking a 5-ft. section of ore, values may be found for the reactions.

$$
\begin{aligned}
R &= \tfrac{1}{3} \times 390 \times 5 = -650 \text{ lb} \\
R_1 &= \tfrac{1}{3} \times 3240 \times 5 = -5400 \text{ lb} \\
R_2 &= \tfrac{2}{3} \times 3240 \times 5 = -10{,}800 \text{ lb} \\
R_5 &= \tfrac{2}{3} \times 390 \times 5 = -1300 \text{ lb} \\
R_3 &= \frac{13{,}450 \times 7 \times 5}{7 + 5} = +38{,}900 \text{ lb} \\
R_4 &= \frac{13{,}350 \times 5 \times 5}{7 + 5} = +27{,}800 \text{ lb}
\end{aligned}
$$

The reactions R, R_1, R_2, and R_5 require members to resist tension; R_3 and R_5 cause compression in the posts.

SELECTION OF MEMBERS

To help hold the structure together tie rods will be located at the following points: (1) top of bin, rods connecting the two inside posts on front and back; (2) the central posts at the ends of the bin.

Point of Application of N. At approximately this point the same pattern of bracing as for the top section will be used.

All these tie rods will have the same diameter. Their selection is based on the maximum tension, which is $R_2 = -10{,}800$ lb.

$$\text{Net area} = 10{,}800 \div 16{,}000 = 0.675 \text{ sq in.}$$

A $1\tfrac{1}{8}$-in.-diameter rod has a net area of 0.693 sq in. after threading. This will be the size of all the rods.

Bearing Plates for Tie Rods. On page 35, the thickness of the plate is found from

$$t = \sqrt{\frac{0.75Pa}{(a - d)s}}$$

The bearing stress perpendicular to the grain on Douglas fir (Table 11) is 310 lb per sq in.

$$
\begin{aligned}
\text{Area required} &= (10{,}800 \div 310) + \frac{\pi(1\tfrac{3}{8})^2}{4} \\
&= 36.5 \text{ sq in.}
\end{aligned}
$$

A 6- by 6-in. plate is sufficient.

$$t = \sqrt{\frac{0.75 \times 10{,}800 \times 6}{(6 - 1.375) \times 18{,}000}} = 0.692, \text{ or } \tfrac{3}{4} \text{ in.}$$

The tie-rod plates are 6 by 6 by ¾ in.

Posts. The posts are in compression; therefore their selection is based on a column formula. Above the sill, the front posts have only the compressive stress resulting from friction and the weight of their proportional part of the bin. Below the sill the load is $R_3 = 38{,}900$ lb. On the back posts, the stress both above and below the sill is $R_4 = 27{,}800$ lb. Both front and back posts above the sill will be selected on reaction R_4. The unsupported length of the back post is taken at $16.5 - 5.7 = 10.8$ ft, or 11 ft.

Trying a 6- by 8-in. post,

$$\frac{P}{6 \times 8} = 1200 - 20 \times \frac{11 \times 12}{6} = 760 \text{ lb per sq in.}$$
$$P = 48 \times 760 = 36{,}500 \text{ lb}$$

For the bearing perpendicular to the sill,

$$\text{Area} = 27{,}800 \div 310 = 93 \text{ sq in.}$$

The 6- by 8-in. post is too small. Either an 8- by 12-in. post or a 10- by 10-in. post will provide the area. As the design progresses, the 10 by 10 is found to be most suitable for detailing with other members.

The short posts below the sill will have one dimension equal to 10 in. This details with the sill (to be found later), which in turn details with the upper posts.

Acting as short posts, the compressive area for the front post is

$$38{,}900 \div 900 = 43.3 \text{ sq in. end bearing on the post}$$

and

$$38{,}900 \div 310 = 125.6 \text{ sq in. perpendicular to the sill}$$

The controlling factor is bearing against the sill.

The lower front post is $125.6 \div 10 = 12.6$ in. A 10 by 14-in. post is necessary.

For the back post,

$$27{,}800 \div (310 \times 10) = 8.7 \text{ in.}$$

A 10- by 10-in. post is required.

Floor Beams. The floor beams must be selected on the basis of bending stresses. They are supported at the ends and also near the middle, by a diagonal post perpendicular to the beam. This member is DC in Fig. 85*a*, and the diagonal post is ky in the same figure. The length of yC equals 8.9 ft and yD equals 7.3 ft. For the solution of the stress in the beam, it is assumed that the loading acts like a continuous beam with three supports, carrying a distributed load increasing toward one end. The maximum bending moments and the reactions are very closely approximated by the following solution (see Fig. 85*b*):

Trapezoid $DCFG$ is divided into

$$yCFm = 8.9 \times \frac{7.0 + 10.9}{2} \times 100 \times 5 = 39{,}800 \text{ lb}$$

and

$$DymG = 7.3 \times \frac{3.8 + 7.0}{2} \times 100 \times 5 = 19{,}700 \text{ lb}$$

Average load per foot on yC = 39,800 ÷ 8.9 = 4470 lb
Average load per foot on yD = 19,700 ÷ 7.3 = 2700 lb

The bending moments on the beam are

$$M_1 = M_3 = 0$$

$$M_2 = -\frac{1}{8}\left(\frac{wl^3 + w_1l^3}{l + l_1}\right) \quad \text{ft-lb}$$

$$= -\frac{1}{8}\left(\frac{2700 \times 7.3^3 + 4470 \times 8.9^3}{7.3 + 8.9}\right) = -32{,}500 \text{ lb}$$

The reactions are

$$R_1 = \frac{wl}{2} - \frac{1}{8}\left(\frac{wl^3 + w_1l_1^3}{l^2 + ll_1}\right) \quad \text{ft-lb}$$

$$= \frac{2700 \times 7.3}{2} - \frac{1}{8}\left(\frac{2700 \times 7.3^3 + 4470 \times 8.9^3}{7.3^2 + 7.3 \times 8.9}\right)$$

$$= 5400 \text{ lb}$$

$$R_2 = \frac{1}{8ll_1}(wl^3 + w_1l_1^3) + \frac{1}{2}(wl + w_1l_1)$$

$$= \frac{1}{8 \times 7.3 \times 8.9}(2700 \times 7.3^3 + 4470 \times 8.9^3) + \frac{1}{2}(2700 \times 7.3 + 4470 \times 8.9)$$

$$= 37{,}800 \text{ lb}$$

$$R_3 = \frac{w_1l_1}{2} - \frac{1}{8}\left(\frac{wl^3 + w_1l_1^3}{ll_1 + l_1^2}\right)$$

$$= \frac{4470 \times 8.9}{2} - \frac{1}{8}\left(\frac{2700 \times 7.3^3 + 4470 \times 8.9^3}{7.3 \times 8.9 + 8.9^2}\right)$$

$$= 16{,}300 \text{ lb}$$

Moment M_2 is the maximum negative bending moment. Because of the nature of the loading, the maximum positive moment must be found. It occurs where the shear is zero. This happens twice, once between R_1 and R_2 and once between R_2 and R_3.

From the left end

$$R_1 - wx = 0 \quad \text{and} \quad x = \frac{R_1}{w} = 5400 \div 2700 = 2 \text{ ft}$$

$$M = R_1x - \frac{wx^2}{2} = 5400 \times 2 - \frac{1}{2}(2700 \times 2^2)$$

$$= 5400 \text{ ft-lb}$$

From the right end,

$$R_3 - w_1x_1 = 0 \quad \text{and} \quad x_1 = \frac{R_3}{w_1} = 16{,}300 \div 4470 = 3.65 \text{ ft}$$

$$M = R_3x_1 - \frac{w_1x_1^2}{2} = 16{,}300 \times 3.65 - \frac{1}{2}(4470 \times 3.65^2)$$

$$= 29{,}700 \text{ ft-lb}$$

Neither of the positive moments exceeds the maximum negative moment. If they had, the selection of the beam would have been based on the larger.

$$32{,}500 \times 12 = 390{,}500 \text{ in.-lb}$$

From $M = \frac{1}{6}sbd^2$, the size of the floor beam is found.
If the material is carefully selected, $s = 1800$ lb per sq in. may be used.
Trying a 10- by 12-in. beam,

$$M = \tfrac{1}{6} \times 1800 \times 10 \times 12^2 = 432{,}000 \text{ in.-lb}$$

A 10- by 10-in. beam is too small; therefore the selection tried must be used. The floor beams will be 10 by 12 in. so that the 10-in. side will detail with the posts.

Diagonal Post. The stress in this member is $R_2 = 37{,}800$ lb from Fig. 85*b*. The length is approximately 8 ft. One dimension is 10 in. It may be assumed as acting like a short post (length is less than 15 times the least dimension). Again it would be desirable to choose a better grade of lumber; in which case $s = 900 + (900 \times 50\%) = 1350$, or 1300 lb per sq in.

$$\text{Area for compression} = 37{,}800 \div 1300 = 29 \text{ sq in.}$$

For bearing perpendicular to the floor beam, the area is

$$37{,}800 \div 310 = 122 \text{ sq in.}$$

One dimension of the post is 10 in ; the other is 122 ÷ 10 = 12.2 in. It probably would be safe to choose a 10- by 12-in. piece. However, a 10- by 14-in. piece will be used.

Because this post, at its lower end, is cut across the grain, it must be examined for diagonal grain bearing. Also the bearing on the sill must be examined. The diagonal post makes an angle of 48 deg with the sill. From page 32,

$$N = \frac{PQ}{P \sin^2 \theta + Q \cos^2 \theta}$$

The angle θ is 48 deg for the bearing on the sill and 42 deg for the bearing against the diagonal cut on the post.

For the sill,

$$N = \frac{1200 \times 310}{1200 \sin^2 48^\circ + 310 \cos^2 48^\circ} = 466 \text{ lb per sq in.}$$

For the post

$$N = \frac{1200 \times 310}{1200 \sin^2 42^\circ + 310 \cos^2 42^\circ} = 525 \text{ lb per sq in.}$$

The diagonal cut on the post has an area of

$$10 \times \frac{14}{\cos 42^\circ} = 189 \text{ sq in.}$$

Area required = 37,800 ÷ 525 = 72 sq in.
Area for sill = 37,800 ÷ 466 = 81 sq in.

The 10- by 14-in. post, after framing, provides sufficient area on the diagonal face for bearing in both cases.

Bolts for holding diagonal post to sill. The stress in the bolts (taken perpendicular to the diagonal post) is

$$37{,}800 \div \tan 48° = 34{,}100 \text{ lb}$$
$$\text{Net area of bolts needed} = 34{,}100 \div 16{,}000 = 2.14 \text{ sq in.}$$

Using two bolts, each has an area equal to 1.07 sq in. Two 1⅜-in.-diameter bolts are needed.

Bolts for connecting floor beams to sill. From Fig. 85*b*, $R_3 = 16{,}300$ lb.

$$16{,}300 \div 16{,}000 = 1.02 \text{ sq in.}$$

One 1⅜-in.-diameter bolt will do.

Flooring. The maximum load on the floor will be at the front of the bin. Taking a column of ore 16.5 ft high, the load per square foot is

$$16.5 \times 1 \times 100 = 1650 \text{ lb acting vertically}$$

Normal to the floor, this becomes

$$1650 \times \sin 48° = 1225 \text{ lb}$$

With 5 ft between beams, and assuming the load as being uniformly distributed on a simple beam, the moment is

$$M = \tfrac{1}{8}wl^2 \times 12 = \tfrac{1}{8} \times 1225 \times 5^2 \times 12 = 45{,}900 \text{ in.-lb}$$

Trying a 4- by 12-in. plank,

$$M = \tfrac{1}{6} \times 1800 \times 12 \times 4^2 = 57{,}600 \text{ in.-lb}$$

The bottom, sides, and ends will be of 4- by 12-in. planks.

Sills. The center line of the diagonal post will be offset from the inner edge of the back post $18.9 \times \tfrac{1}{2} = 9.45$, or 9½ in. (The area of the diagonal face on the post is 189 sq in. and it is 10 in. wide.) To this must be added 5 in. to obtain the distance from the center of the 10- by 10-in. bottom back post. The force acting here is the vertical component of $R_2 = 37{,}800$ (see Fig. 85*b*).

$$37{,}800 \times \sin 48° = 28{,}100 \text{ lb}$$

On the other end of the sill the floor beam has a reaction of 16,300 lb (R_3, Fig. 85*b*). The vertical component is

$$16{,}300 \times \sin 48° = 11{,}950 \text{ lb}$$

The diagonal face of the floor beam is 10 in. wide and $12 \div \sin 42° = 17.9$, or 18 in. long. The front posts below the sill are 10 by 14 in. Thus the center line of the floor beam intersects the top of the sill at $18 \times \tfrac{1}{2} + 14 \times \tfrac{1}{2} = 16$ in. from the center of the lower post. The distance between the reactions (center of lower back post to center of front post) is 12 ft + ½ × 10 in. + ½ × 14 in. = 13 ft. (This is not absolutely correct. Exact values can be found only after assembling the data in the form of a sketch. However, the procedure followed is sufficiently close.)

We now have the data for finding the stress on the sill. The data are as follows: the length between center lines is 13 ft; the vertical component from diagonal post is 28,100 lb, acting 14½ in. from the end of the beam; and the vertical component from the floor beam is 11,950 lb, acting at 16 in. from the end. By taking moments, the reaction at the lower back post is found to be the greater.

$$\begin{aligned}\text{Reaction at back post} &= \frac{11{,}950 \times 1.33 + 28{,}100 \times 11.8}{13} \\ &= 26{,}800 \text{ lb}\end{aligned}$$

The maximum moment is under the diagonal post.

$$M = 26{,}800 \times 14.5 = 389{,}000 \text{ in.-lb}$$

To detail with other members, the width of the sill must be 10 in. Then

$$d = \sqrt{\frac{6 \times 389{,}000}{1800 \times 10}} = 11.4 \text{ in.}$$

Allowing a 1-in. dap, top and bottom of the sill,

$$11.4 + 2 \times 1 = 13.4 \text{ in.}$$

For the sills, a 10- by 14-in. timber is needed. Checking for shear at the edge of the back post (load = 26,800 lb),

$$26{,}800 \div (10 \times 310) = 8.6 \text{ in.}$$

But 10 × 14 = 140 sq in. are available.

Washers or bearing plates for bolts. At the upper end, 1⅜-in. bolts are used for the diagonal post.

Area for one bolt = 34,100 ÷ 2 = 17,050 lb
Diameter of bolthole in plate = 1⅜ + ¼ = 1⅝ in.
Area of hole = 2.07 sq in.
Net area for bearing = 17,050 ÷ 310 = 55 sq in.
Total area = 55 + 2 = 57 sq in.

A plate 7½ by 7½ in. is satisfactory.

$$\begin{aligned}t = \sqrt{\frac{0.75Pa}{(a-d)s}} &= \sqrt{\frac{0.75 \times 17{,}050 \times 7.5}{(7.5 - 1.6) \times 18{,}000}} \\ &= 0.95, \text{ or } 1 \text{ in.}\end{aligned}$$

The plates are 7½ by 7½ by 1 in.

For the two bolts, a plate 7½ by 16 by 1 in. may be used.

For the lower end,

$$\text{Vertical component} = 17{,}050 \times \cos 42^\circ = 12{,}650 \text{ lb}$$
$$\text{Horizontal component} = 17{,}050 \times \sin 42^\circ = 11{,}420 \text{ lb}$$
$$\text{Area for bearing} = 12{,}650 \div 310 = 41 \text{ sq in.}$$

A plate 7 by 7 in. has sufficient area.

The depth of dap for the horizontal component is

$$\text{Area} = 11{,}420 \div 1800 = 6.3 \text{ sq in.}$$
$$\text{Depth} = 6.3 \div 10 = 0.63 \text{ in., or } \tfrac{3}{4} \text{ in.}$$

Two angle washers 7 by 7 by ¾ in. are needed.

To tie the floor beam to the sill, the upper back post to the sill, and the upper front post to the sill, 1⅜-in. bolts are used. The same size bearing plates as just calculated are used.

Remaining Timber. The bracing on the sides, ends, and lower posts does not have any definitely calculable stress. It is required to "tie" the structure rigidly together. Such material will be made of 6- by 8-in. timber, except as shown in Fig. 86.

Other Bolts. The remaining bolts may be 1 in. in diameter with standard washers, except as shown in the figure. Chute posts are attached to the lateral bracing by ⅞- by 12-in. lag screws. The diagonal bracing for the posts is also fastened with lag screws.

Footings. Concrete footings will prove more durable than wooden sills. A safe concrete mix for this purpose is 1: 2: 4. This mix will safely stand a compressive stress of 400 lb per sq in. This is low, but because of the exposed condition and the possible use of poor aggregate and sand, it is better to use the 1: 2: 4. The loads on the footings are taken equivalent to $R_3 = 38{,}900$ lb and $R_4 = 27{,}800$ lb. Bearing on the soil will be taken at 8000 lb per sq ft. Actually the bearing depends on the nature of the ground (sand, soil, rock, or other material). The value assumed is for compact sand.

Front Posts. The dimensions are

$$\text{Area for post bearing} = 38{,}900 \div 400 = 97 \text{ sq in.}$$

The front post provides 140 sq in.; a bearing plate is not needed.

The area of the top of the footing should be at least twice the area of the column or base plate (see page 98).

$$\text{Area of top} = (2 \times 10 \times 14) \div 144 = 1.94 \text{ sq ft}$$

Proportioning this to the 10- by 14-in. post, the top of the footing is 1.65 by 1.18 ft or 1 ft 8 in. by 1 ft 2 in.

$$\text{Area of bottom} = 38{,}900 \div 8000 = 4.86 \text{ sq ft}$$

Proportioning to the top gives 2.61 ft by 1.78 ft or 2 ft 7½ in. by 1 ft 10½ in.

The depth is found from

$$d = \tfrac{1}{2}(b - a) \tan 60^\circ$$
$$d = \tfrac{1}{2}(2.61 - \tfrac{14}{12}) \times \tan 60^\circ = 1.25 \text{ ft} = 1 \text{ ft } 3 \text{ in.}$$

Back Posts. The back-post footings are found in the same way. The dimensions are

Top = 1 ft 2 in. by 1 ft 2 in.
Bottom = 1 ft $10\frac{1}{2}$ in. by 1 ft $10\frac{1}{2}$ in.
Depth = 11 in.

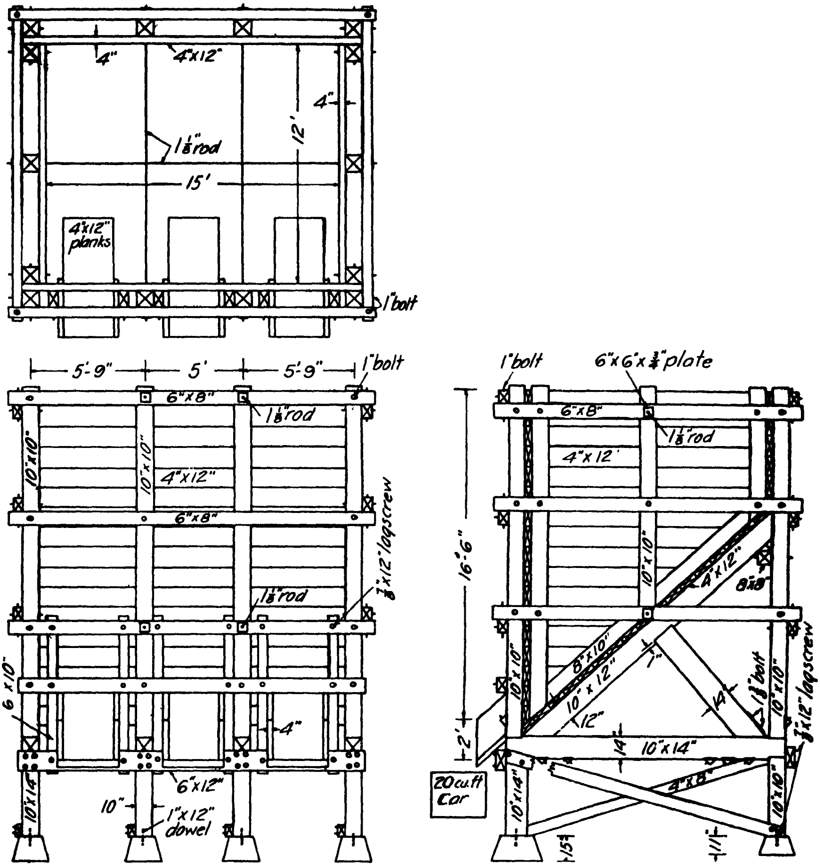

FIG. 86. Assembled rectangular bin. (*Courtesy of Engineering and Mining Journal.*)

The proportions of sand, gravel, and cement are

$$\text{Cement} = \frac{11}{1 + 2 + 4} = \frac{11}{7} = 1.57 \text{ bbl per cu yd of concrete}$$

$$\text{Sand} = 1.57 \times 2 \times \frac{3.8}{27} = 0.44 \text{ cu yd per cu yd of concrete}$$

$$\text{Gravel} = 1.57 \times 4 \times \frac{3.8}{27} = 0.88 \text{ cu yd per cu yd of concrete}$$

The posts will be held to the footings by a 1-in.-diameter iron rod, extending 6 in. into both post and concrete.

Figure 86 shows the assembled bin.

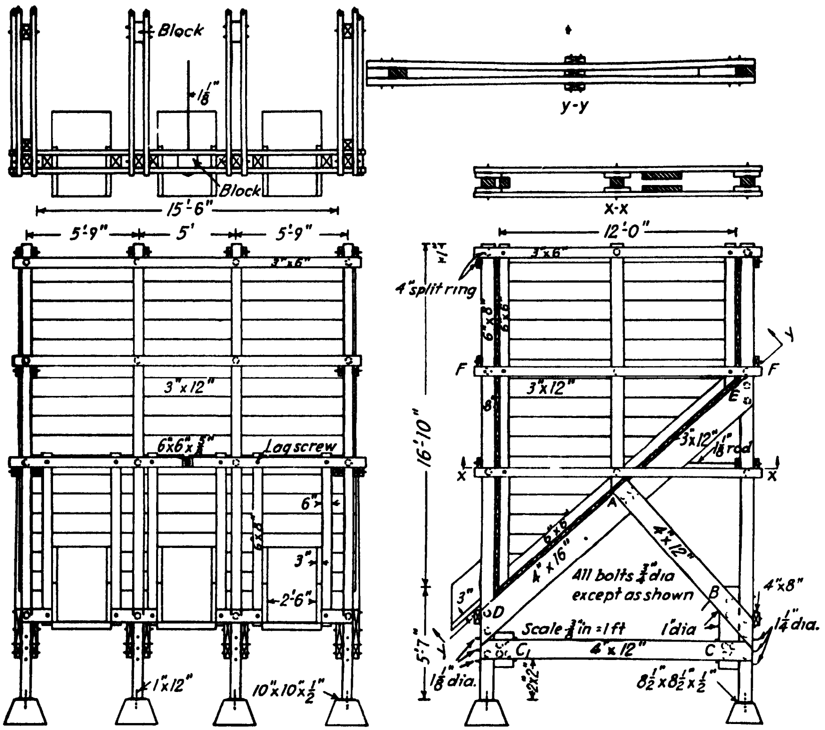

FIG. 87. Ore bin designed with timber connectors. (*Courtesy of Engineering and Mining Journal.*)

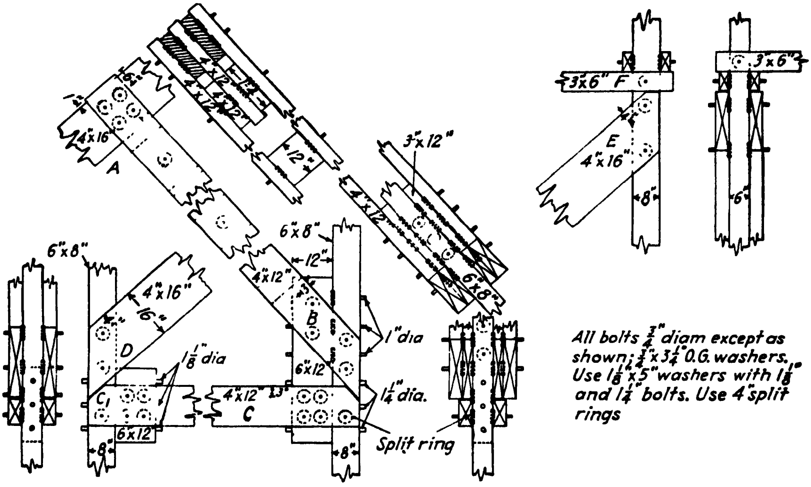

FIG. 88. Joint details for Fig. 87. (*Courtesy of Engineering and Mining Journal.*)

Timber-connector Ore Bin.[1] In Figs. 87 and 88 is shown the ore bin of Fig. 86 designed with Teco timber connectors.

This type of construction has other advantages in addition to a substantial saving in material. It will be noted that planks instead of large-dimensional solid timbers are used. These are much easier to obtain; and a higher unit stress may be used because these relatively thin members are subject to better selection.

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[1] STALEY, W. W., Design of Wooden Ore Bin with Timber Connectors, *Eng. Mining J.*, **140**, No. 9, 45, (1939).

CHAPTER VI

SKIPS AND CAGES

Because of the safety features involved, the selection of cage members requires a high safety factor. If a skip is to have a cage suspended from it, or vice versa (arrangements which are rather common, but which would not appear to be good practice), then at least the same factor of safety used for the cage should be applied to the members making up the skip.

According to Peele[1] the factor of safety for cages is 10. He recommends the following unit stresses:

Tension (iron and steel) = 6000 lb per sq in.

Compression (iron and steel) $= \frac{P}{A} = 6000 - 70\frac{l}{r}$

Rivets and pins:
  Single shear = 5000 lb per sq in.
  Double shear = 10,000 lb per sq in.
Bolts:
  Single shear = 3500 lb per sq in.
  Double shear = 7000 lb per sq in.

The clearance between the cage deck and the shaft timbers should be at least 2 in. Between the sides of the guides and the guide shoes, the clearance is ¼ to ½ in. on each side; and between the face of the guide and the shoe it is ½ to 1 in.[2]

The selection of the various members that constitute the cage is based on simple strength of materials—tension, bending, and shear. Not less than two rivets or bolts per connection should be used. Those parts (guide shoes, etc.) which are subject to wear and which require occasional replacement are usually fastened with bolts. Welding may be used instead of rivets. In fact, many cages and skips are of welded construction. For guidance in designing cages or skips, "Mine Plant" by Tillson[1] should be consulted.

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[1] Peele, R. (ed.), "Mining Engineers Handbook," 3d ed., Sec. 12, p. 102, John Wiley & Sons, Inc., New York.

[2] Eaton, L., "Practical Mine Development and Equipment," p. 260, McGraw-Hill Book Company, Inc., New York.

Figure 89 shows a common type of cage.

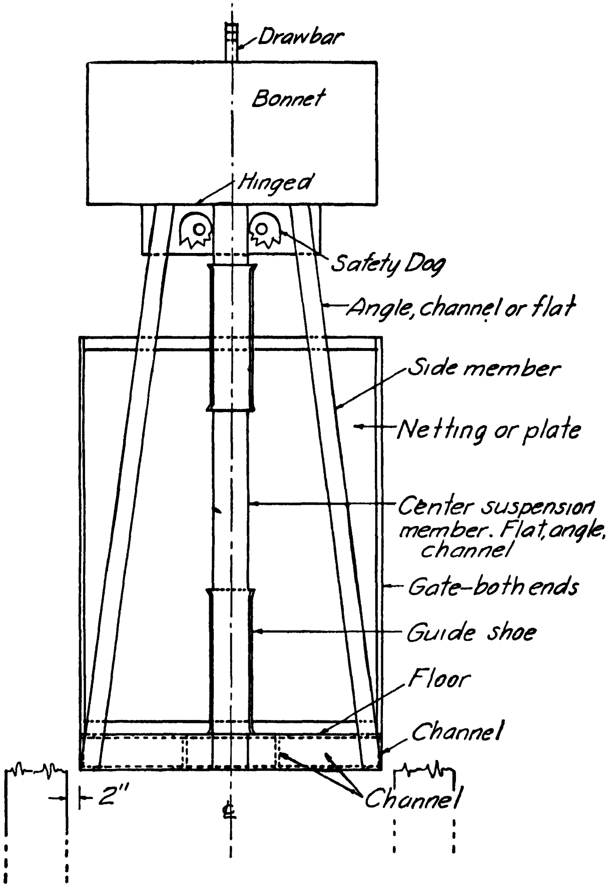

Fig. 89. Common type of small cage.

The capacity of the skip should be about 10 per cent greater than the volume of the load to be handled. This will help prevent spillage down the shaft; when at all possible a measuring pocket should be used for loading the skip. The depth of the skip should be greater than either of the other two dimensions.

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[1] Tillson, B. F., "Mine Plant," p. 88–122, The American Institute of Mining and Metallurgical Engineers (1938).

Cages and skips made of aluminum or nickel steel will save 25 to 50 per cent of the receptacle weight when compared with mild steel. This will permit a considerable lowering of the rope size and reduces the power for accelerating the load; or these conditions may be maintained and the capacity increased. Sufficient aluminum cages and skips are in operation to demonstrate their applicability.[1]

For estimating purposes, the weight of an empty steel skip with bridle, etc., is about one-third the weight of the contents.

SKIPS

An inspection of available data suggests that the design of very few skips has been based on any particular principle. On making inquiries it developed that most engineers have followed some previous design that usage had proved more or less satisfactory. Sharp[2] and Egan[3] have presented two entirely different approaches to the subject. These authors in the discussion following the presentation of their papers before the South African Institute of Engineers, showed no sort of agreement on the basic principles involved.

There is little opportunity for disagreement in the design of the skip body. Rather, it is the guiding and dumping that should be carefully investigated. Both Sharp and Egan mention examples of the practical application at South African gold mines of their designs where only minor and inconsequential repairs to the skip, guide tracks, rollers, or horns were required after as much as 3 years' service. This would indicate that either of their methods was satisfactory.

In designing the dumping arrangement, difficulties that may arise in the shop from trying to reproduce the drafting board figure must be kept in mind. Sharp's method seems to be the most workable if entrance velocities of 10 ft per sec or lower are not exceeded. Common entrance velocities are much lower than this; in fact, they do not greatly exceed 2 ft per sec.[4]

If high entrance velocities are to be used, then Egan's method should be considered. The procedure to be followed in this book, with some modifications, is that of Sharp.

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[1] *Ibid.*

[2] SHARP, G. W., The Tipping and Guiding of Vertical Skips, *J. S. African Inst. Engrs.*, November, 1922, p. 54; February, 1923, p. 144; June, 1923, p. 296.

[3] EGAN, A. L., The Design of Tips for Skips for Vertical Shafts, *J. S. African Inst. Engrs.* p. 66, November, 1925; p. 90, December, 1925; June, 1926, p. 250.

[4] DOW, H. W., Latest Development in Mine Hoisting, "Mining Practice Volume," *Trans. AIMME*, **163**, 381 (1945).

Briefly, the difference between the two is that Sharp only indirectly considers the effect of the rotating velocity of the skip about the pivot and bases his solution on establishing a relationship between, on the one hand, the distance from the pivot to the center of the wheel (designated by *C*) and the radii of the compound dumping track curve; the diameter of the wheel and roller; the curvature of the horns and their location on the skip body; and the location of the roller. This is done so that shock is eliminated and the points of greatest guide pressure are indicated. Egan, on the other hand, uses the rotating forces set up by the skip moving about the pivot. He derives a basic curve and data that applies, with modifications, to any set of conditions. His method is not so easily transferred to the shop.

Other designers use the involute of a circle, a plain circle, or an ellipse for the track curve.

Types of Skips. Figure 90 shows the various shapes that have been used for skip bodies. The square-cornered ones are more easily made and repaired. But less trouble is claimed with the rounded corners when sticky material is handled. It will be noted that in some designs the bottom is narrower than the top or that the front slopes inward. There are several reasons for this: (1) The center of gravity is moved away from the pivot, thus lessening the tendency to overturn while hoisting if a latch is not used. (2) The discharge angle is reached with less vertical travel. A drainage hole is occasionally located in the center of the bottom when wet material must be handled.

When selecting the dimensions for a skip, the long, narrow type should be avoided when at all possible. This type is difficult to load and to discharge in addition to being unstable. When existing shaft conditions make an exceedingly long skip necessary, then the bottom dump should be considered (see page 236).

Most skips are of the so-called "Kimberly type," Fig. 90, *i.e.*, a pair of wheels engage a curved track that causes the skip to overturn and discharge. Special conditions have resulted in some radically different designs. At the Hecla Mine, Burke, Idaho, it is necessary to lower the ore from the Star Mine workings to the main haulage level where it is hauled about 2 miles to the Hecla shaft. The nature of the ore requires that the skip be turned completely upside down to ensure prompt removal of the contents. A very ingenious adaptation of a four-bar linkage was applied to the problem by R. W. Neyman, the mine superintendent. This skip will be discussed later. The Anaconda Copper Mining Company, Butte, Montana, is confronted with the necessity of using very long, narrow skips. In some instances the existing headframes are not high enough to permit the most economical use of the conventional type of skip dump. They have developed a bottom dumping skip which has proved so satisfactory that they contemplate converting most of their skips to that type. This skip could

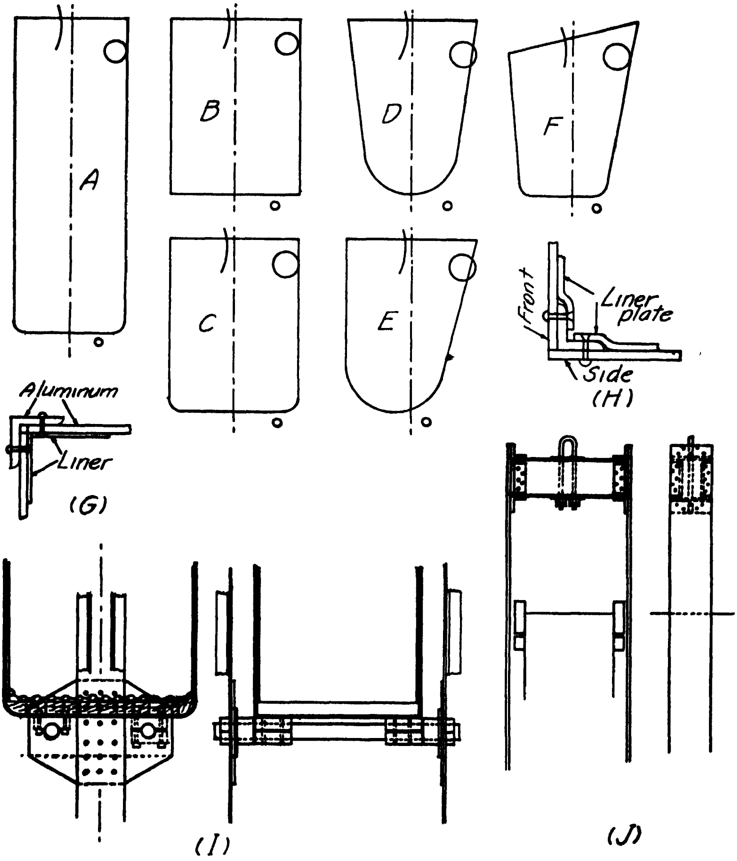

FIG. 90. Various types of skips.

be considered for handling coal when breakage must be kept to a minimum.

To reduce vibration and wear on the guides, small, pneumatic tired wheels have been used to replace the usual steel guide shoes.[1]

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[1] SMITH, W., Pneumatic Guides Provide Smoother Skip Travel, *Eng. Mining J.*, **147,** No. 4, 106 (1946).

DESIGN OF CONVENTIONAL TYPE OF SKIP DUMP

By conventional type is meant the usual wheel and curved track with horns or skid bars on the skip that engage rollers, thus ensuring the skip returning to the shaft right side up. This is commonly known as the "Kimberly skip."

The ultimate object is to decide upon a track curvature that will result in the least sudden shock to the guides, rope, and drawbar or bridle. This will be accomplished when the following conditions are met or closely approached.

1. The path of the center of gravity or wheel follows a flat, slowly changing curve. (The vertical movement is much greater than the horizontal distance.)
2. At the instant the center of gravity is vertically above the pivot (on the pivot path), the radius of the curve changes so that the horizontal movement is somewhat greater than the vertical travel. This second curve may or may not end in a short horizontal tangent. (This will usually depend on the distance C as defined later.)
3. To reduce the shock at the instant the horns engage the roller and a little later leave the roller, the roller must have a definite location with respect to the wheel when the wheel has reached the maximum horizontal distance from the pivot (*i.e.*, the line connecting the pivot and the wheel is horizontal and movement of the wheel has momentarily become zero).
4. The horns must have a curvature that keeps the shock angle, resulting from the engagement, at the smallest possible value.
5. The engagement of the wheel with the second wheel track and the lifting of the horns from the rollers must be as near simultaneous as possible.

Sharp, by the tedious process of trying a large number of combinations and drawing the necessary shock and work diagrams arrived at the following relationship (see Fig. 91).

C = distance from center of pivot to center of wheel, ft
$R_1 = 1.5C$, ft
$R_2 = 0.4C$, ft
$D = 0.15C \times 12$, in.
$d = 1.25 \times D$, in.
$x = 3d$, in.
$r = 2d - D/2 - \frac{1}{8}$ in., in.
C/H and W/H to be as large as possible.
P/H to be as small as is safe (if latches or split guides are used, this ratio can be quite small).

R_3 and R_5, and the other dimensions in Fig. 91*a* can be varied from the values shown without particularly affecting the stresses resulting from dumping the skip.

The right-hand track below the position of the wheel as shown in the figure is suggested as ensuring the return of the skip to its proper

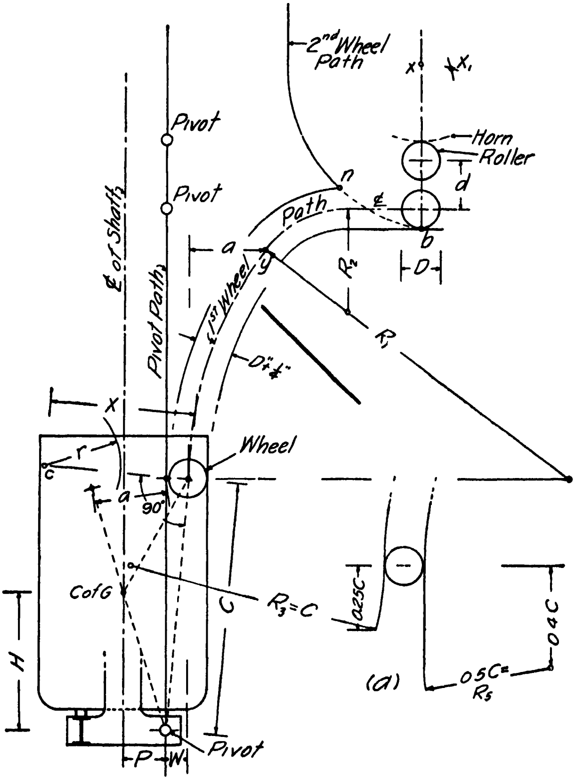


FIG. 91. Kimberly type of skip dump. (*After Sharp.*)

position on the crosshead. The left-hand track is sometimes hinged and backed with a strong spring.

A method that differs from Sharp's is used for locating the second wheel path.

As a check on the drafting and the location of the nose *n* a supplementary model of the skip should be carefully drawn on tracing paper. The wheel, pivot, center of gravity, and horn are located on this model. By working the model along the curve, the important points are checked (contact of horn with roller, nose, etc.). This is also the most convenient way to plot the center of gravity path. A sufficiently large scale should be selected so as to reduce inaccuracies of drafting.

Figure 92 further emphasizes the reasons influencing Sharp toward selecting the type of curve that results from using his ratios of *C*. Figure 92*b*, the circular curve, is taken as unity; the relative reactions at the bottom of the drawbar are indicated by the values. The rate of change of the center of gravity of the load is also shown by the bottom of drawbar curve. This is favorable in *a* and *c*, but is too sudden in *b*.

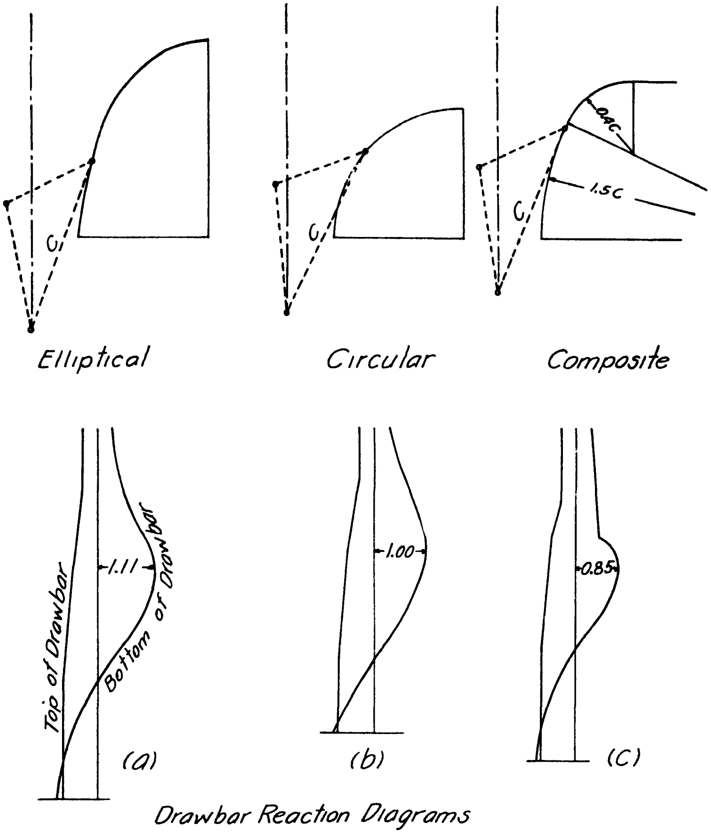


FIG. 92. Drawbar reactions. (*After Sharp.*)

Figure 93 is reproduced from Sharp's article to show the points at which shock occurs if too small a roller and wheel are adopted and if the horn is improperly designed and located. The shaded portion is the friction resulting from the guides. It is found by taking 40 per

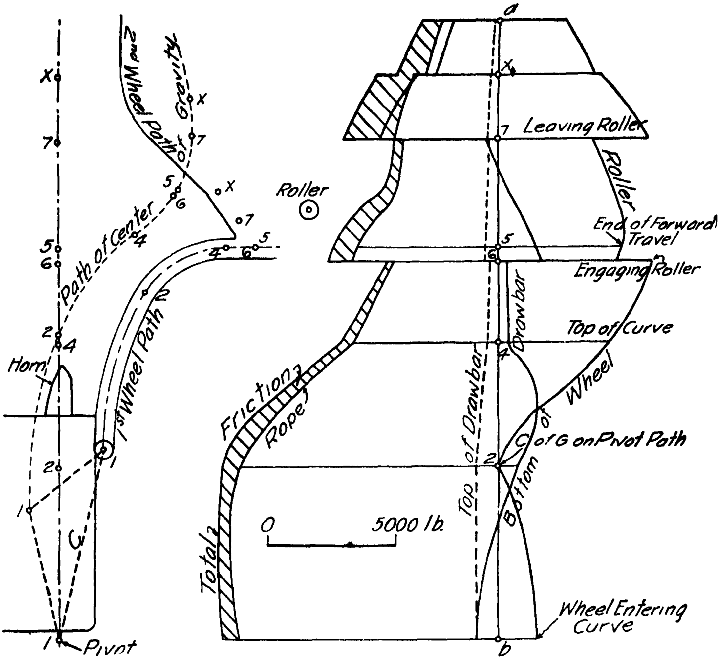
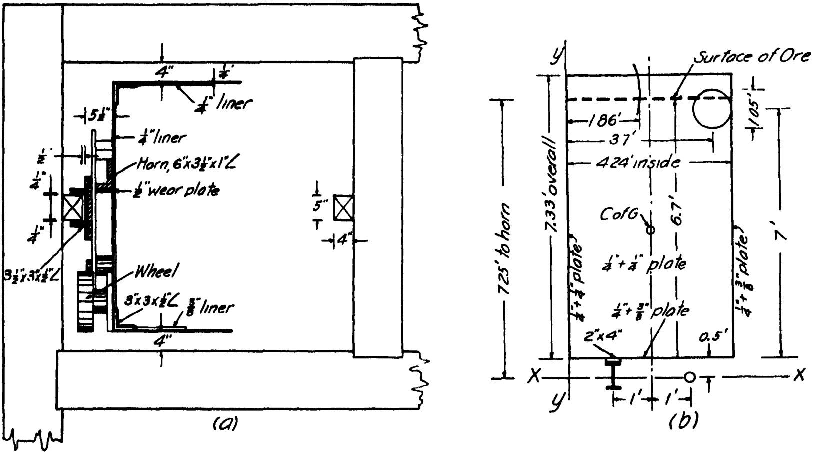


FIG. 93. Work diagram for Kimberly skip. (*After Sharp.*)



(a)

(b)

FIG. 94. Design of a skip.

cent (friction coefficient, 0.4) of the sum of top and bottom guide reactions at the various positions. The shock results almost entirely from the improper contact angle of the horn with the roller. If the horn is designed and located as suggested in Fig. 91, the shock is practically eliminated (see Fig. 98 also).

The method of determining the reactions and stresses shown in Fig. 93 will be demonstrated later.

Designing of a Skip and Dumping Track. A skip with dumping arrangements will be designed for the shaft compartment and capacity shown in Fig. 67. Figure 94*a* is a sketch showing the customary clearances and other data for the skip: the approximate dimensions for calculating the location of the center of gravity is given in Fig. 94*b*. As the design of the skip and the dumping arrangements progress, many of these measurements undergo minor changes. The capacity of the skip is to be 5 tons.

From the data given in Fig. 94*a* the approximate dimensions are found as follows:

After making allowance for the guides, drawbar, etc., the inside width is 3 ft 2 in. = 3.17 ft. The breadth is

$$60 \text{ in.} - 2 \times 4 \text{ in.} - [(\tfrac{1}{4} \text{ in.} + \tfrac{1}{4} \text{ in.}) + (\tfrac{1}{4} \text{ in.} + \tfrac{3}{8} \text{ in.})] = 4 \text{ ft } 2\tfrac{7}{8} \text{ in.} = 4.24 \text{ ft.}$$

Cross-sectional area = 3.17 × 4.24 = 13.45 sq ft
5 tons at 18 cu ft per ton = 90 cu ft
Adding 10% = 90 + 0.9 = 99 cu ft
Depth = 99 ÷ 13.45 = 7.35, or 7 ft 4 in.

The inside dimensions are

Width = 3 ft 2 in. = 3.17 ft
Breadth = 4 ft $2\frac{7}{8}$ in. = 4.24 ft
Depth = 7 ft 4 in. = 7.33 ft

Location of Center of Gravity

Depth of ore in skip = 90 ÷ 13.45 = 6.7 ft
C = center of pivot to center of wheel = 7 ft (assumed)
D = diameter of wheel = $0.15C = 0.15 \times 7 = 1.05 = 12\frac{1}{2}$ in.
$d = 1.25D = 1.25 \times 12\frac{1}{2} = 1$ ft $3\frac{5}{8}$ in.
$x = 3d = 3 \times 1$ ft $3\frac{5}{8}$ in. = 3 ft $10\frac{7}{8}$ in.
$r = 2d - D/2 - \frac{1}{8}$ in. $= 2 \times 1$ ft $3\frac{5}{8}$ in. $- 12.5/2 - \frac{1}{8}$ in.
= 2 ft $\frac{7}{8}$ in.

For the present, the horn is assumed to be made from 4 × 4 × $\frac{3}{4}$-in. angles, 15 in. long. (This is later increased to the size shown in Fig. 94*a*.)

To find the center of gravity, a vertical section of ore 1 ft thick is assumed. The weight of this ore is

$$6.7 \times 4.24 \times 1 = 28.4 \text{ cu ft}$$
$$28.4 \div 18 = 1.576 \text{ tons} = 3152 \text{ lb}$$
$$\text{Weight of wheel} = \pi \frac{1.05^2}{4} \times 0.25 = 106, \text{ or } 100 \text{ lb}$$

The weight of horn and wearing plate is

Angle = 23.2 lb
Plate = 8.5 lb
Total = 32 lb

The weight of pivot journal and axle (diameter assumed at 9 in. and length ½ ft) is 108 lb.

Crossbar wearing plate on skip = 2 × 4 × 12 = 27 lb

The coordinate axes are taken as follows (see Fig. 94*b*):

Axis *xy* along the *outside* face of the side plate.
Axis *xx* through the center of the pivot.

The center of gravity is determined by finding the distance from the coordinate axis to the center of gravity of each of the component parts making up the skip body (horn; wheel; front plates; side plates; back plates; bottom plates; pivot journal and axle; crossbar rest for skip; corner angles (side and bottom); reinforcing straps at top and bottom; and ore). With the exception of the angles, the members have a symmetrical section so that the center of gravity is at the center of each; the x and y distances are then easily found. The weight of each of these sections is multiplied by its x and y coordinates and then summed up. This value, when divided by the total weight of the section, gives the $\bar{x}$ or $\bar{y}$ coordinate of the center of gravity.

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3} \ldots$$
$$\bar{y} = \frac{w_1y_1 + w_2y_2 + w_3y_3}{w_1 + w_2 + w_3} \ldots$$

It was found that

$$\bar{x} = 2.223 \text{ ft from } yy$$
$$\bar{y} = 3 \text{ ft } 9\tfrac{3}{4} \text{ in. from } xx$$

This distance from the outside face of the side plate to the center line of the guides (skips) is

$$\tfrac{1}{4} + \tfrac{1}{4} + (\tfrac{1}{2} \times 4.24) = 2.16 \text{ ft}$$

The center of gravity is 2.223 − 2.16 = 0.063 ft = ¾ in. off center line toward the pivot.

By making the bottom of the skip 2½ in. less than the top, on the pivot side, the center of gravity falls, for all practical purposes, on the center line.

From the data given in Fig. 91,

$$R_1 = 1.5C = 1.5 \times 7 = 10.5 \text{ ft} = 10 \text{ ft } 6 \text{ in.}$$
$$R_2 = 0.4C = 0.4 \times 7 = 2.8 \text{ ft} = 2 \text{ ft } 9\tfrac{5}{8} \text{ in.}$$

Figure 95 shows a sketch of the skip and track designed by using the ratios of C as suggested by Sharp.

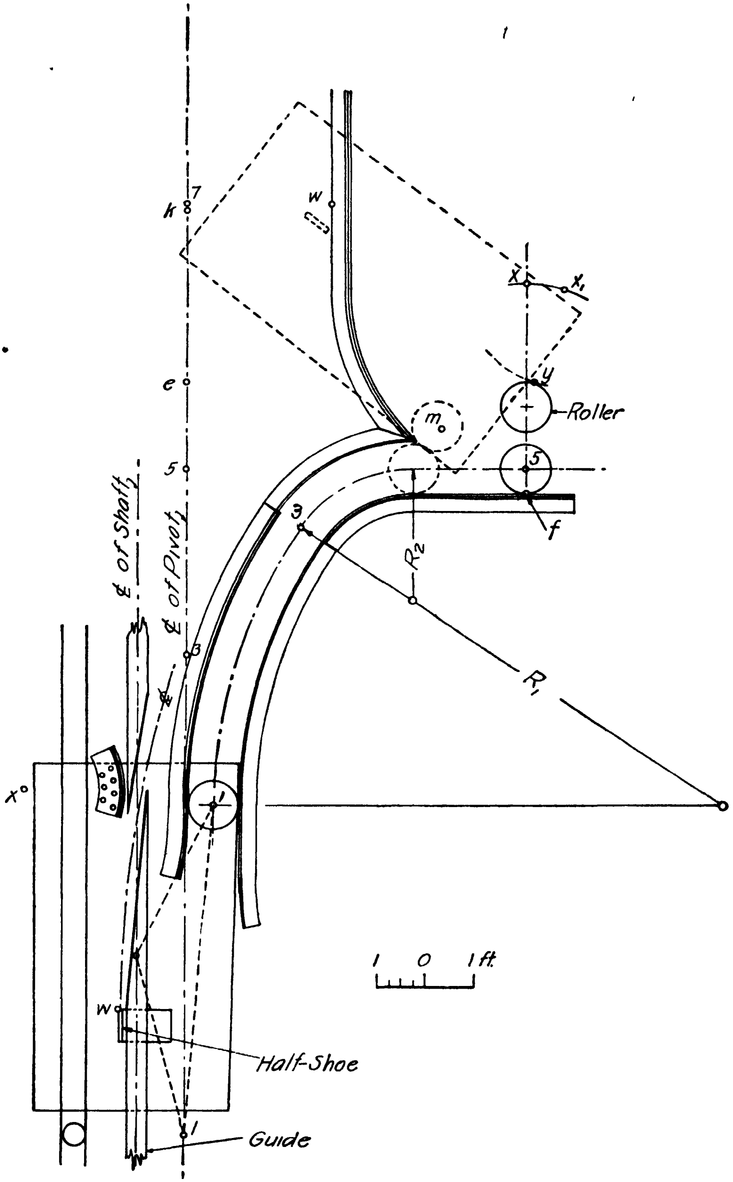


Fig. 95. Design of Kimberly-type skip.

The second wheel track was found in the following manner. The position 5-5 represents a horizontal line connecting the centers of the pivot and wheel when the skip has reached its maximum horizontal forward travel. Because of the horizontal section at the end of the first wheel track, the wheel reaction is vertical (always normal to the track). (If the first wheel track ends with a curve, the wheel reaction will point up and to the left at an angle depending on the degree of curvature.) The center line, 5-x, through the wheel and roller is always normal to the track if shock is to be avoided. The distance 5-4 was found to be 3 ft 10⅞ in. An arc x-x_1 is described from the roller center. The pivot is moved up its path until opposite the top of the roller, point e. This gives the maximum forward movement y of the horn on the roller. Taking the distance e-4, with e as a center, intersect arc x-x_1, thus locating x_1. With x_1 as a center and distance x_1-f describe an arc to the left. This is the second wheel path. As the pivot moves up its path the wheel will be lifted up this arc. The second wheel path is started where the arc cuts the top track of the first wheel path. This forms the nose, or point. A model of the skip moved up the track will indicate any errors in calculations or drafting.

The second wheel path becomes vertical at the point of tangency to a line parallel to the pivot path.

Length of Horn. The horn must have sufficient extension in its direction toward the bottom of the skip so that it will remain in contact with the roller when the skip is in its farthermost forward position, which is the length of the line connecting the pivot and the point of tangency of the horn with the roller. This point is where a line connecting the center of the roller and x_1 crosses the edge of the roller (point y). Using the pivot e as a center and the distance e-y as a radius, an arc from the pivot on the skip is described intersecting the horn on the skip. To provide for slight inaccuracies, the horn extends 3 in. below this point. To find the minimum extension of the horn in the forward direction, an arc through the center of the wheel is turned concentric with the second wheel-path arc. With the distance x-5 (5 on the pivot path) and y as a center, locate k on the pivot path. With k as a center and distance 5-5 (pivot to wheel center) cut the concentric arc from x_1 at m. Now, with the distance y-m and the wheel center on the skip as a center, describe an arc cutting the horn on the skip. This point is extended 3 in.

If the skip is exceptionally long with narrow breadth, it is customary to provide a latch or half shoe to prevent overturning in the shaft. When a half shoe is used, it is necessary to split the guides to allow a passageway for the half shoe.

The location of the half shoe must be such that it will not contact the first or second wheel paths. Less weakening of the guides would result if the shoe could be placed near the top of the skip. At this location it will contact the dumping track. The approximate location near the bottom of the skip may be found by taking the distance from 7 on the pivot path to the closest edge of the material forming the second wheel path. This distance is then laid off on the skip model from the pivot as a center, which is the top left-hand corner of the half shoe. There is a 1-in. clearance between the half shoe and the guide. The model is passed over its path and the path of the shoe observed. If it does not clear the tracks, the position is altered.

The location of the opening in the guides presents some difficulties. The mathematical location of the opening is complex. The quickest way is by trial and error. A closely approximate center line of the path of the shoe through the guide may be found as follows:

On a line even with the top of the half shoe and perpendicular to the pivot path, a distance equal to twice the distance from the center of R_1 to the center of the half shoe is measured. With this point as a center an arc through the center line at the top of the shoe is turned through the guide. This arc follows very closely the path traced by the shoe as the skip is moved along the dumping track. Enough of the guide must be removed on each side of this center line to permit free passage of the half shoe. The face of the half shoe is 8 in. long and about 3 in. wide (this latter dimension depends on the thickness of the guides used). It is 1 in. thick, ½ in. of which is a wearing plate. There is 1 in. clearance between the face and the guide.

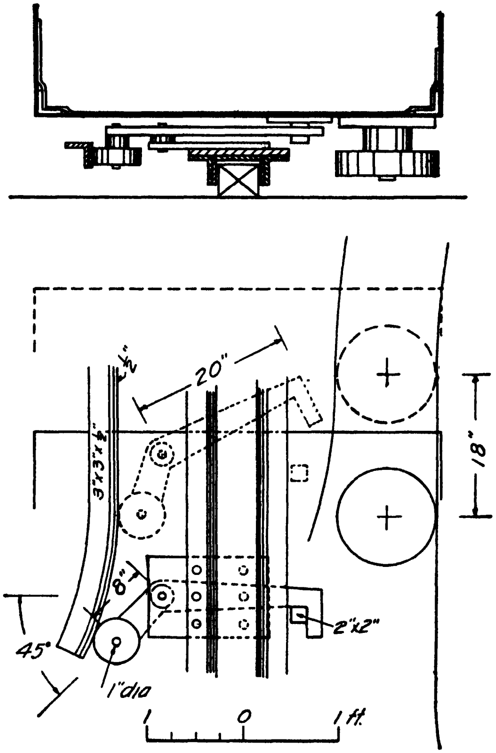


Fig. 96. Skip-latching device.

The preceeding method weakens the guides at almost the spot where the maximum guide pressure comes during the dumping operation (see Fig. 98). A 6-in. wheel on the crossbar, engaging vertical metal guides, as shown in Fig. 95, will greatly strengthen the design. Some arrangements provide these extra guides for the entire dumping operation even though half shoes are not used.

Instead of half shoes with the resulting split guides, a latch is often used. This arrangement is shown in Fig. 96. The latch is 20 in. from the center of the pivot to the edge of the nose. The arm carrying the 6-in. roller is 8 in. center to center, and it makes an angle of 45 deg with the long arm. In order to have the latch return to the lug, the weight on the long-arm side of the pivot must considerably overbalance that on the roller side. For the present example, the ratio is almost 2:1; therefore the latch should readily fall into position on the downward journey of the skip.

Enclosed dirtproof bearings are desirable for the latch pivot and roller. It would be well to have the dumping wheel and roller mounted in the same way.

The latch starts to unlock when the center of the wheel is 18 in. below the start of the dumping curve.

Ordinarily no latch or half shoe is necessary. If the center of gravity is well away from the pivot, the skip not unduly long as compared with its cross-sectional dimension, and the shaft is in reasonably good alignment, no locking attachment need be provided.

Point 3, Fig. 95, shows the position where the center of gravity is vertically above the pivot.

Stresses Resulting from Dumping the Skip. As the skip passes through the dumping cycle, stresses result in the rope, guides (top and bottom at the guide shoes), pivot, wheel, and roller. An allowance must be made for guide friction also. Sharp determines these stresses by use of what is termed the "skip triangle." This triangle is formed by the center of gravity, pivot, and wheel center. The weight of skip—empty or full—is assumed as acting vertically downward at the center of gravity. To find the stress in the rope, the approximate distance between the upper and lower crossheads must be known (pivot-to-rope attachment). This distance may be assumed at $2C$.

The reaction at the wheel is always assumed normal to the track at the particular instant under consideration. This is also done at the roller-horn contact.

The positions of the skip that are usually selected are the following: entering curve; center of gravity on pivot path; top of curve; end of track (horizontal movement); engaging roller (simultaneous with end of track); leaving roller; and engaging second wheel track (simultaneous with leaving roller). These may become the critical shock points.

Figure 97 shows the stress diagrams for the various positions. At position 1 the skip is just entering the curve; *ab* is the weight of the loaded skip (15,000 lb). At the point representing the center of gravity, there are two unknowns, *bd* and *da*; *ab* is known. The stress diagram (1) is started with these three by the usual methods of graphic statics. Next, *bc* and *cd* are drawn; *ay* is drawn parallel to the line connecting the upper drawbar with the pivot. It intersects the horizontal line *cy*. A diagram for each of the positions, 1 to 9, is drawn as shown in Fig. 97. The following stresses are thus found:

Vertical component of *ay* = rope pull, lb

Horizontal component of *ay* = top guide pressure, lb
cy = bottom guide pressure, lb
ca = pivot reaction, lb
bc = wheel reaction, lb

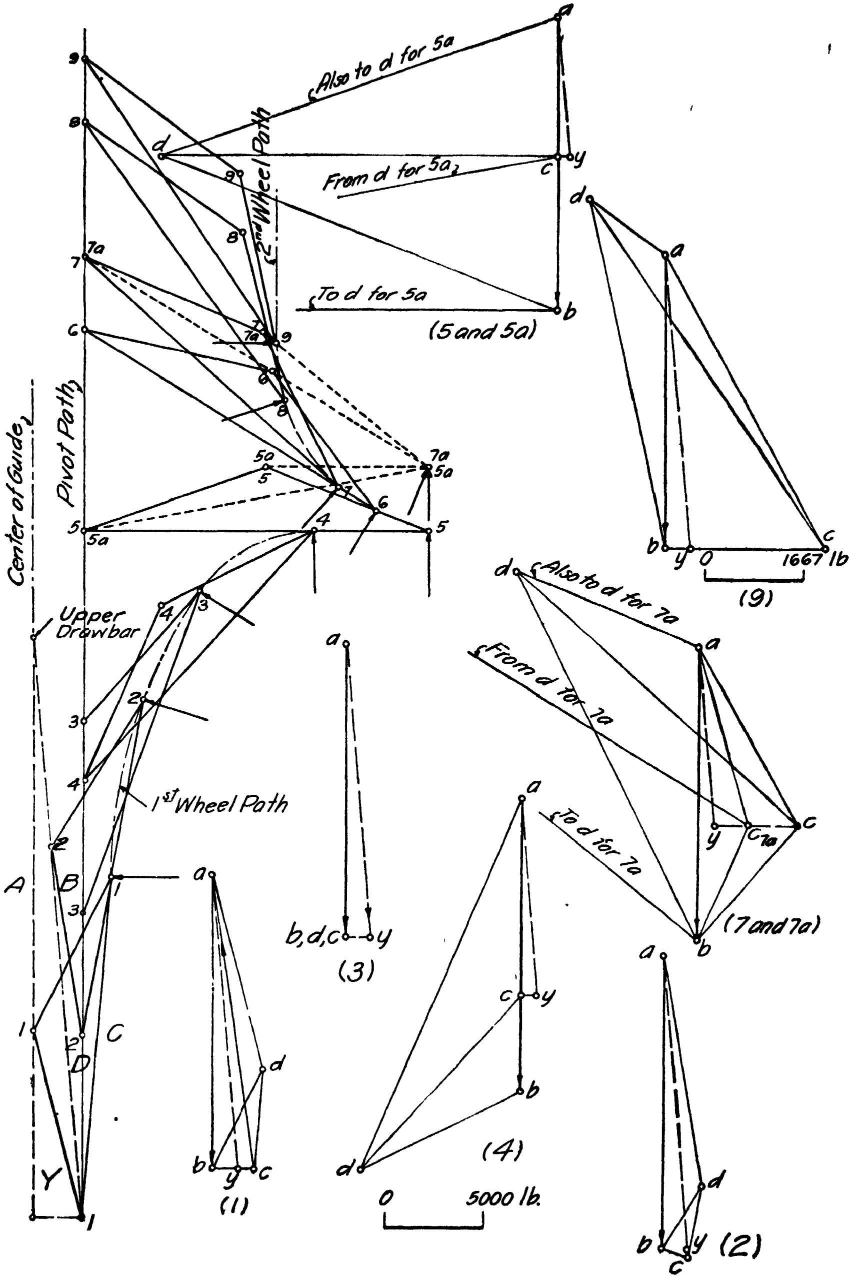


Fig. 97. Stress diagram for skip dump. (*After Sharp.*)

The reactions given in Table 36 are plotted in Fig. 98. The friction is taken at 40 per cent of the sum of the top and bottom guide pressures. If sufficient points are taken, a curve is obtained instead of the broken lines. Some authorities make an allowance for the ore as it is removed from the skip (end of horizontal travel and above). This would appear to be an unnecessary refinement as the skip and dumping arrangement would be selected on the basis of the maximum travel along the wheel path. Such stresses will result from the loaded skip.

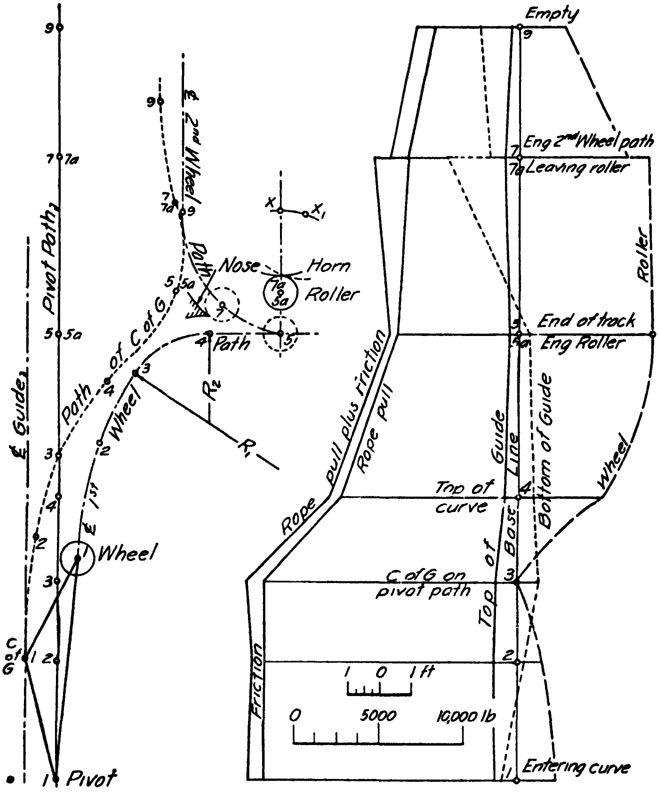


FIG. 98. Work diagram for Kimberly-type skip.

COMPLETELY OVERTURNING SKIP

To meet an unusual condition at their Star Mine, R. W. Neyman[1] applied the principle of a four-bar linkage in designing a skip for lowering the ore. During the dumping operation, the skip turns completely upside down. The design selected was necessary because the ore is sticky and shaft space for the dumping operation was limited. Ore may be lowered, at present, a distance of 2000 ft. Over this distance 150 tons per hr have been handled. Through a distance of 1300 ft one skip per minute, including loading and dumping time, has been realized. The entire operation runs with exceptional smoothness.

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[1] R. W. Neyman, general superintendent, Hecla Mining Company, Burke, Idaho. Personal communication.

The Star hoist, which is located underground, was designed to handle a gross weight of 13,000 lb at a rope speed of 1400 ft per min. A skip was designed to satisfy these requirements. It is 30 in. by 56 in. with an over-all depth of 78 in. The bottom is curved to a 24-in. radius, the sides flaring out tangentially toward the top. The walls next to the guides are vertical and parallel. With the exception of the guide shoes, liners, and arm bearings, the skip is of welded construction. The material is corrosion-resistant Cor-Ten steel.

TABLE 36. SKIP REACTIONS
Pounds

Condition	Rope pull	Guidepressure		Reaction		Friction
		Top	Bottom	Pivot	Wheel or roller	
Entering curve (1)	15,000	1350	900	15,250	2150	900
Position (2).............	15,000	1400	450	15,500	1500	740
C of *G* on pivot (3).......	15,000	1300	1300	15,000	0	1040
Top of curve (4)..........	10,100	850	850	10,100	4950	680
End of forward travel (5)*..	7,150	600	600	7,150	7850	480
Engaging roller (5*a*)....	7,150	600	600	7,150	7850	480
Engaging second track (7)..	6,600	850	4250	10,450	7700	2040
Leaving roller (7*a*)........	6,600	850	1750	9,450	6400	1040
Empty (9).............	5,000	420	2300	5,700	2700	1090

* From this point on, more refined calculations would correct for reduction of the load through discharging the contents. This is usually assumed as occurring at a constant rate.

The shell is $\frac{1}{4}$-in. plate reinforced on the corners with angle irons welded to the shell. A $\frac{3}{4}$-in. soft rubber lining is covered by an abrasion-resisting plate. This plate is $\frac{3}{16}$ in. thick on the curved and sloping sides, and $\frac{1}{16}$ in. thick on the vertical walls.

A safety catch is provided on each side of the skip to prevent overturning in the shaft. Retractable guides are provided at the various pockets. It has the following safety features: An electric eye to prevent overrunning when dumping, and green and red lights on the hoist deck to indicate the position of the dump guides, whether in the shaft or in the retracted position.

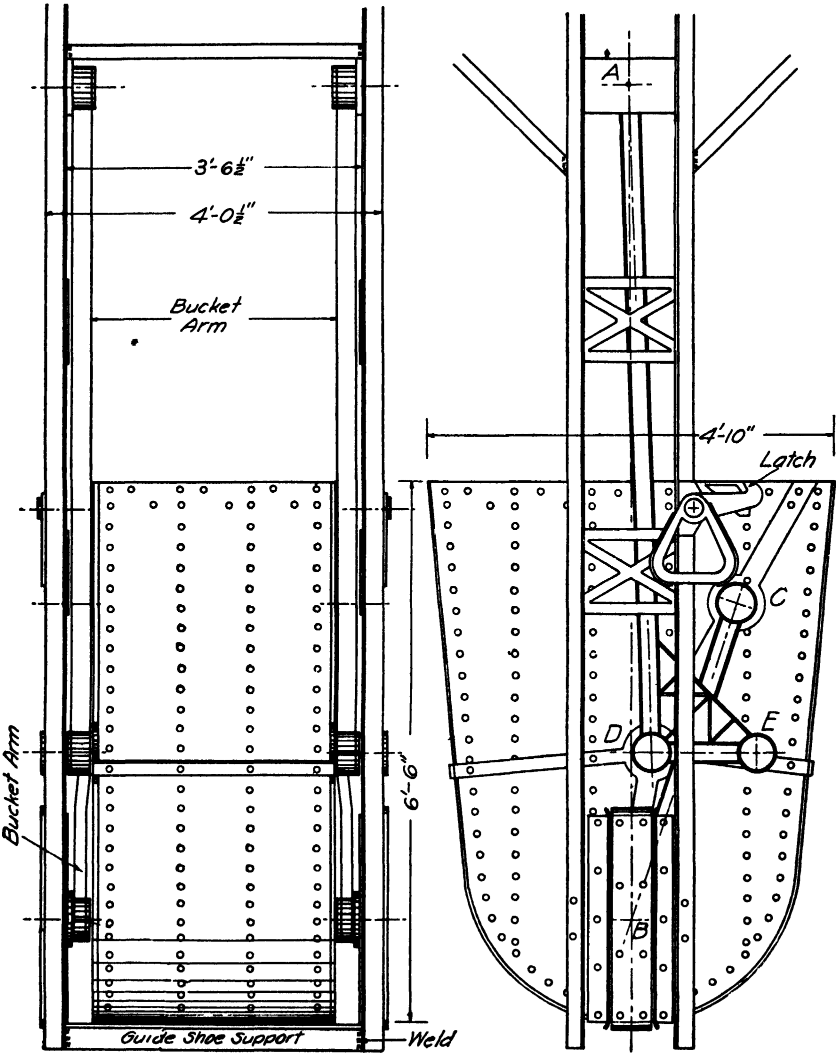

Fig. 99. Overturning skip, Star mine type.

The Star skip is shown in Fig. 99. Figure 100 gives some of the essential details of the dumping track and retracting mechanism. The safety latch, to prevent possible overturning of the skip during lowering, is shown in Fig. 101. Just as the roller enters the track, the latch moves into a groove cut in the guide. This is brought about by the overbalancing effect of the latch arm and the roller track engages the side of the latch. While the skip is running in the shaft, contact with the guides keeps the arm in the locked position.

Figure 102 gives the measurements of the linkage for the Star skip. Connection *D* is set 3 in. off the center of gravity. This aids in returning the skip to its normal position after dumping. It also tends to prevent premature overturning during travel. Arrows at *C* and *D* indicate the movement when *E* is moved. The arm *AB* is the fixed arm. Point *A* is attached at the top of the drawbar and *B* at the bottom. The trunnions *C* and *D* are attached to the skip. Arm *DE* is given a length that is suitable to keep the track from interfering with operations in the shaft. It is advisable to keep it as short as is convenient in order to reduce the effect of the stresses. The relative location of *B*, *D*, and *C* will depend on the skip body selected and on the space available for revolving the skip. They should fall as nearly as possible on the same line as the width of the arms will permit. As the spacing off the center of gravity increases, an increasingly greater force at *E* is required to turn the skip about *D*.

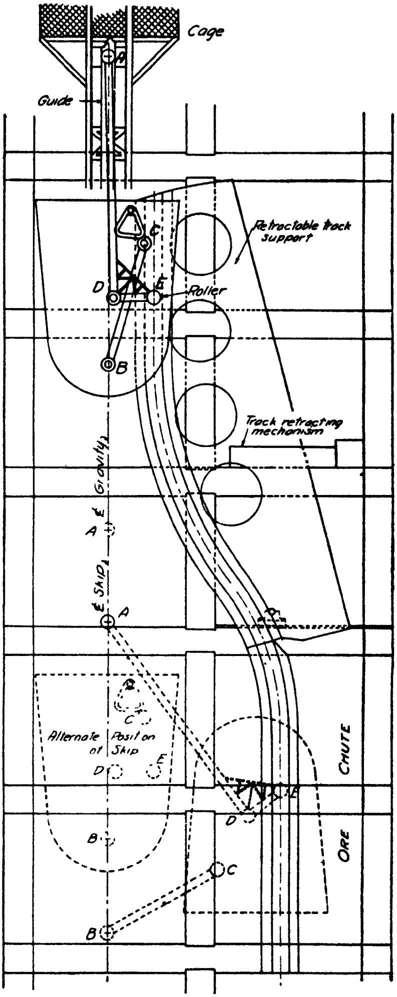
Cage
Guide
A
C
Retractable track support
D
E
Roller
B
Track retracting mechanism
℄ Gravity
A
℄ Skip
A
C
Alternate Position of Skip
D
E
B
E
D
C
B
CHUTE
ORE

FIG. 100. Details of Star skip dump.

After the linkage has been selected, the ultimate attitude of the skip must be decided upon. This will range from being revolved through 180 deg (upside down) to some angle at which the ore will just discharge. As previously pointed out, this latter angle may be partly controlled by sloping the front of the skip. The link *CD* is most suitable to choose as a reference. Its relationship to the upright position and the final position of the skip is easily determined. The distance from the shaft center line (or other reference point) that E finally reaches to put the skip in the desired position must be known. This establishes the second tangent to the track curve. The roller approaches the beginning of the track curve on the first tangent. These two tangents are connected by a reverse curve. As was true with the Kimberly type of skip, the roller must gradually take to the curve. If the point of reversal for the reverse curve is taken about two-thirds to three-fourths the distance from the point of tangency of the first tangent, a suitable curve is obtained (see Fig. 104).

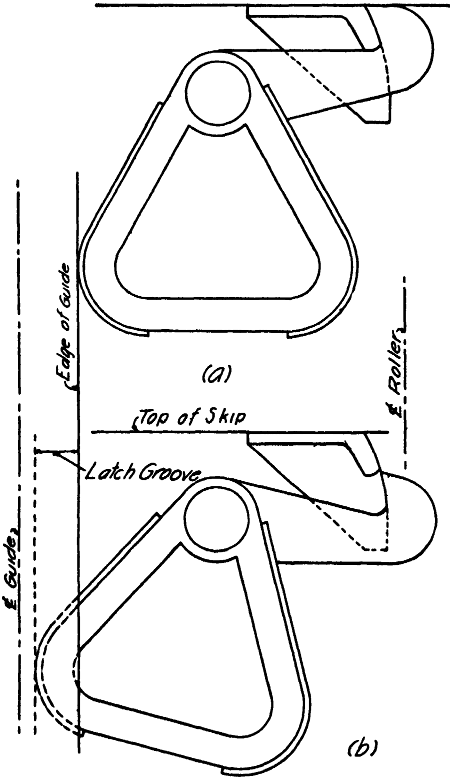

Fig. 101. Safety latch on Star skip.

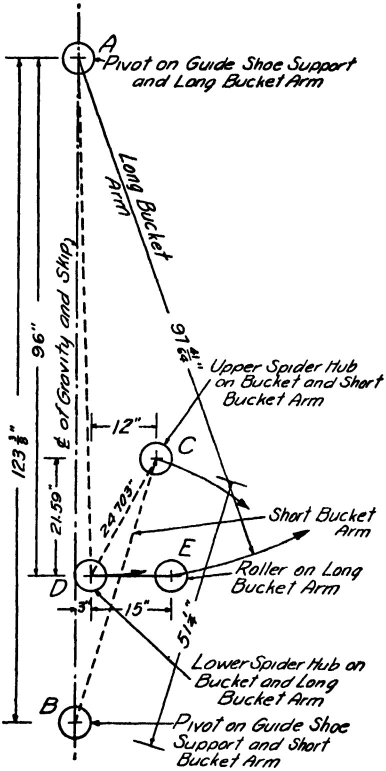
A
Pivot on Guide Shoe Support and Long Bucket Arm
Long Bucket Arm
91 41/64"
Upper Spider Hub on Bucket and Short Bucket Arm
℄ of Gravity and Skip
96"
123 1/8"
12"
C
24.703"
21.59"
Short Bucket Arm
E
Roller on Long Bucket Arm
D
3"
15"
51 1/4"
Lower Spider Hub on Bucket and Long Bucket Arm
B
Pivot on Guide Shoe Support and Short Bucket Arm

Fig. 102. Linkage for designing Star skip.

One important feature of a self-contained linkage system is that movement of the system as a whole has no effect on the relative movement of the links making up the system. Therefore, the various positions of the individual members may be found without the added complication of moving the roller along the track and moving AB along the guides.

Figure 103 is the linkage; AB is the fixed link; BC is the short arm and is pivoted at B on the drawbar and C on the skip; AD is the long arm, pivoted at A on the drawbar and at D on the bucket; DC is the link connecting the two arms, pivoted at D and C on the bucket; and, finally, DE is the roller arm and has no effect on the linkage. A few salient features of the movement of this setup will be given. The arc DD' is swung from A. As D advances along its arc, C advances at a more rapid rate along CC''. Numbers 1, 2, 3, 4, 5, 6, and x and D' represent various positions of D. If CD is set off on the compass and arcs are swung from these positions of D, the location of C on arc CC''

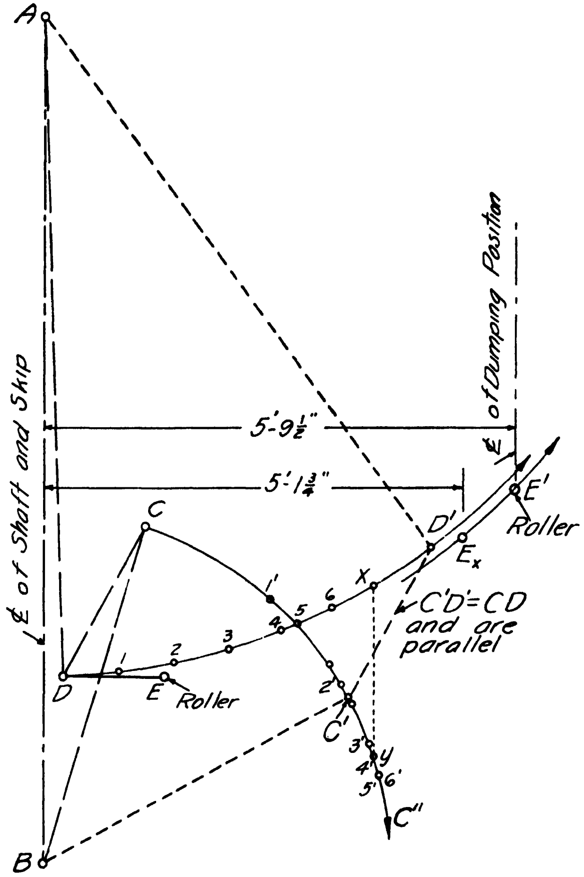

FIG. 103. Movement of linkage, Star skip.

is found. Such positions are shown by 1′, 2′, 3′, 4′, 5′, 6′, *y*, and *C*′. Point 5′ is as far as *C* will travel in its downward journey. At this instant *D* (5) is at the intersection of the arcs. As *D* continues toward *D*′, *C* returns back along its former path. The position of the roller is found by laying off *DE* from *D*′.

It has been stated that link *CD* may be used for the reference lines in determining the slope of the skip for various positions of *E* along the track. When *C* revolves 180 deg about *D*, the skip will be exactly upside down. Any angle less than 180 deg will leave the skip inclined. The object is to find a position for *D*′ that will give the desired tipping angle. This may be done by a very complicated and tedious mathematical process. A much quicker and simpler way is by a cut-and-try graphical solution. If the distance of *E* from the reference base (shaft center line, for example) is not *accurately* found (within less than ½ in.), the final attitude of the skip will vary radically from that desired; for instance, the path shown in Fig. 104*a* has the skip in a vertical position at the end of the track. An error on the first calculation made the distance 1¾ in. too large. When the end of the curve was reached, the skip had passed beyond the vertical position almost 40 deg.

In Fig. 103, *C*′*D*′ is *CD* revolved through 180 deg. It is located by drawing a line parallel to *CD* and making *C*′*D*′ = *CD* in length. Set *CD* on the dividers, and move at right angles to *CD* until two points are found, one on each arc with the connecting line parallel to *CD*. Now, with *DE* lay off *D*′*E*′ on arc *E*. This represents the center of the roller. The distance of *E*′ from the shaft center line is 5 ft 9½ in. Figure 104*a* shows the track curve and successive positions of the bucket. The linkage arms are indicated for positions 1 and 5. Returning to Fig. 103, *xy* is the position for link *CD* to tip the skip to a position suitable for most conditions. The tangent distance from the shaft center line is 5 ft 1¾ in. Figure 104*b* shows the layout. This skip is ascending instead of descending like the Star skip.

Additional dimensions of the Star skip are

Diameter of trunnion axles = 5¼ in.
Diameter of roller = 5¾ in.
Diameter of latch axle = 3 in.
Short bucket arm = 3- by 1¾- in. channel
Long bucket arm = 3- by 3-in. channel
Roller arm = 3- by 3-in. channel.
Other arms = 3- by ½-in. plate
Drawbar (for one side) two 3- by 2½- by ½-in. angles

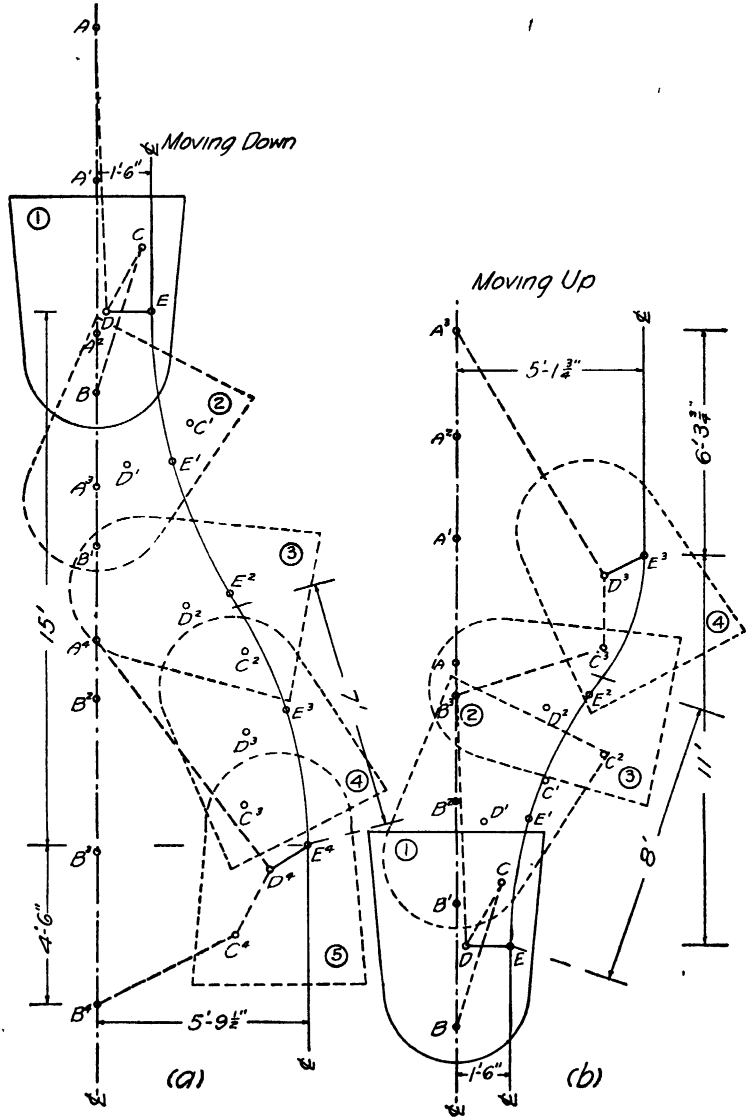

FIG. 104. Overturning positions for Star-type skip.

To locate positions of the skip while it is traveling along the track, it is best to make a model on tracing paper and to move the model. Various positions are occupied, and the skip outline is pricked through onto the drawing.

Referring to Fig. 104*a*, the locating of position 2 will be discussed. On the shaft center line, *A′* is located at any desired position. With *A′* as a center, an arc *AE* is swung to cut the track at *E′*. An arc with radius *AD* is swung from *A′* and an arc with radius *DE* is swung from *E′*, their intersection locating *D′*. Below *A′*, *B′* is located a distance equal to *AB*. Taking the distance *BC* and *B′* as a center, and distance *CD* with *D′* as a center, intersecting arcs are swung. This locates *C′*. If the skip model is now placed so that *C* and *D* correspond to *C′* and *D′*, the outline shown by position 2 is found.

BOTTOM DUMP SKIP (ANACONDA TYPE)[1]

This type of skip is shown in Fig. 105. Many of the minor details in regard to material size and measurements that depend on shaft dimensions are not given. The figure is presented primarily to illustrate the features involved in opening the gate. Permanent parts of the skip are fastened by welding. Inside liner plates are attached by bolts having countersunk heads. The inside corners are rounded by means of the liner plates.

Referring to Fig. 105*a*, the important feature is the movable arm *BDE*. This arm is pivoted at *E* and is limited in its movement to the right by an arm stop. The roller *D* engages the track shown in Fig. 105*b*. At *C* is the fixed end of the chain cable, which passes over the sheave *A* and is attached to the gate. To reduce maintenance costs, the idler sheave at *B* has been adopted. On the original designs the chain cable was attached directly to the end of the arm. After using this design about 3 years, it was determined that repairs could be reduced if the idler sheave was inserted.

The traveling distance of *B* about *E*, to raise the gate the desired distance, is found by cut-and-try methods, as the circular coverage on *A* and *B* is a variable. All the moving parts connected with the gate-raising mechanism are mounted on ball bearings.

Figure 105*b* shows that with a 6-ft vertical movement of the roller *D* along the track, the gate is raised 4 ft 1½ in. The horizontal movement of the roller (distance between tangents of the reverse curve) is 1 ft 2 in. Position 1 is at the beginning of the operation; at position 5 the gate is completely opened.

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[1] Data reproduced by permission of the Anaconda Copper Mining Company, Butte, Montana.

The size of the material making up the roller-arm assembly, trunnions, axles, etc., may be found in the following way: The weight of the gate is found. To this is added the equivalent frictional resistance in pounds of pressure against the gate (see the chapter on ore bins). Half of this total would be the load on each arm. From the gate the chain cable passes over the top wheel, *A*, the arm roller *B*, and is

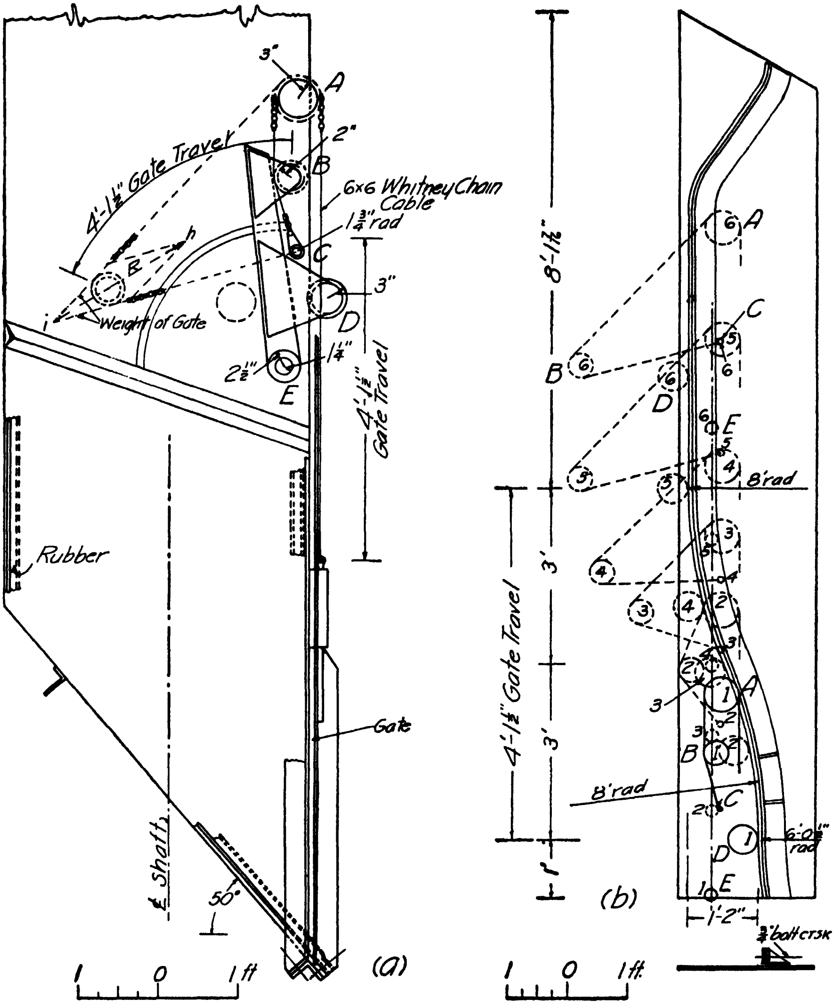

Fig. 105. Anaconda bottom dump skip.

anchored at *C*. The resultant force is found by laying off the load, to a suitable scale, on the cable line on both sides of *B*. A force diagram is constructed with the diagonal (resultant) passing through *B*. This is shown in Fig. 105*a* at *i-h*. The resultant *R* is resolved into components normal to the arm and parallel to it. The force acting at *D*, *E*, and *B* and the bending in the arm may be found by moments.

INCLINED SHAFTS

Skips and dumping arrangements for inclined shafts have been selected in much the same way as vertical arrangements. After a design has been found to give satisfaction, it is widely adopted. A. L. Egan[1] by a series of graphical diagrams suggests a method for designing inclined skips. The location of the center of gravity is important; if improperly located the skip may not return from the dumping tracks to the track proper. Egan suggests the following rules:

> The empty and full centers of gravity should be as nearly coincident as possible, by keeping the wheels high relative to the floor. The bridle trunnions should be so placed that the forces *bm* and *aq* read in the direction shown in *A*.
>
> The tip should be so designed that the paths of the centers of gravity do not change suddenly in direction and the slope of these paths, after the skip is fully tipped, should not exceed the angle of the shaft.
>
> The path of the empty center of gravity should not slope at an angle lower than that of the shaft, at any point in the tip. Under the above conditions, a skip will not overturn, lift up in front, or fail to descend properly.

In the present text all forces are drawn in a clockwise direction. The figure referred to by Egan is thus redrawn in Fig. 106. Here, *da* is the reaction at the front wheel and *cd* the reaction at the back wheel. Figure 107 shows the same skip but with the bridle trunnions located nearer the rear, in the dumping position. It will be noted that the tendency is for the back wheels to lift up when the skip is loaded; and the reaction is zero when empty. Figure 108 shows the skip ascending with the trunnion nearer the rear than in Fig. 106.[2]

Egan's deductions appear to be possible to apply on shafts up to moderately steep slopes (say, 45 to 50 deg). On steep slopes his

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[1] Egan, A. L., The Design of Skips and Tips for Inclined Shafts, "Witwatersrand Mining Practice," Watermeyer and Hoffenberg (eds.), p. 290, Transvaal Chamber of Mines, Johannesburg, South Africa.

[2] Figures 106, 107, and 108 are after Egan, and are his Figs. *A*, *B*, and *E*.

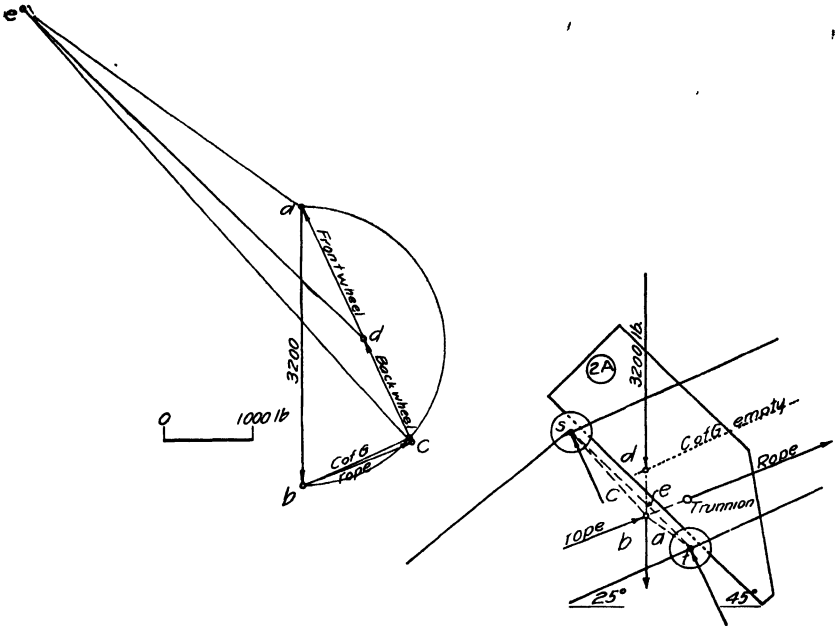

FIG. 106. Stress diagram for inclined shaft dump. (*After Egan.*)

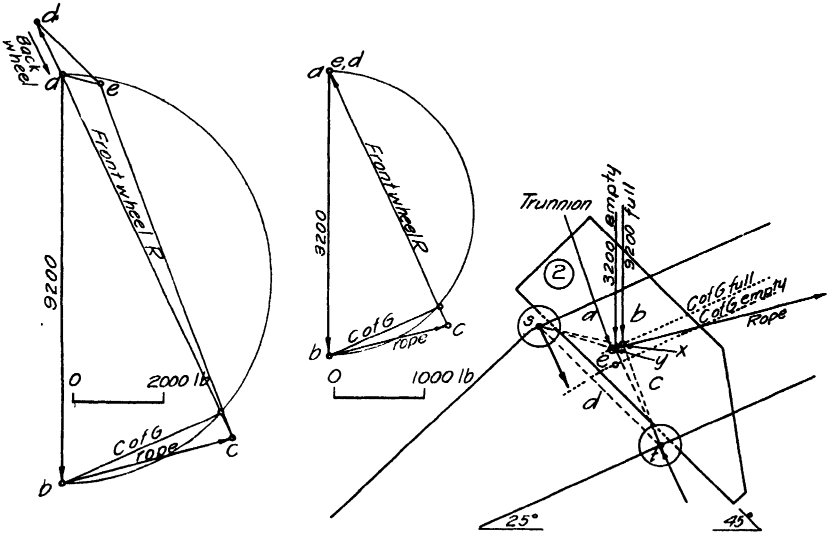

FIG. 107. Stress diagram for inclined shaft dump. (*After Egan.*)

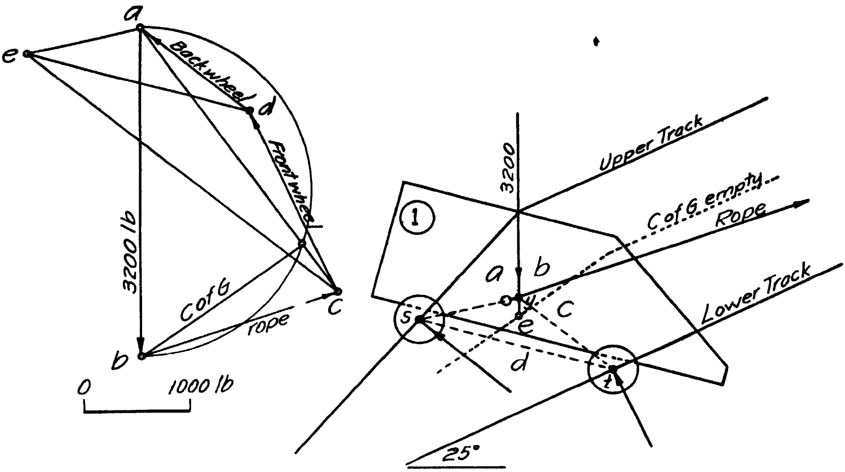

FIG. 108. Stress diagram for inclined shaft dump. (*After* **Egan.**)

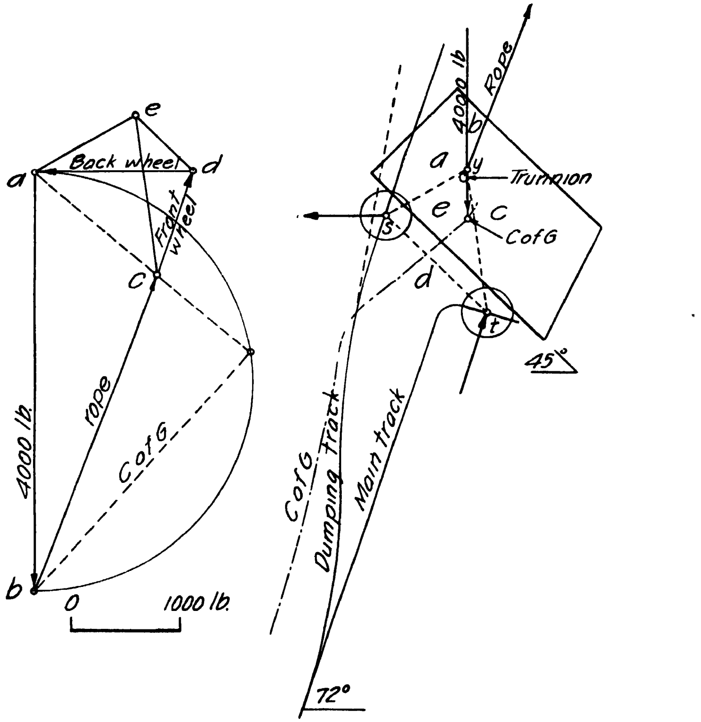

FIG. 109. Stress diagram for steeply inclined shaft.

proposed rules are difficult to meet. Figure 109 shows a shaft inclined at 72 deg. The back-wheel reaction indicates that the wheel is lifting. If the dumping track is continued along the dashed line, a somewhat better center of gravity path is obtained. On a steep shaft the bridle trunnions cannot conveniently be placed near the front of the skip. Also, because of the dumping angle (usually not less than 45 deg), the path of the center of gravity is difficult to maintain parallel to the tracks.

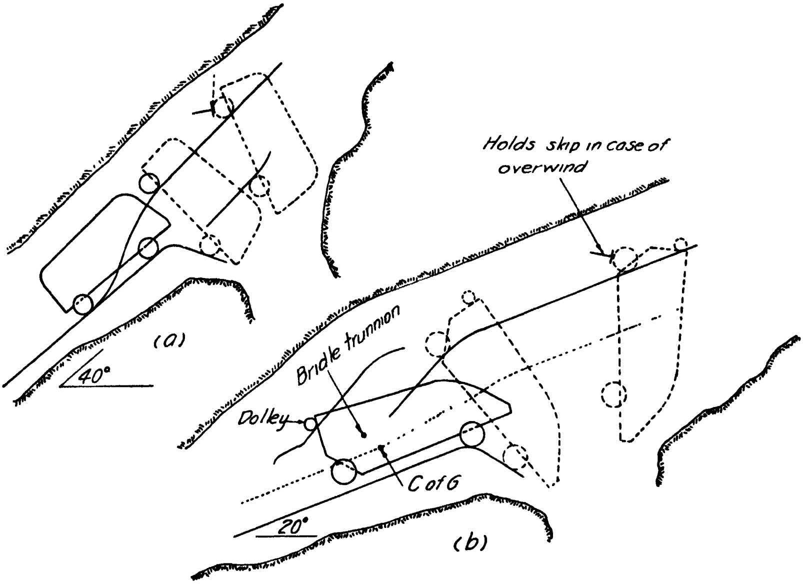

FIG. 110. Method of dumping inclined shaft skips. (*After "Witwatersrand Mining Practice."*)

Figure 110,[1] shows two types of skips and arrangements used on the Rand in South Africa. It will be noted that these skips were apparently designed to have a very low center of gravity.

Several types of skips are shown in Fig. 111;[2] Fig. 111*a* has a particularly low center of gravity. It is practically impossible to derail this skip. In Fig. 111*b* is shown the usual types of rear wheels for engaging the dumping tracks; Fig. 111*c* is an end dump skip for lowering material; Fig. 111*d* shows a common type of skip with various details. The dimensions of the bridle, crossbar, and axles are not given as they vary from skip to skip, depending on the load. These available. The low-carbon copper-nickel high-tensile steel is most desirable. The yield point is 50,000 lb per sq in. in sections up to ½ in. in thickness. Carbon content usually does not exceed 0.12 per cent. They have excellent welding and forming qualities. Copper-nickel structural steels are much more desirable than the 3½ per cent nickel-steel shapes. For liner plates, steel 8660 is better than 3160. It is cheaper and has equally good abrasion-resisting properties.

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[1] Watermeyer and Hoffenberg (eds.), *op. cit.*, pp. 295, 297.

[2] TILLSON, B. F., "Mine Plant" Fig. XIV-11, 38, 41, American Institute of Mining and Metallurgical Engineers (1938).

Copper-nickel steels are sold under various trade names. Among these are Yoloy, Youngstown Sheet and Tube Company; Hi Steel, Inland Steel Company; RDS, Republic Steel Company; Hi Tensile, American Rolling Mills; Mayari, Bethlehem Steel Company; and Corten, Carnegie-Illinois Steel Company.

1. Levack Mine No. 2 shaft
    a. Skips
        (1) Body plates and structural shapes—SAE 2320 specification (Ni-Cu steel when available)
        (2) Forgings—Inco RR Specification No. 2 (0.30 C, 2¾% Ni)
        (3) Steel castings—Inco RR Specification No. 4 (0.20 C, 2% Ni)
    b. Cages
    Same as for skips except that the No. 14 gauge plate used on the cage body is made of Yoloy.
2. Creighton No. 6, Stobie No. 7, and Murray No. 2 shafts. Due to the difficulty of obtaining nickel-steel angles and other structural shapes for the building of this equipment, mild steel of structural grade was substituted. Material used in the manufacture of these skips and cages was
    a. Angles and other structural shapes—carbon steel of structural grade (Ni-Cu steel when available)
    b. Body plates—SAE 2320 steel specification (Ni-Cu steel when available)
    c. Liner plates—SAE 3160 steel specification (A8660 available in U.S.)
    d. Steel forgings—Inco RR Specification No. 2B (0.30C, 2¾% Ni)
    e. Steel castings—Inco RR Specification No. 4 (0.20 C, 2% Ni)
    f. In this case also the No. 14 gauge plate used in the cage construction was of Yoloy (Yoloy is Ni-Cu steel, 2% Ni, 1% Cu).

TONNAGES HOISTED IN LIGHTWEIGHT SKIPS

Mine	Skip No.	Skip No.	Skip No.
Frood No. 3	SK-75 5,577,506	SK-76 5,716,047	SK-77 6,710,563
Creighton No. 3	SK-84 3,306,044	SK-85 3,151,808	SK-86 3,295,434
Creighton No. 5	SK-81 1,959,694	SK-82 1,928,618	SK-83 1,811,687
Levack No. 2	SK-88 1,696,335	SK-89 1,890,024	SK-90 1,961,680
Garson No. 2	SK-94 681,924	SK-95 783,633	SK-96 672,014

Each shaft has two skips in service with one as a stand-by. When a skip has hoisted from 200,000 to 250,000 tons, it is taken out of service for inspection and repairs. In this way, the skips are in regular rotation. The tabulation above includes tonnages to the last skip change in each case.

An analysis of repair costs of the two types of skips and cages over a period of 3 years for each type, indicates that no reduction in cost has been obtained following the adoption of the lighter type of equipment.

The pitting of the nickel steel, presumed to be the result of acid water corrosion and which was discussed at considerable length when the skips and cages were first installed, although still in evidence has not proved to be a serious factor in the maintenance of the equipment. The use of protective coatings of paint, *viz.*, Peck-o-dite, Ano-dite, and in some cases red lead, is no doubt responsible in some measure for this.

A saving in weight of 25 to 33 per cent is indicated by using nickel-copper steel.

Through the courtesy of the Aluminum Company of America[1] the following information is presented:

The use of aluminum alloys in skips and cages will reduce dead weight from 30 to 60 per cent, giving increased capacity, increased speed, decreased power consumption, or decreased hoisting-machinery costs. In many cases, corrosion-resistant aluminum alloys will result in reduced depreciation and maintenance costs.

Leading mines throughout the United States and Canada are using aluminum skips and cages in the production of gold, silver, lead, copper, iron, coal, and salt.

Design. The average skip or cage is a tailor-made job built to suit local conditions of operation and equipment. The shape and the arrangement of members of both skips and cages is usually fixed within narrow limits by shaft clearances, loading and dumping facilities, and hoisting equipment. The problem is to build a container to fit these conditions and a sling to support and guide that container. In changing from steel to aluminum, it is seldom possible to make major changes in design but by a careful study of the problem it is frequently possible to effect substantial savings of weight and material costs by changes in details.

The necessity of weight reduction and the desire to save material cost justify more extensive design effort than has been given to most skips and cages. Experience has demonstrated that a direct size for size substitution of strong alloy aluminum parts for steel will usually provide adequate strength and durability. Proper stiffeners will frequently permit a reduction in plate thicknesses with an increase in strength and a decrease of weight. Carefully designed connection details will eliminate waste and possibility of trouble.

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[1] Personal communication from P. V. Faragher.

Abrasion. Skips and cages are subjected to wear by materials handled and by contact with guides and dumping mechanism. Strong alloy aluminum has proved adequately resistant to abrasion by coal, salt, and similar materials. Most heavy metal ores are extremely hard and heavy. Where such materials constantly fall upon or slide over the surfaces of the skip, the aluminum must be protected by replaceable liners of aluminum, steel, wood, or rubber. This applies to the skip bottom and those portions of the sides which are subject to the direct fall of material during loading and to the dumping lip. Protection of these parts is usually necessary on all-steel skips so that no extra expense is involved. It is customary to provide the slings of skips and cages with removable cast-iron shoes, which engage the guide rails fixed on the walls of the shaft. Such shoes should be retained on aluminum skips. Dumping horns or roller should be of steel.

Corrosion. Experience has shown that the resistance of certain aluminum alloys to most types of corrosion encountered is superior to that of other construction materials. This fact tends toward decreased maintenance and replacement costs. In spite of this it must never be assumed that unprotected aluminum will solve all corrosion troubles. For each installation it is desirable that the type of exposure and the nature of the corrosive elements be thoroughly investigated. Every aluminum skip and cage must have adequate paint protection.

Materials. The most generally economical aluminum alloy for *structural parts* of skips and cages is 61ST or 14ST. Plates and shapes of this alloy have good strength and corrosion resistance and are standard material available on short notice. Its toughness and resilience gives it good resistance to static or impact loads. Plate and sheet of 27ST has been used to good advantage where its hardness is desirable to resist abrasion. In some cases it will give satisfactory service as a replacement material for steel liner plates. In conditions of exposure to severe corrosion, particularly sodium chloride, 53ST has desirable qualities but it is definitely inferior to 61ST or 14ST in general strength characteristics.

For *cast parts*, alloy 220-T-4 can be used where maximum strength and shock resistance is required. Alloy 195 in various tempers has reasonable good strength and is lower in price. It may be used for parts where cast iron has proved satisfactory. No aluminum casting alloy has a strength equal to that of high-grade steel castings and the thickness of aluminum castings must be increased over that necessary for the same item in cast steel. Given proper design, a 220-T-4 casting will weigh about 50 per cent as much as a steel casting of equal strength.

Rivets and bolts of steel will usually prove satisfactory. If corrosion conditions are severe, cold driven 53SW rivets up to ¾ in. in diameter can be used. For such rivets, the shear strength is about half of that of a steel rivet. Aluminum bolts should be of alloy 24ST.

Certain parts that should remain in steel are mentioned above. In addition to these it is usually desirable to use steel for miscellaneous hardware, wheels, shafts, and safety devices. Where such steel parts are in contact with aluminum, galvanizing or cadmium plating is desirable. Bushings should be used on all points where there is movement of a shaft in aluminum castings or plate. Such bushings may be bronze or steel as the service demands.

Fabrication. The fabrication of aluminum equipment must be done by competent workmen with good tools. The general procedure for steel fabrication should be followed but heat-treated aluminum alloys must not be welded, flame-cut, or heated. Where sharp forming of shapes or bars is necessary, this work should be done at our Massena plant. Where plates must be cut to irregular shapes, our Alcoa plant can do the work if furnished with accurate drawings. It is uneconomical and may be detrimental to their strength for the average isolated mine shop or small fabricating plant to attempt hot forming of heat-treated aluminum alloys.

In riveting it is desirable that holes in highly stressed members be subpunched and reamed. If hot steel rivets are used, standard methods should be

TABLE 37*a*.—CAPACITY AND WEIGHT OF SKIPS

Capacity		Mine	Weight of skip, lb.
Cu. ft.	Lb.		
60	......	United Verde	3,950
80	......	United Verde	4,800
100	......	United Verde	5,800
112	......	United Verde	12,000
160	......	United Verde	8,000
...	12,200	New Jersey Zinc Company	5,500
..	10,000	Boston and Montana	7,200
...	13,400	Newport Mining Company	4,500
...	22,000	(Skip plus yoke) New Orleans	15,100

TABLE 37*b*. CAPACITY AND WEIGHT OF COMBINED SKIPS AND CAGES

Capacity of skip, lb.	Weight of skip, lb.	Weight of cage, lb.	Mine
4,000	3,050	1,550	Butte Mines
6,000	4,000	1,000	Butte Mines
8,000	5,880	3,000	Butte Mines
12,000	6,500	3,300	Butte Mines
12,000	8,700	3,900	Butte Mines
12,000	9,000	3,500	Butte Mines
14,000	14,000	3,600	Butte Mines
14,000	8,000	1,800	North Butte Mining Co.

TABLE 37*c*. WEIGHTS OF CAGES

Single-deck cages

Size, ft.	Weight, lb.	Car, lb.	Contents, lb.	Total load in cage, lb.
4 × 3	2000 to 4000	1000	2200	3200 to 6000
4 × 3	5000	2500	6500	9000
4 × 5	900 to 1400	. . .		2000
4 × 5	1400 to 1800	.	.	3000
4 × 5	2800 to 3000		. . . .	4000
4 × 8	4700 to 6000	.	. . .	9000
6 × 11½	5600	2500	6000	8500

Two-deck cages

Weight of top deck, lb.	Weight of lower deck, lb.	Load per car, lb.
3600	2000	2500
2600	2000	2500
2400	960	2500

followed. If cold-driven 53 ST rivets are used, a flat-cone type of head will facilitate the ease and speed of driving.

Pressed bushings in aluminum castings should have a fit of 0.001 in. per inch of bushing diameter. For bushings in wrought products, the fit should be 0.002 in. per inch of diameter.

Painting. All parts should be thoroughly cleaned before painting. The use of Deoxidine No. 6 or Rusticide No. 50 followed by a fresh-water rinse affords an effective means of cleaning aluminum parts.

Before assembly, all aluminum parts should be given a thorough coat of a zinc chromate iron oxide priming paint. All steel parts should be given a thorough coat of a bitumastic paint. This is particularly important at points where steel and aluminum are in contact. A finishing coat of a bitumastic vehicle with aluminum powder gives excellent protection. All wood parts should be thoroughly painted with aluminum paint before assembling.

Table 37*d* gives data on aluminum-alloy skips and cages.[1] The cages and skips shown there were used at gold, silver-lead, copper, salt, coal, and iron mines. This equipment proved very resistant to corrosion when unpainted; after painting little corrosion was noted. Table 37*a*, *b*, and *c* gives data on steel skips and cages.

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[1] Aluminum Company of America.

TABLE 37*d*. ALUMINUM-ALLOY CAGES AND SKIPS*

Size, ft	Capacity	Weight, lb		Weight, saving on aluminum, per cent	Material	Remarks
		Aluminum	Steel			
		1550	3100	50	17ST plate, bar and shapes; steel rivets	Increased operating depth from 2000 to 3000-ft level
	12 men	2980	5240	43	17ST plate and bar; 17ST and 25ST shapes; steel rivets	Power saving; increased safety
	4 tons	5100	8100	37	17ST shapes and plates; steel rivets	Power saving hoisting ore from 4000-ft level; steel wear plates
	2½ tons	2957	5515	46	17ST plate and shapes; steel rivets	Combination skip and cage; increased operating depth
5½ × 10	60 men†	8265	11,500	28	17ST plate, shapes, bolts; steel and 17S rivets	Doubled speed getting men in and out of mine
		1175	1885	27	....	Crosshead
4⅔ × 4⅔		1650	2800	41	53ST shapes; 53ST and 4S plates; 53SW rivets	Salt mine; cage
	7 cu yd	2900	.	50	27ST plate; 17ST shapes; steel rivets	Increased speed; decreased power, skip
	7 cu yd	4170	10,170	59	53ST shapes and plate; 53SW rivets	Corrosion resistance, weight saving, freedom from rust contamination, salt mine
		4450	10,950	59	17ST shapes and plates; steel rivets	Increased speed; decreased power; combination skip and cage
		4000	6000	60	17ST shapes and plate; steel rivets	Increased speed and safety

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* See also KERR, J. E., The Role of Nickel in Mining Equipment, *Eng. Mining J.*, **140**, No. 11, 47, (1939).

† The capacity of the steel cage was only 30 men.

CHAPTER VII

SAFETY CATCHES

Most state mining regulations require that cages or conveyances handling men be provided with a safety device for stopping the cage in case of rope failure.

In many cases these safety catches, or "dogs," as they are commonly known, function with more or less indifferent success. The main reason for their failure to operate is probably lack of consistant inspection to see that the mechanism is free of dirt and caked or hardened grease; that the actuating springs have retained their resiliency with age; and in some cases that the device was properly designed and tested at the start. There is some evidence that the guides wear to the extent that the dog teeth can not get sufficient "bite" to bring the cage to a stop. Guides should be replaced when they have worn down 1 in. on each face. Occasionally, accidents occur because the rope breaks so far above the cage that the spring is unable to overcome the inertia of the rope before the cage has gained too great a downward velocity.

There has been some doubt expressed as to rope inertia being responsible for the safety mechanism not functioning. If the rope should break when the cage is descending, both rope and cage should have the same velocity. Windage and friction would slow the cage down so that the rope should really gain a little, thus providing slack for the dog springs to act. It is very possible, however, that the end of the rope catches on the shaft timber and momentarily withdraws the dogs from the guides. Sufficient velocity is then gained to prevent the dogs again from slowing and stopping the cage. A most difficult part of the design is to provide a positive means of interrupting the connection between the drawbar and the dog shaft after the dogs have once acted.

A comprehensive investigation and excellent report has been made to the Minister of Mines, Province of Ontario, Canada, by Dean C. R. Young, chairman of the committee appointed by the Minister of Mines.[1]

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[1] Report of the committee appointed by the government of the Province of

This report covers the subject of hoisting in a very thorough and detailed manner.

The Ontario Department of Mines requires that the following conditions be satisfied before they approve the safety mechanism.[1]

1. Stop the cage if the rope breaks even when the cage is descending at the equivalent of maximum hoisting speed.
2. Dogs must not turn over and through the guides.
3. Dog teeth will not clog up with chips or other refuse.
4. Will compensate for guide wear.
5. Trailing rope will not disengage dogs.
6. Dog teeth stop cage with a deceleration of not more than $2g$ when the cage is descending at maximum hoisting speed. W. O. Tower suggests that the mechanism be designed for the maximum man-load to meet these conditions.

The Young Committee report might well be studied by anyone contemplating the design of safety dogs. After the design is completed, it should be tested to determine what changes may be necessary. The committee found that 12.1 per cent of the 116 tests reported failed. Some of the tests were experimental. The reasons for these failures were

1. Guides bent outward under pressure of dogs (pressure was applied too near the inner edge of guide).
2. Teeth that were shorter than $\frac{5}{8}$ in.
3. Shallow teeth that were completely clogged with wood spoil.
4. Spruce guides and poor guide joints.
5. Dogs forced *through* the guides. A wider tail piece was needed and possibly an additional stop to limit rotation of dogs.
6. Drag of rope trying to accelerate sheave and hoist drum. Springs could not overcome this resistance. (The trailing rope effect is the principal difficulty left.)
7. Dog teeth clogged up with wood. A splitter for preventing this condition has been devised.

Summarizing these failures, five resulted from dogs turning over; three from clogging teeth; two from guides buckling; two from faulty guide joints; one from teeth that were too short; and one from trailing rope.

Ontario to enquire into matters arising out of the accident at the Paymaster Consolidated Mines, Ltd., Feb. 2, 1945. Ontario Department of Mines, Toronto, Ontario, Canada, Mar. 25, 1946.

[1] Personal communication from W. O. Tower, chief inspector of mines, Toronto 2, Ontario.

Influence of Velocity on Test Failures. Safety mechanisms were found to function successfully at velocities of more than 2000 ft per min. They worked just as well at high as at low velocities.

Ratio of Spring Force to Weight of Conveyance. The Young Committee found no apparent connection between the initial spring force and the weight of the cage. The ratio of the initial spring force to the weight of the loaded conveyance varies from about 15 to 100 per cent. The average ratio would be about 40 to 50 per cent. For the empty cage this ratio would be proportionally greater.

Number of Dog Teeth. Dogs with two to three long teeth gave consistently better results than those having numerous shallow teeth (see Fig. 114).

Stops to Limit Rotation of Dogs. There are a number of ways to limit the rotation of the dogs.

1. Cage stops
2. Guide stops
3 Linkage stops
4. Dog-shaft stops
5. Combination of these

One method seems to be as good as another. Guide stops probably provide better self-adjustment for guide wear; one of the other stops should be used with a guide stop in order to prevent complete rotation of the dogs and stop through the guide.

Width of Dog Face. There was no definite relationship between failure and width of face. The main thing is to see that the dog does not engage too near the edge of the guide.

The following conclusions were made by the Young Committee, so far as safety mechanisms were concerned.

1. Timber guides are frequently inadequately connected to the dividers, or horizontal shaft timbers.

2. Safety dogs effectively stopped a falling conveyance under initial velocity in 87.9 per cent of 116 drop tests carried out in Ontario since the Paymaster Mine accident. The cause of failure in the unsuccessful cases in decending order of frequency were clogging of the teeth of the dogs, buckling of guides, unsuitable or badly connected guides, and a trailing rope.

3. Safety dogs acted as effectively for rope speeds of from 1200 to 2000 ft per min as for lower speeds.

4. Dogs with a considerable number of small, shallow teeth are less satisfactory than those with a few deep teeth. The addition of a feature for clearing away wood spoil by the dog itself is meritorious.

5. An acceptable ratio of the rate of deceleration to acceleration of gravity is 3.0. Cage safeties should be designed to keep within this limit.

The depth of tooth penetration into the guide before the guide stop meets the guide is 1 to 1¼ in.

Lancaster[1] has presented data for designing the dog for any given set of conditions. This method is still more or less in the realm of experimentation. The mechanism should be tested before final acceptance. Figure 113 gives the general features of his design, which has proved satisfactory in practice. In *a* the dogs are held in a retracted position by the torsion springs shown in *d*. The loose links limit the rotation of the dogs away from the guides. The full pull is on the rope in *a*. If the load is removed from the rope, the leaf spring moves the drawbar downward, causing the links to pull the dogs into contact with the guides. The pawl locks the dog in this position and prevents it from rotating away from the guide.[2] This is shown in

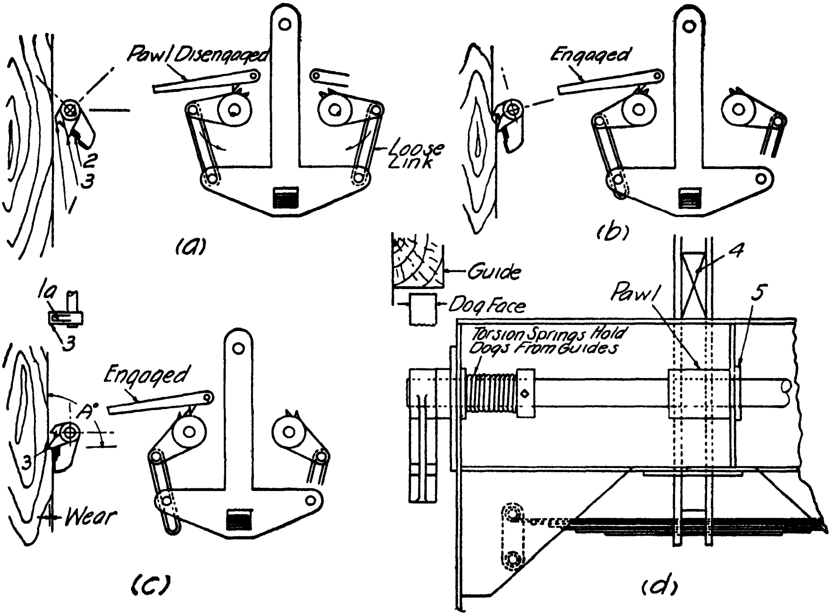

FIG. 113. Safety dog and drawbar arrangement. (*Courtesy of Canadian Mining Journal.*)

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[1] LANCASTER, A. W., Mine Cage Safety Dog with Predictable Deceleration, *Can. Mining J.*, November, 1946, p. 1039.

[2] If a cage is suspended from a skip, with the dogs on the skip, it is recommended that the pawls be left off; it is difficult to disengage them if they act. *Ibid.* December, 1946, p. 1118.

Fig. 113*b*. Because of the weight of the cage, the tooth marked 3 is forced into the guide; it is locked in this position by the pawl. If by any chance the trailing rope catches and raises the drawbar, the loose linkage has no effect on disengaging the dogs. The pawl prevents the torsion spring from rotating the dogs away from the guides. Figure 113*c* shows the dogs fully engaged and locked in position by the pawl. The possible guide wear and the compensation for it by the long tooth 3 are shown also. When the guides have lost a total of 2 in. in width, they should be replaced. In *a* the point 2 is a knife-edge splitter intended to prevent the teeth from clogging up with wood chips. The angle *A*, Fig. 113*c*, will be explained later. In view *d*, 4 is a filler to prevent material from falling between the drawbar and the dog shaft; an additional bearing 5 is used to help stiffen the dog shaft.

Lancaster has approached the deceleration control by investigating the angle the projected tooth area makes with the guide. This is *A* in Fig. 113*c*. He used the action of the bit in an ordinary carpenter's plane as an analogy. His data are for British Columbia fir (Douglas fir). Experimentally, the average wood resistance was determined for several values of *A* keeping the area of the tooth constant. The probable deceleration may be found from the formula:[1]

$$\mathrm{KE} = \frac{1}{2} mv^2 = \frac{W}{2g} v^2$$

$$v = \sqrt{2gh}$$

$$\mathrm{KE} = \frac{W}{2g} (\sqrt{2gh})^2 = Wh \text{ or } Ws$$

Let s_f = distance for free fall, ft

v_f = velocity for free fall distance s_f, ft per sec

g = acceleration of gravity, free fall, ft per sec^2

s_s = distance traveled after dogs engage, ft

v_s = velocity at start of engagement, ft per sec

g_s = deceleration for stopping, ft per sec^2

$v_f = \sqrt{2gs_f}$

$v_s = \sqrt{2g_s s_s}$

but

$$v_f = v_s$$

therefore

$$\sqrt{2gs_f} = \sqrt{2g_s s_s}$$

squaring,

$$2gs_f = 2g_s s_s$$

and

$$\frac{s_f}{s_s} g = gs$$

or

$$\frac{s}{s_1} = \frac{g_s}{g} = \text{number of } g\text{'s of gravity}$$

As previously stated, this value should not exceed 2.

$$\text{Kinetic energy} = KE = Ws$$

and

$$P = \frac{\text{KE}}{s_1} + W$$

where W = weight of cage plus load lb
s = free fall to point where dogs engage guides, ft
s_1 = stopping distance ft (distance traveled after dogs engage)
P = wood resistance lb

The ratio of deceleration to the acceleration of gravity is found from

$$\frac{s}{s_1} = \text{number of } g$$

Also

$$R = \frac{P}{a} = \text{ultimate wood resistance} \quad \text{lb per sq in.}$$

a = projected area of tooth in contact with wood

This is for all four dogs. Values of R are given in Table 38.[1]

TABLE 38. RESISTANCE OF WOOD GUIDES TO DESTRUCTION BY SAFETY DOGS
(Pounds per square inch of projected tooth area)

Angle A	Pressure	Angle A	Pressure	Angle A	Pressure	Angle A	Pressure
90°	2700	78°	1600	65°	1200	55°	800
84°	2100	72°	1400	60°	1000	50°	600

The guides used in Table 38 were of British Columbia fir. Test data were obtained for 65 deg, 78 deg, and 90 deg. Other values are by interpolation. Types of dogs used are shown in Fig. 114.[2]

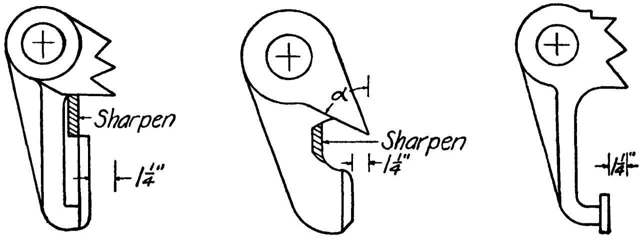

FIG. 114. Safety dogs. (*Young Committee Report.*)

Using Lancaster's data, the application to the selection of the dog size is shown in the following example. Weight of cage and contents

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[1] LANCASTER, A. W., *op. cit.*, Table III. The values were found experimentally.
[2] *Young Committee Report*, $C8$, $C9$, $B1$ of Fig. 6.

= 10,000 lb; maximum descending velocity = 1200 ft per min = 20 ft per sec; angle A = 90 deg, maximum deceleration not to exceed $2g$. What should be the total area of the four control teeth?

EXAMPLE

$$v = 20 \text{ ft per sec}$$

$$s = \frac{v^2}{2g} = \frac{20^2}{2 \times 32} = 6.25 \text{ ft}$$

$$\frac{s}{s_1} = 2$$

$$s_1 = \frac{6.25}{2} = 3.125 \text{ ft}$$

$$\text{KE} = Ws = 10{,}000 \times 6.25 = 62{,}500 \text{ ft-lb}$$

$$P = \frac{\text{KE}}{s_1} + W = \frac{62{,}500}{3.125} + 10{,}000 = 29{,}960 \text{ lb}$$

$$a = \frac{P}{R} = \frac{29{,}960}{2700} = 11.1 \text{ sq in.}$$

This would be the maximum area permissible. A greater area would slow the cage at more than $2g$.

$$\text{Area of one tooth} = \frac{11.1}{4} = 2.77 \text{ sq in.}$$

Assuming that the total depth of tooth is 1¼ in., the width becomes

$$\frac{2.77}{1.25} = 2.22, \text{ or } 2\tfrac{1}{4} \text{ in.}$$

Under actual test it has been found that the cage travels about 1 to 1.5 ft after the dogs start turning into position before they actually engage. The inclusion of this correction would make the area a little less than found above.

The preceding calculations will provide a starting basis. The dog should be tested before its final adoption.

If necessary, a more desirable tooth depth and width can be had by decreasing angle A.

EXAMPLE

Using A = 78 deg,

$$a = \frac{P}{R} = \frac{29{,}960}{1600} = 18.7 \text{ sq in.}$$

$$\text{Area for one dog} = \frac{18.7}{4} = 4.67 \text{ sq in.}$$

If tooth is 1¼ in. deep,

$$\text{Width} = \frac{4.67}{1.25} = 3.74 \text{ in.}$$

By using a reduced area for the first tooth to engage, the slowing-down operation will be more gradual. Such a dog is shown in 1*a* in Fig. 113*c*.

There are in general use three types of springs for operating the safety dogs.

1. Semielliptic springs (flat leaves)
2. Coiled torsion springs made of circular spring wire
3. Coiled tension (extension) or compression springs of circular spring wire

In addition to the previous suggestions, the following rules should be considered when beginning the design:

1. Inspection of the entire mechanism, from drawbar to spring to dogs, should be provided for.
2. Positive means of lubricating the moving parts must be included.
3. The spring should be enclosed in a dustproof waterproof housing when possible or if shaft conditions warrant.
4. A method for testing the spring and freedom of the other moving parts should be provided.

General Considerations for the Design. There is a scarcity of published information that actually gives necessary details. Peele[1] has a little general data; but the mining companies seem to have adopted a somewhat different line of approach.

The upper crosshead is designed as a simple beam with a concentrated load at the center. The bending moment is

$$M = \tfrac{1}{4}Pl \qquad \text{in.-lb}$$

where P = the load on the rope (weight of cage and contents), lb
l = length of crosshead, in.

The net area of the drawbar is found from $P \div 6000$ (see Skips and Cages).

Side members for one side are selected from one half the cage and contents plus one-half the weight of that part of the cage below the crosshead.

Safety Dogs. Peele recommends that the dimensions of the dogs, dog shaft, and dog-shaft bearings be large enough so that two of the dogs will support the load. Each linkage, dog shaft, etc., is then designed to take one-half the load. The extension of the dog shaft beyond the plate or center of bearing supporting it is considered as a cantilever beam with a concentrated load at the center of the dog. The concentrated load is

$$W = 0.5\,\frac{P}{\cos\alpha} \qquad \text{(see Fig. 115}f\text{)}$$

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[1] PEELE, R., "Mining Engineers' Handbook," 3d ed., Sec. 12, p. 102, John Wiley & Sons, Inc., New York.

where α = angle between vertical (guide) and line through center of shaft to point of contact of dog with guide
= 45 deg (this assumed value closely approximates actual conditions)

If the method suggested by Lancaster (previously discussed) is used, then P should probably be based on $P = KE/s_1 + W$.

Inserting the 45 deg,

$$W = \frac{0.5P}{\cos 45^\circ} = 0.708P \qquad \text{lb}$$

and the bending moment on the cantilever is

$$M = 0.708\,Pa \qquad \text{in.-lb}$$

where a = horizontal distance between center line of dog and center of dog-shaft bearing

$M = \dfrac{sI}{c} = sS$, in.-lb

S = section modulus $= \dfrac{I}{c}$

s = 6000 lb per sq in.

$I = \dfrac{\pi d^4}{64}$ for a circular section (dog shaft)

$c = \dfrac{d}{2}$

$$S = \frac{0.708Pa}{6000} \qquad \text{in.}^3$$

then

$$\frac{\dfrac{\pi d^4}{64}}{d/2} = \frac{0.708Pa}{6000}$$

$$d = \sqrt[3]{0.0012Pa} \qquad \text{in.}$$

Springs for Operating Dogs. Peele[1] suggests that the spring or springs be designed to support one-third the weight of small cages and one-tenth the weight of large cages (unloaded). This does not seem to be followed in practice.[2] The Anaconda Copper Mining Company has designed its springs for about two-thirds the weight of the cage, and the Sunshine Mining Company, Kellogg, Idaho, for one-half. The average Canadian practice comes close to one-half. Whatever weight is decided upon, it must be such that the springs do not cause the dogs to grip the guides while the cage is in motion. This would be apt to happen during lowering if the springs were too strong.

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[1] *Ibid.*

[2] Personal communications from W. R. Catrow, Anaconda Copper Mining Company; Sunshine Mining Company; the Young Committee Report.

Force Acting on the Spring. Before the force acting on the spring may be found, the type of spring (elliptical, torsion, compression, or tension, and number of springs) must be decided upon. Figure 115 illustrates, diagrammatically, the different possibilities. There are others, but those shown are in common use.

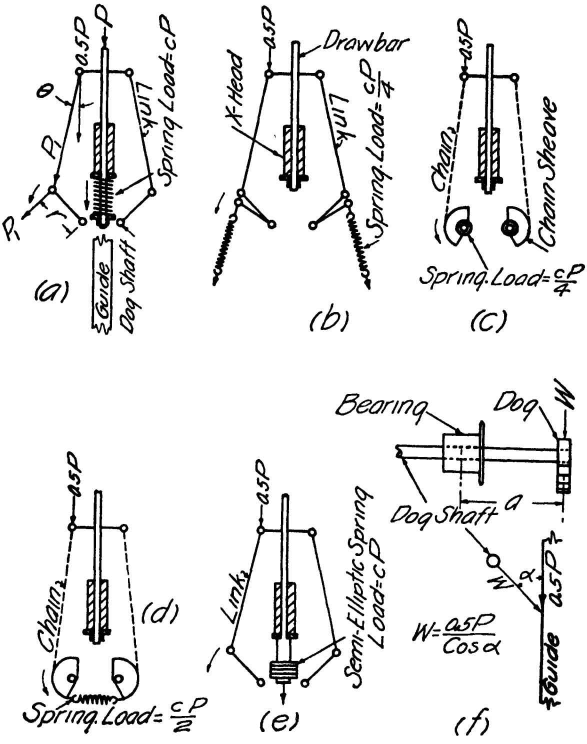

FIG. 115. Types of springs for operating safety dogs.

In the figure,

P = weight supported by safety mechanism (cage empty) lb

$P_1 = \dfrac{0.5P}{\cos\theta}$ (see Fig. 115a) lb

P_1r = turning moment on dog shaft in.-lb

The force P is multiplied by the percentage factor c, decided upon for the weight of the cage to be supported by each spring.

In Fig. 115*a*, there is only one spring; therefore, the load is cP; in *b* there are four springs and the load on each is $\frac{1}{4}cP$; *c* has four springs also; *d* has two springs, so that the load is $\frac{1}{2}cP$; and *e* has one semi-elliptic spring and the load is cP. To be described later is the Hollinger setup, which uses compression springs on the drawbar and counteracting torsion springs on the dog shafts (see also Fig. 113).

The diameter of the dog shaft to resist the turning moment P_1r must be checked against the shaft found to resist the cantilever action previously mentioned. The larger is selected.

Inertia of the drawbar, links, chain sheaves, etc., and friction of the parts, all tend to slow down the action of the spring. Some designers have included such retarding action in their calculations. This would seem unnecessary if the spring were designed on the basis of *c* not less than 50 per cent.

Formulas for Springs. The formulas to be given are those commonly used.[1] In actual practice the constants are, in some cases, made equal to unity; and the working load value for spring steel varies considerably from that suggested by the various handbooks. However, if these formulas are used, the results should provide a close guide toward manufacturing the spring. It is the practice of most spring manufacturers to make up a sample spring and to test it against the specifications. The final product is then based on this test.

Helical Compression or Tension Springs (*Without Initial Tension*). The following formulas are for round wire. They are the ones commonly used for this type of spring.

$$P = 0.3927\,\frac{Sd^3}{D - d} \qquad \text{lb}$$

$$f = \frac{S\pi}{G}\,\frac{(D - d)^2}{d} \qquad \text{in.}$$

$$H = n(d + f) + I = \text{free unloaded height} \qquad \text{in.}$$

$$h = nd + I = \text{loaded height} \qquad \text{in.}$$

$$l = n\pi D \qquad \text{in. (approximately)} = \sqrt{(\pi D_1)^2 + p^2} \qquad \text{in. (more exactly)}$$

where d = diameter of spring wire, in.

D = outside diameter of coil, in.

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[1] KENT, "Mechanical Engineers Handbook," John Wiley & Sons, Inc., New York.

MARKS, "Mechanical Engineers Handbook," 3d ed., 1930. McGraw-Hill Book Company, Inc., New York.

BERARD, S. J., and E. O. WATERS, "Elements of Machine Design," 2d ed., D. Van Nostrand Company, Inc., New York.

"Mechanical Springs—Their Engineering and Design," Wallace Barnes Company, Bristol, Connecticut.

S = maximum fiber stress in section; or stress resulting from load P, lb per sq in. (direct shear and effect of curvature ignored)
P = compression or tensile load on spring (load to close or extend spring), lb
D_1 = pitch diameter of coil, in.
I = inactive portion of length, due to flattened ends, or loops, in.
= about $1\frac{1}{4}d$ for flat ends
n = number of active helical turns or coils
f = extension or compression of one coil with load P. Usually space (pitch) between coils of compression spring closed solid under load P, in.
fn = extension or compression of n coils with load P, in.
l = length of wire to form active spring, in. (to this must be added the inactive part to obtain total length)
p = pitch of coils, under no load, in.
G = torsional modulus of elasticity
= average value of 12,000,000 for practical design

The stress values usually used for designing are

Hot-coiled { 80,000 to 95,000 lb per sq in. for compression
60,000 lb per sq in. for tension
Cold-coiled { 90,000 lb per sq in. for compression
80,000 lb per sq in. for tension

Torsion Helical Springs, Round Wire

$$P = \frac{\pi d^3 S}{32rK} \quad \text{lb}$$

$$ra = \frac{2rlS}{dEK}$$

$$l = \sqrt{(\pi D_1)^2 + p^2} \quad \text{in.}$$

$$S = \frac{adEK}{2l} \quad \text{lb per sq in.}$$

where P = safe load, lb
d = diameter of spring wire, in.
S = safe stress in bending, lb per sq in., or equals stress resulting from a given load
D_1 = pitch diameter of coil, in.
r = pitch radius of coil, in.
l = effective length of spring wire, in.
K = spring constant = 1 for dog springs
p = pitch of coils, in.
a = circular deflection of spring in radians (usually between 60 deg and 90 deg)

When torsion springs are contemplated, the design should always be undertaken on the basis that the spring will be coiled up when under the load. If this cannot be done, the spring must be heat-treated after forming the coil.

As a torsion spring is loaded, its coil diameter changes. Inside and outside clearances must be provided to permit these changes in diameter. This change is approximately equal to the number of revolutions through which the coil is twisted, divided by the number of coils expressed as a percentage. At least this percentage change in diameter must be allowed for. For example, if the spring has 15 coils, is 5 in. inside diameter, and is turned through one revolution, the allowance is $\frac{1}{15} \times 100 = 6.67$ per cent and

$$6.67 \text{ per cent} \times 5 \text{ in.} = 0.33 \text{ in.}$$

The maximum diameter of the shaft or arbor about which the spring was coiled would be $5 - 0.33 = 4.67$ in.

When the load is put on a torsion spring, the longitudinal distance between coils also changes. The minimum allowance for this may be found as previously explained for diameter clearance. In the foregoing example, where the spring was assumed as being wound up, a pitch clearance of 6.67 per cent would be allowed.

Semielliptic Springs. This type of spring is shown in Figs. 113*d* and 117

$$P = \frac{nSbt^2}{6l} \quad \text{lb.}$$

$$S = \frac{6Pl}{nbt^2} \quad \text{lb per sq in.}$$

$$y = \frac{6Pl^3}{Enbt^3} \quad \text{in. for a spring with one full-length leaf}$$

$$y = \frac{5.5Pl^3}{Enbt^3} \quad \text{in.} \qquad \text{with more than one full leaf}$$

By substituting the expression for P in y,

$$y = \frac{l^2S}{Et} \quad \text{in.}$$

where P = one-half load at center of spring, lb
S = maximum unit stress in spring under load P; or maximum allowable unit stress, lb per sq in.
l = one-half effective length of spring, in. (this is the length under the load P)
= $(L - x) \times \frac{1}{2}$, where L equals total length and x equals width of strap or clip at center; if there is no restraining band at the center, $l = L/2$
E = modulus of elasticity = 30,000,000
n = number of leaves
b = width of leaf, in.
t = thickness of leaf, in.
y = deflection of end of spring, in.

The value of S may be taken at 80,000 to 85,000 lb per sq in. [The spring in Fig. 117 was designed for a load of 3000 lb ($P = 1500$), and $S = 126{,}000$ lb per sq in.]

When selecting a spring, the quickest method is to assume values for the inside diameter of the coil and the diameter of the wire. Before choosing the inside dimensions of the coil, sufficient allowance must be made for the member which passes through the coil (dog shaft, or drawbar). After deciding upon an acceptable value for P and S for the type of steel the formulas may be applied.

EXAMPLE

A spring for the design shown in Fig. 115*a* is to be selected. The linkage connecting the drawbar to the dog shaft is such that the drawbar will move a vertical distance of 8 in. The spring will surround a 3-in.-diameter sleeve through which the drawbar passes. Value of P is 1200 lb. For the steel, a value of 80,000 lb per sq in. is assumed.

This spring will be designed by the helical compression formula.

Assuming that the inside diameter of the coil is 3¼ in. and the wire $\frac{9}{16}$-in. in diameter,

$$P = 0.3927 \times \frac{80{,}000 \times (\frac{9}{16})^3}{(3\frac{1}{4} + 2 \times \frac{9}{16}) - \frac{9}{16}} = 1470 \text{ lb}$$

This is larger than the tentative P decided upon. But a ½-in. wire gives $P = 1050$ lb. Using the $\frac{9}{16}$ in. wire,

$$f = \frac{80{,}000 \times 3.14 \times (3\frac{3}{16})^2}{12{,}000{,}000 \times \frac{9}{16}} = 0.54 \text{ in.}$$

Therefore, one coil deflects 0.54 in. for the 1470-lb load.

For the 1200-lb load, $\frac{1200}{1470} \times 0.54 = 0.44$ in.

Number of coils $= n = 8 \div 0.44 = 18.15$, or 18 coils

$$h = nd + I$$

Assuming two inactive coils,

$$h = 18 \times \tfrac{9}{16} + 2 \times \tfrac{9}{16} = 11\tfrac{1}{4} \text{ in.}$$
$$H = 18(\tfrac{9}{16} + 0.44) + 2 \times \tfrac{9}{16} = 19.18 \text{ in.}$$
$$\text{Movement of drawbar} = 19.18 - 11.25 = 7.92 \text{ in.}$$

Selection of a Tension Helical Spring. A spring is to be selected for the type shown by Fig. 115*b*. The total spring load is 2000 lb. Extension is 6 in. per spring. Assume $S = 80{,}000$ lb per sq in. There are four springs; therefore the load on each is $\frac{1}{4} \times 2000 = 500$ lb. The outside diameter of the spring is taken at 3 in. Trying a $\frac{3}{8}$-in.-diameter wire,

$$P = 0.3927 \times \frac{80{,}000 \times (\frac{3}{8})^3}{3 - \frac{3}{8}} = 631 \text{ lb}$$
$$f = \frac{80{,}000 \times 3.14 \times (3 - \frac{3}{8})^2}{12{,}000{,}000 \times \frac{3}{8}} = 0.385 \text{ in.}$$

For the 500-lb, load

$$f = \frac{500}{631} \times 0.385 = 0.305 \text{ in.}$$
$$n = 6 \div 0.305 = 19.7, \text{ or } 20 \text{ coils}$$
$$h = 20 \times \tfrac{3}{8} = 7\tfrac{1}{2} \text{ in.}$$

Extended or loaded length $= n(d + f) = 20 \times (0.375 + 0.305) = 13.6$ in.

The spring may be attached to a dog-shaft sheave by nut and to the cage by nut on threaded ends of wire, or by a loop on each end of the spring.

A ⅜-in. wire after threading has a net area of 0.068 sq in.

The load on the net section is $500/0.068 = 7350$ lb per sq in., which is safe.

Helical Torsion Spring. A spring for the type of safety mechanism shown in Fig. 113c may be determined by the following procedure: The spring would be mounted on the dog shaft as shown in Fig. 118. Total force exerted by the four springs is 2000 lb. The inside diameter of the coil equals 3½ in.; the diameter of the wire is ½-in. The spring is to rotate through an angle of 95 deg. The load on each spring is $\frac{1}{4} \times 2000 = 500$ lb. The pitch is $\frac{9}{16}$ in. $= 0.5625$ in. There are 12 coils assumed.

Length of wire

$$\text{Pitch diameter of coil} = 3\tfrac{1}{2} + \tfrac{1}{2} = 4 \text{ in.}$$

$$l = \sqrt{(\pi 4)^2 + 0.5625^2} = 12.6 \text{ in. length of one coil}$$

For 12 coils, $l = 12 \times 12.6 = 151.2$ in.

From $ra = 2rlS/dEK$,

$$S = \frac{dEKra}{2rl} = \frac{dEKa}{2l}$$

$$K = 1$$

$$a = \text{circular deflection in radians}$$

$$= 95^\circ \div 57.3 = 1.655 \text{ radians}$$

$$S = \frac{\tfrac{1}{2} \times 30{,}000{,}000 \times 1.655 \times 1}{2 \times 151.2} = 82{,}200 \text{ lb per sq in.}$$

$$P = \frac{3.14 \times (\tfrac{1}{2})^3 \times 82{,}200}{32 \times 2 \times 1} = 504 \text{ lb}$$

The spring assumed, which has 12 coils, 3½ in. inside diameter, ½-in. wire, $\frac{9}{16}$-in. pitch of coils, and 95 deg rotation, provides the 500 lb per spring required.[1]

Another approach to the above example will be given. As in the previous examples, S will be taken at 80,000 lb per sq in. The inside diameter will depend on the diameter of the dog shaft and spring bushing (see Fig. 118*F*).

$$P = \frac{3.14 \times (\tfrac{1}{2})^3 \times 80{,}000}{32 \times 2} = 492 \text{ lb}$$

$$a = 1.655 \text{ radians}$$

$$l = \frac{\tfrac{1}{2} \times 30{,}000{,}000 \times 1.655}{2 \times 80{,}000} = 155 \text{ in.}$$

$$n = \text{number of coils} = l \div \pi D$$

$$= 155 \div 3.14 \times 4 = 12.32$$

A 95-deg turn $= 95^\circ/360^\circ \times 3.14 \times 4 = 0.332$ turns of the coil. Spacing of coils,

$$0.332/12.32 \times 100 = 2.69\%$$

$$2.69\% \times 3\tfrac{1}{2} = 0.0942 \text{ in.} \quad \text{between coils.}$$

$$\text{Pitch of coils} = 0.5 + 0.0942 = 0.5942 \text{ in., or } \tfrac{5}{8} \text{ in.}$$

---

[1] This is the method used by the engineers of the Anaconda Copper Mining Company. The wire diameter was assumed; other data were approximated, and the suitability of the selection was checked as shown. The bending stress of 100,000 lb per sq in. was specified to allow the manufacturer some leeway and to avoid curvature correction.

The spring would be 12 coils, 3½ in. inside diameter, ½-in. wire, with a coil pitch of ⅝ in.

The spring manufacturer would use the data obtained in the foregoing examples as a guide. His first product might not check the data exactly. This would not be serious, as the value of P is much lower than the actual weight of the cage. This would permit a somewhat stronger spring. A somewhat weaker spring would not ordinarily prove detrimental. The total deflection must be carefully decided upon so that the drawbar transmits its pressure onto the crosshead without danger of compressing or extending the spring beyond its determined deflection. The closed height of the compression spring should bring the coils not quite in contact with each other.

Semielliptical Spring. A spring for the type of design shown in Fig. 115*e* will be selected. The spring will be made as follows: These dimensions will be checked back against the weight of the cage. The cage weighs 4500 lb. Total deflection is 5 in.; at this deflection the extended length is 37 in. The spring consists of seven ¼-in. leaves, 3 in. wide; three leaves are full length.

From $y = \frac{l^2S}{Et}$,

$$S = \frac{Ety}{l^2} = \frac{30{,}000{,}000 \times \frac{1}{4} \times 5}{18.5^2} = 110{,}000 \text{ lb per sq in.}$$

$$P = \frac{7 \times 110{,}000 \times 3 \times 0.25^2}{6 \times 18.5} = 1300 \text{ lb}$$

With this setup the total load on the spring will be 2 × 1300 = 2600 lb, which is about 58 per cent of the cage weight.

This spring is well within the limits of carefully selected and tested spring steel.

Figure 116[1] shows the design of a compression spring used at the North Star mine, Grass Valley, California. It is used on a double-deck cage. The mechanism held under 3000 ft of 1⅜-in. rope that had fallen down the shaft and piled on top the cage.

In Fig. 117 is shown the semielliptic spring design used at the Sunshine Mining Company, Kellogg, Idaho.[2] No dimensions are shown; safety designs for cages are a tailor-made job for each set of conditions. In reproducing this design, some liberties have been taken with accepted drafting practice in order to emphasize certain detail.

The standardized design used by the Anaconda Copper Mining Company[3] is shown in Figures 118, 119, 120, 121, 122, 123, and 124.

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[1] TILLSON, B. F., "Mine Plant," Fig. XIV-4, p. 89, American Institute of Mining and Metallurgical Engineers (1938).

[2] By permission of The Sunshine Mining Company.

[3] By permission of E. R. Borcherdt, research engineer.

This mechanism will stop the cage within a distance of 1 in. of the slacked-rope position. Attention is called to the sleeve in Fig. 118*F* and 119*F*. It prevents the dogs from rotating beyond their effective position. (This is a dog-shaft stop.)

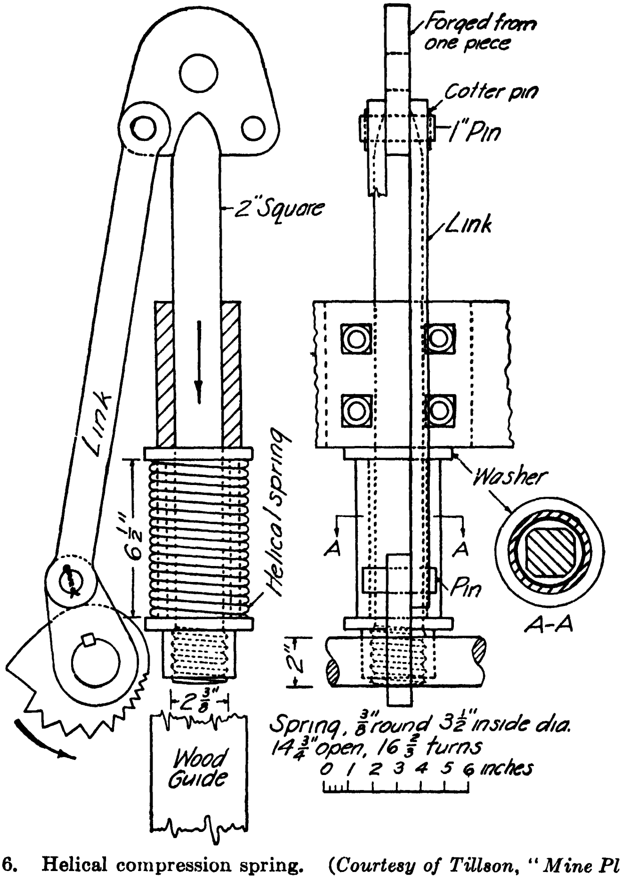


FIG. 116. Helical compression spring. (*Courtesy of Tillson, "Mine Plant."*)

The teeth of the dog shown in Fig. 124, show but slight tendency to clog, and their sharpness aids the springs in bringing the dogs into the engaged position.

Figure 121 shows the type of crosshead and side plates used on the Anaconda cages. In Fig. 122 are shown the essential features of the dogs and housing installed on the cage. The torsion spring is illustrated by Fig. 123.

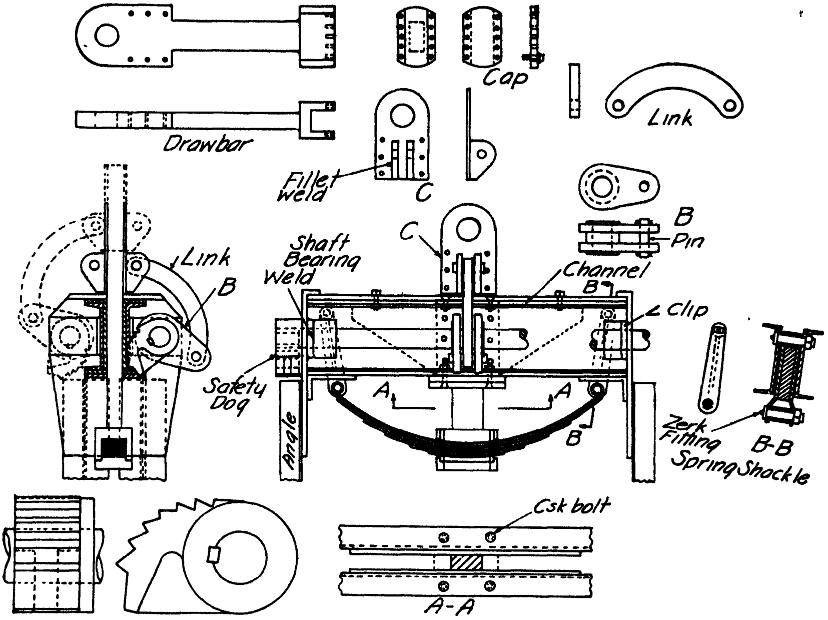


FIG. 117. The Sunshine Mining Company safety mechanism.

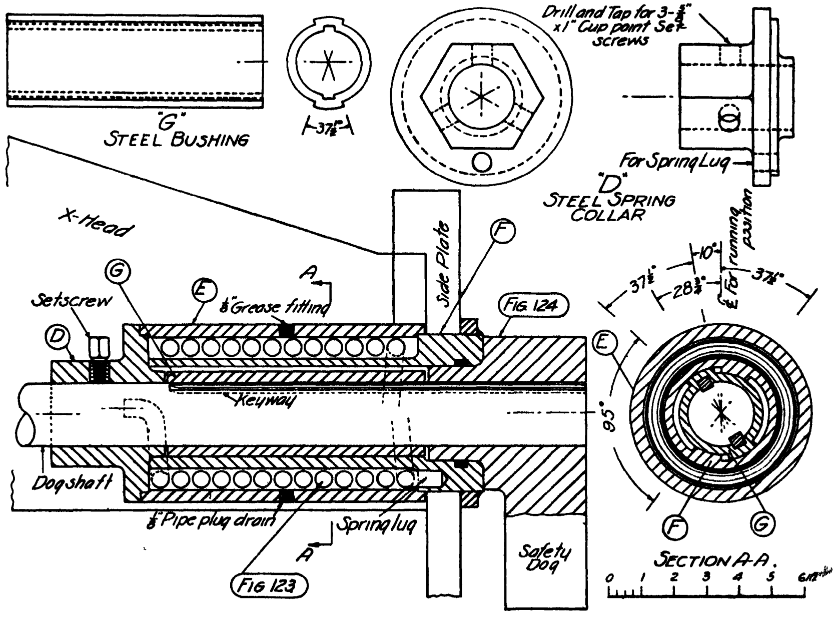


FIG. 118. Assembled safety mechanism, Anaconda type.

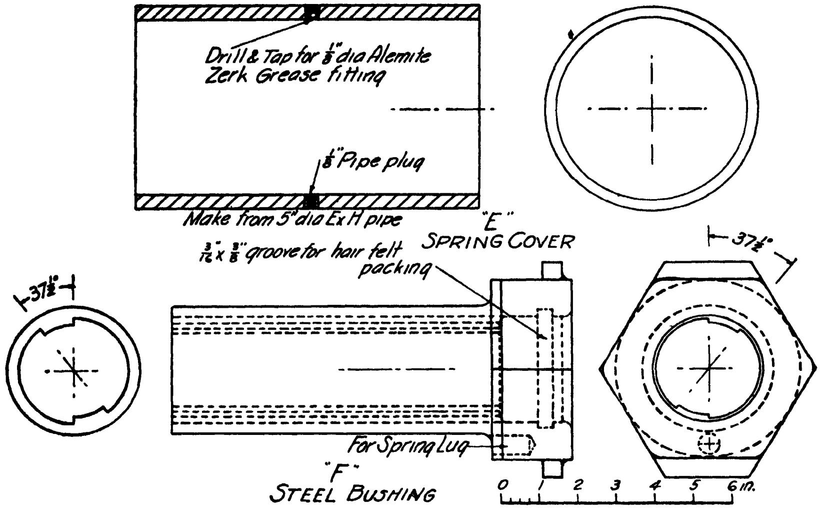

Drill & Tap for 1/8" dia Alemite Zerk Grease fitting
1/8" Pipe plug
Make from 5" dia Ex H pipe
"E"
SPRING COVER
3/16" x 3/8" groove for hair felt packing
37½°
37½°
For Spring Lug
"F"
STEEL BUSHING
0 1 2 3 4 5 6 in.

FIG. 119. Bushing and spring cover, Anaconda type.

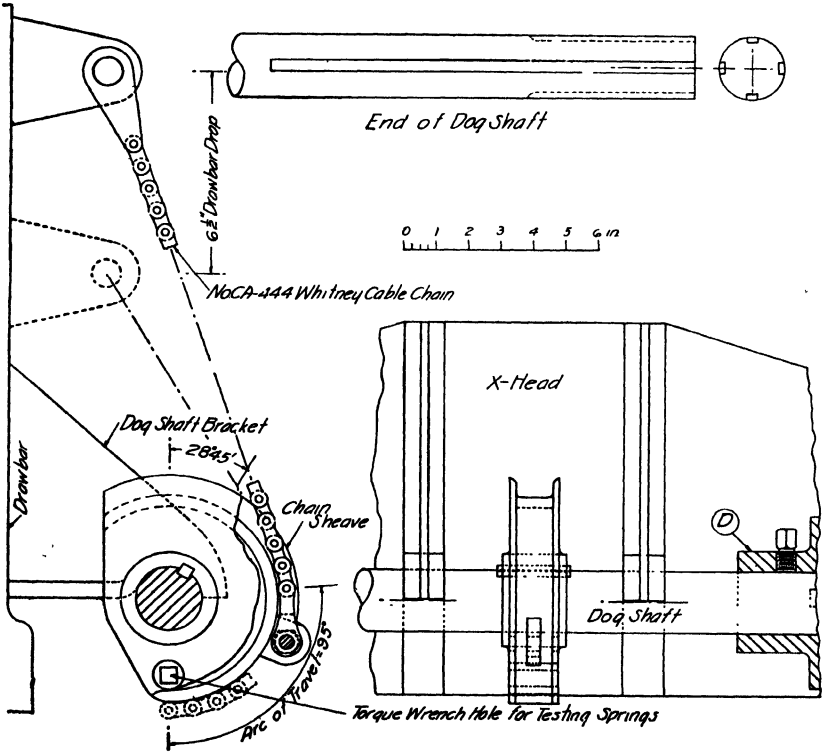


FIG. 120. Dog-shaft sheave, Anaconda type.

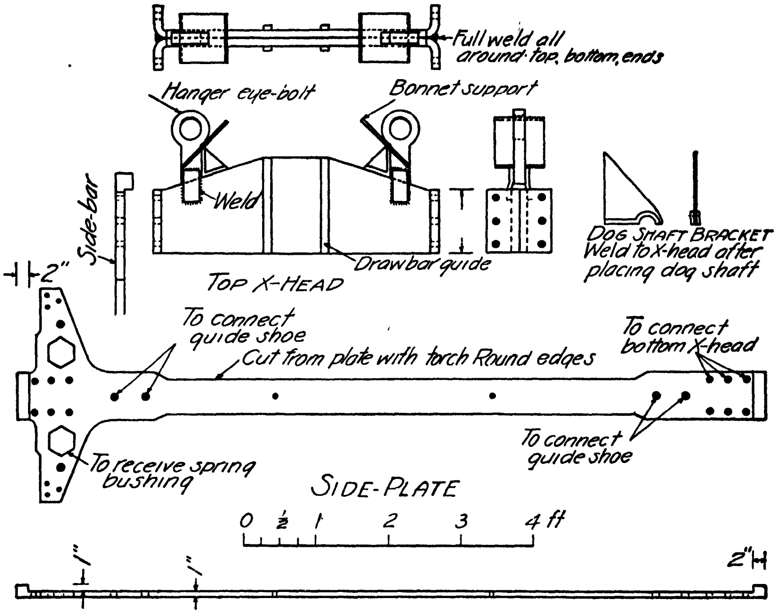


FIG. 121. Crosshead and side plate, Anaconda cage.

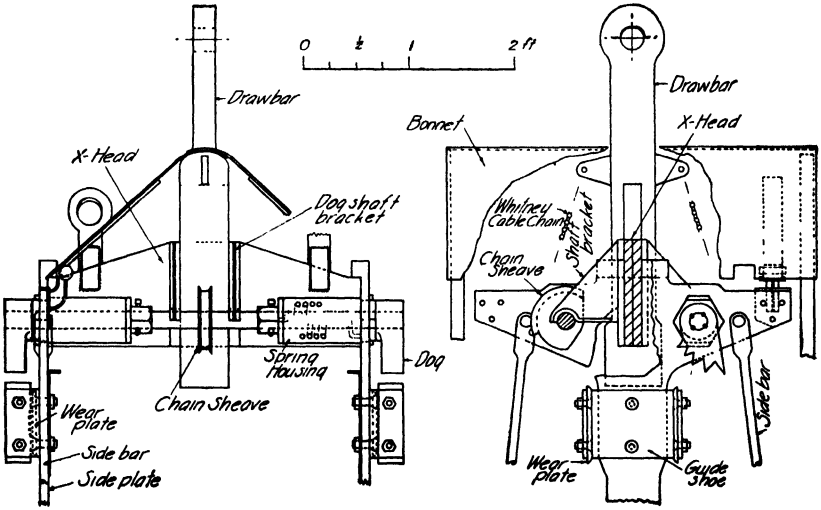
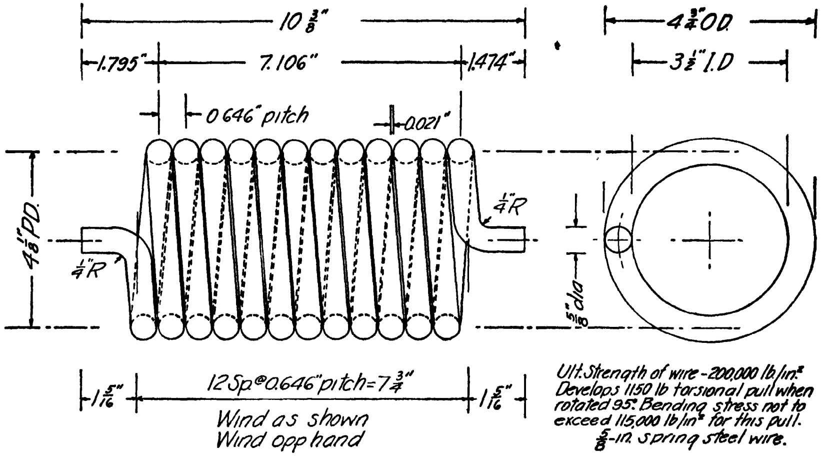
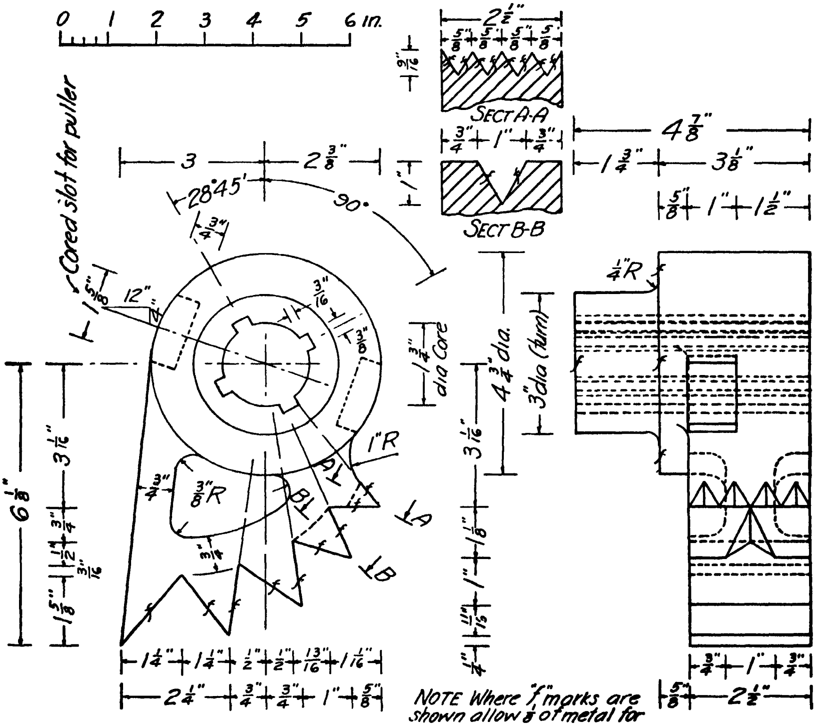


FIG. 122. Assembled safety dog, Anaconda cage.



Ult. Strength of wire - 200,000 lb/in.² Develops 1150 lb torsional pull when rotated 95°. Bending stress not to exceed 115,000 lb/in.² for this pull. ⅝-in. spring steel wire.

FIG. 123. Safety-dog spring, Anaconda type.



NOTE Where "f" marks are shown allow ⅛ of metal for machining

CAST STEEL - Make one as shown—One opposite hand

FIG. 124. Safety dog, Anaconda type.

In Fig. 125 is shown the safety mechanism in use at the Hollinger Consolidated Gold Mines, Ltd., Timmins, Ontario.[1] The following description of the mechanism explains its operation.[2]

There are two links which connect the top of the cage and the plates to which the rope is attached. These links take the load. In the center of the crosshead is the drawbar with its springs. In case of failure of the links, the drawbar will take the load. Its main purpose is to carry the spring tension. There are three coil springs, set one inside the other, and all enclosed in a 10-in.-pipe housing (see also Fig. 126). The full travel of the springs is $4\frac{5}{8}$ in. and the tension is 2800 lb. From the drawbar there are two arms that engage levers on the dog shafts (see Fig. 126). If the rope breaks, the springs pull the drawbar down. The two arms press against the levers on the dog shafts and rotate the dogs so that they engage the guides. These arms are connected to the dog shaft by a slip connection. If the rope

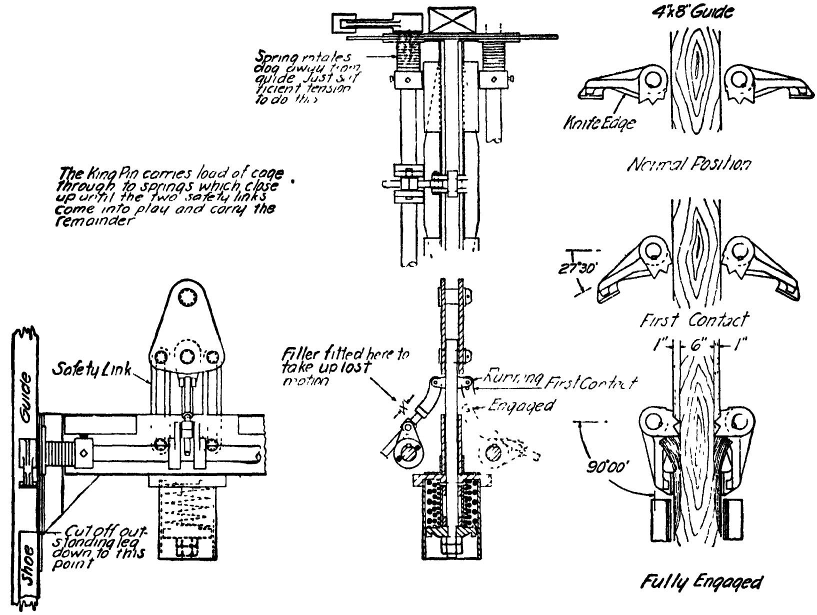

Spring rotates dog away from guide Just sufficient tension to do this
The King Pin carries load of cage through to springs which close up until the two safety links come into play and carry the remainder
Safety Link
Guide
Shoe
Cut off out-standing leg down to this point
Filler fitted here to take up lost motion
Running
First Contact
Engaged
4½x8" Guide
Knife Edge
Normal Position
27°30'
First Contact
1"
6"
1"
90°00'
Fully Engaged

FIG. 125. Hollinger Consolidated safety mechanism.

---

[1] By permission of J. W. Dougherty, general superintendent, Hollinger Consolidated Gold Mines, Ltd., Timmins, Ontario.

[2] PRITCHARD, W. H. (master mechanic), Origin, Type and Operating Mechanism of Cage Safety Dogs Used at the Hollinger Mine (1945).

catches in the shaft and raises the drawbar, the arms simply slide through these connections to the dog shafts. They can rotate the dog shaft in the engaging direction only. Each dog shaft has a coil torsion spring on each end. One end of the spring fits in a hole in the cage; the other fits in a collar on the shaft. There is just enough tension wound on the spring to be sure that the dogs will be free of the guides. Figure 127 shows the detail of the dogs. Figure 125 also shows the action of the dogs on the guides.

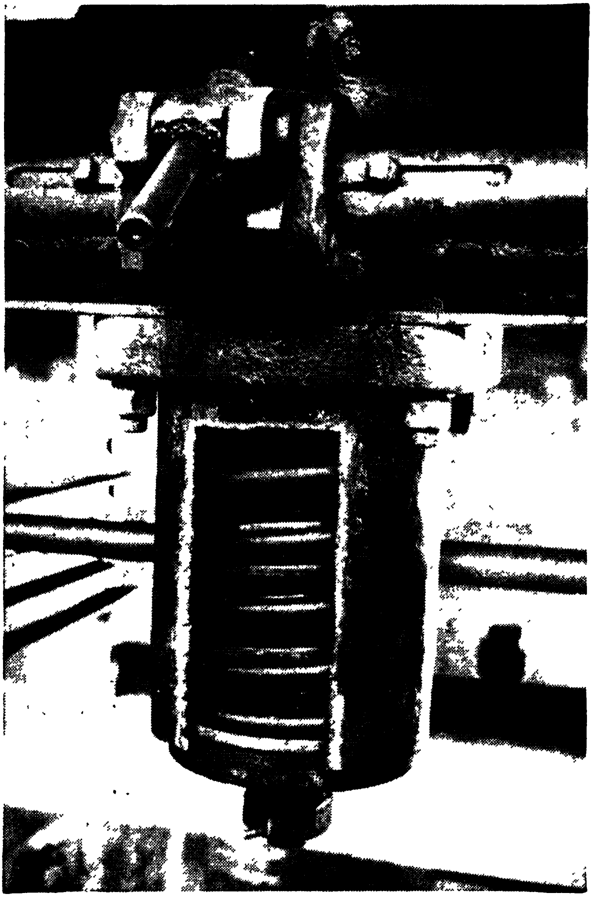


FIG. 126. Drawbar spring, Hollinger type.

The Hollinger mechanism has consistantly functioned under repeated testing. It is interesting to give the results of one such test.

Weight of cage	5393 lb
Load of sand	4400 lb
Total	9793 lb

The cage was dropped 9 ft before tripping the dog mechanism (see Fig. 128; chain is attached to the tripping device). It traveled an additional 1 ft 7 in. before the first tooth engaged the guide. This drop of 10 ft 7 in. was equivalent to a velocity of 1562 ft per min. The cage stopped in 6 ft 3 in. after the first tooth engagement.

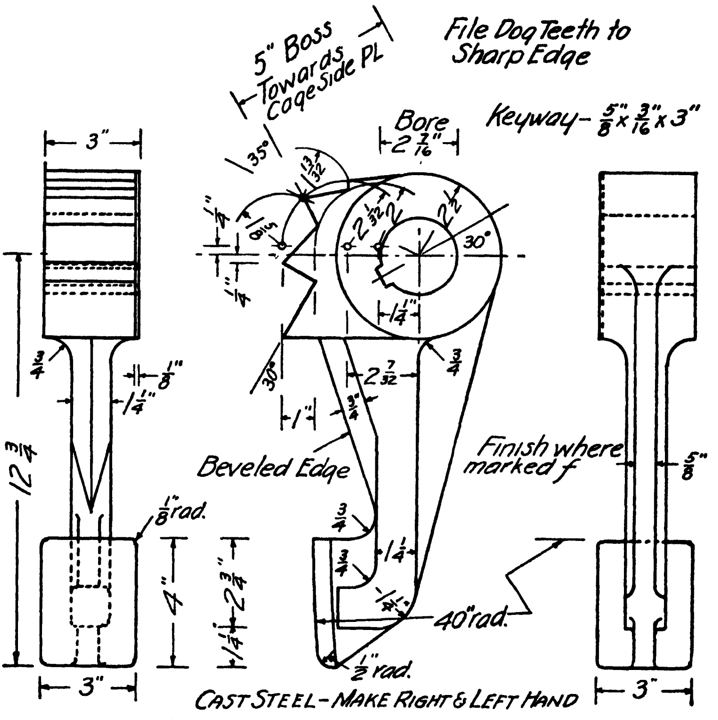


FIG. 127. Safety dog, Hollinger type.

$$\frac{s}{s_1} = \frac{10 \text{ ft } 7 \text{ in.}}{6 \text{ ft } 3 \text{ in.}} = 1.69g\text{'s}$$

Figure 129 shows the possible penetration of teeth into the guides. In Fig. 130 is shown the action of the dogs on the guides after a drop test.

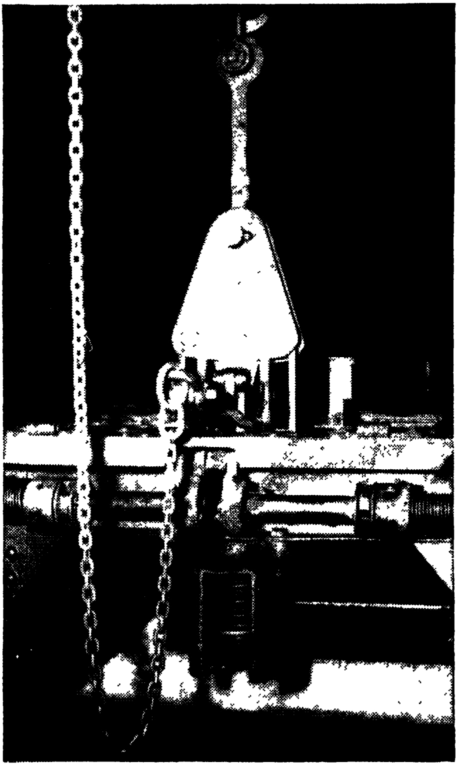


FIG. 128. Hollinger testing device.

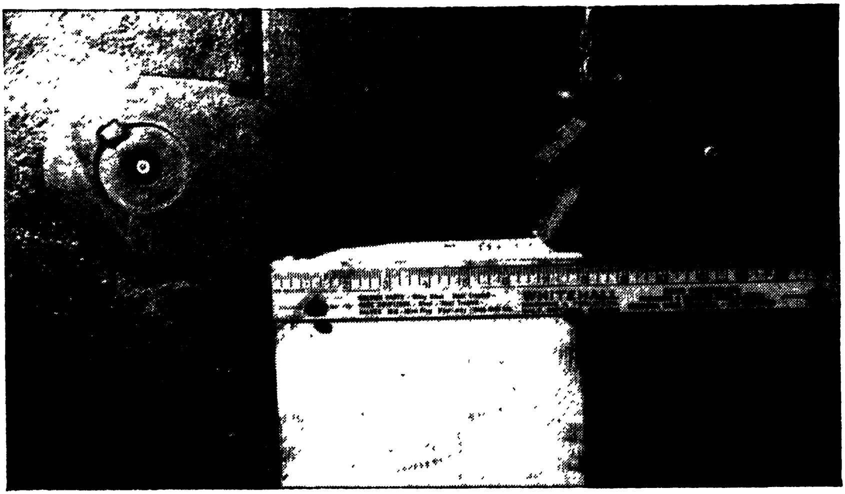


FIG. 129. Penetration of dogs, Hollinger type of dog.

In Fig. 131 is shown the John Inglis mine-cage safety device.[1] This cage is equipped with the King wedge brakes and the Sweet overspeed governor. Both attachments have been successfully tested and are being received with considerable favor. The figure shows, more or less diagrammatically, the functioning of the safety mechanism. When the cage is suspended on the rope, drawbar *A* pulls the wedge attached to *B* to its normal position, as shown. This compresses the springs *D* (one at each end) and brings *B* against *E*, which is part of the crosshead. If the rope breaks, *B* is forced downward by the springs *D*. The wedges *C*, which are carried on the ends of *B*, because of their downward motion cause the lower end of *F* to move out. This movement of *F* brings the spur wheels *G* into contact with the guides. There are four levers *F* and four wheels *G*. Contact of the teeth on *G* with the guides causes *G* to rotate, winding up the cable *J*. Upward motion of *J* raises the wedges *K*, causing them to wedge tightly against the guides. This slows the cage down and brings it to a stop. Upon raising the drawbar *A*, wedge *C* is raised and spring *L* forces the wedges down, thus releasing the cage. Spring *H* moves wheel *G* away from the guides.

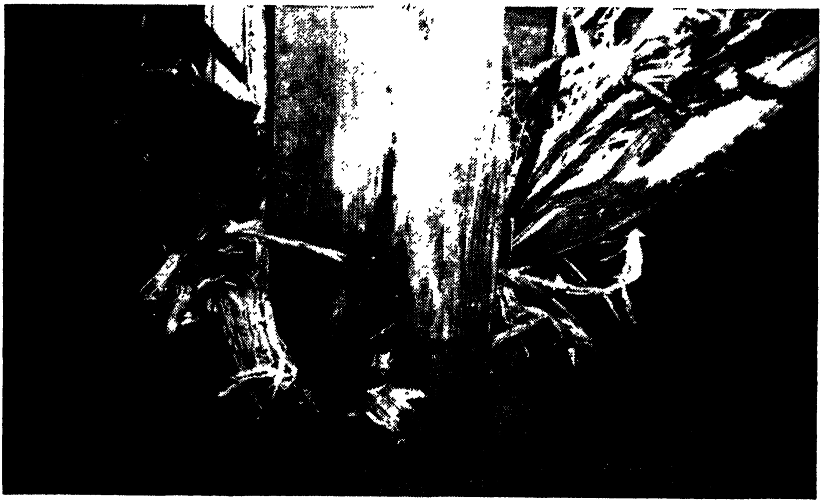


FIG. 130. Effect of dogs on guide, Hollinger safety dogs.

The governor wheel shown in Fig. 131*a* is a spur wheel running in contact with the face of one guide. It connects to a ball type of governor. The governor is set for some definite cage speed. When this is exceeded, the latch *M*, (Fig. 131*c*) is tripped. This releases the lock bars *N*. The inner ends drop under the effect of spring *O*. Springs *D* force wedge *C* down by pushing *N* out of its locking position. This results in the engagement of wedges *K*, as previously described.

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[1] By permission of John Inglis Company, Ltd., Toronto, Canada. This is a patented device.

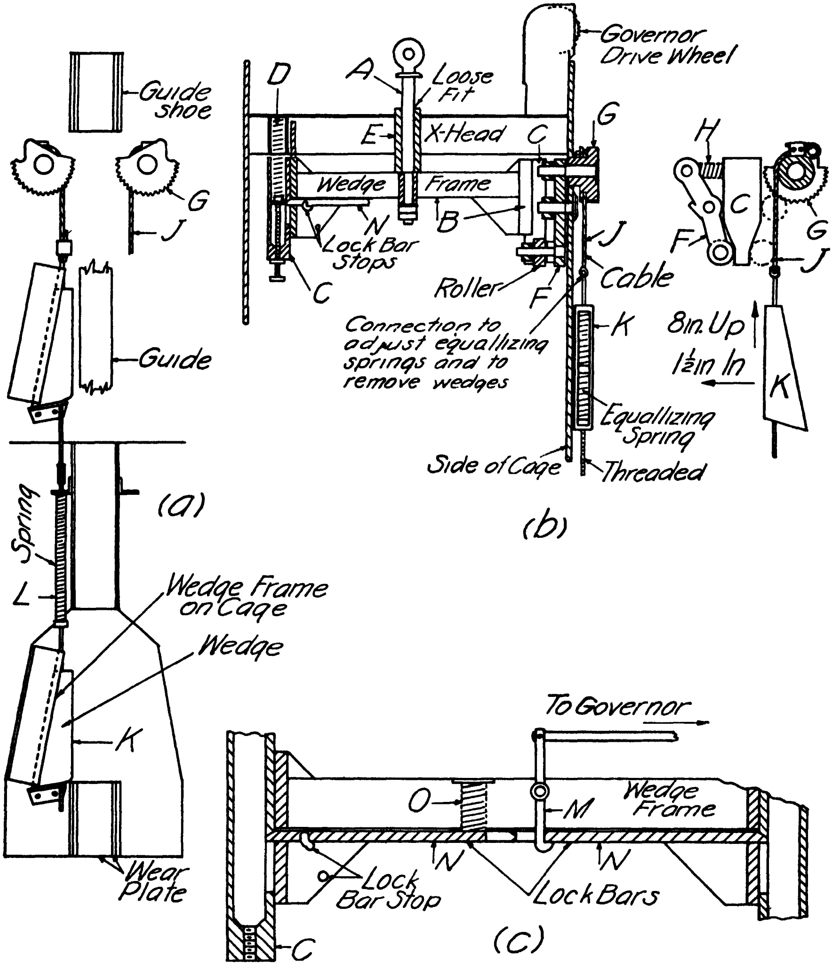

FIG. 131. John Inglis Company safety mechanism and overspeed governor.

Springs *D* are adjusted to 500 lb each. Equalizing springs are enclosed in the wedges *K*.

For light cages, only the upper wedges are required.

All moving parts are provided with positive lubricating connections.

Safety Mechanism for Inclined Shafts. Safety dogs on conveyances used in inclined shafts are not very common. It is difficult to devise an arrangement that will work on the steel rails. A patent[1] has been issued to J. T. Tackett for a device that works as follows:

1. An additional pair of rails are placed inside those on which the skip runs.
2. A pair of serated dogs are connected by linkage and compression coil springs to the drawbar.
3. If the cable breaks, the dogs are forced against the rails (see paragraph 1) by the coil springs.
4. To prevent the action in (3) from spreading the rails, a flange-plate extends from the bottom of the skip on the outside of the dog rails.

Tillson[2] shows a type of dog similar to those used on vertical cages. An extra wide wood sill is used for the steel rails. Just inside the rail one safety dog on each side of the skip is arranged to engage the wooden sill. A hook bends from the skip bottom under the sill just below the dogs. This is to prevent the dogs from lifting the skip. This arrangement must be carefully designed with regard to clearances. Wear has a tendency to bring the dogs above the rail base, where they will not function on the steel.

It is rather common practice to install wood guides like those used in vertical shafts; the conventional type of dogs and drawbar attachments are then used. The usual steel rails and wheels are used to support the cage or skip.

Other companies prefer to rely on careful inspection of the hoisting rope. Still others use an extra large factor of safety for the rope.

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[1] United States Pat. No. 2,297,589, Sept. 29, 1942, to J. T. Tackett, Oroville, California.

[2] TILLSON, *op. cit.*, Fig. XIV-39, p. 99.

CHAPTER VIII

MINE HOISTS

The mining engineer will in general be interested in the selection of a hoist for one of two reasons: the installation of new equipment, or the use of old hoists on new jobs. In either case it is desirable for him to determine the duty cycle and the size of the hoist and motor. This is probably more easily done when new equipment is installed, because of aid obtained from the manufacturer, than when old equipment must be adjusted to new operations. To ensure economical operations the drums should be of proper diameter and length and the motor capable of handling the load without undue heating.

Because of competition, hoist manufacturers may show a tendency to adhere too closely to the allowable minimum for drum size and motor capacity. The mine operator must be able to comprehend their reasons and to foresee, within reasonable limits, just what effect a later change in operations will have on the equipment. Such an understanding may best be had if the actual procedure for selecting the hoist is followed. It is possible to make the actual installation check the mine's requirements so closely that a small change in operating conditions (increased skip capacity, shorter operating cycle, excessive hoisting out of balance, etc.) would prove fatal to the equipment.

The duty cycles of two hoists are seldom the same. If an occasion arises in which it appears desirable to use a hoist motor at a shaft other than that for which it was selected, the effect of the new cycle on the motor must be known. A few seconds' change in the accelerating, retarding, constant speed, or rest periods will cause a radical change in the peak-horsepower requirements.[1]

Hoisting Systems. There are numerous methods whereby hoisting can be accomplished. We may use: (1) unbalanced, (2) counterweighted, (3) partly balanced, or (4) balanced methods. In general a hoist may be balanced either by a tail rope or by a particular drum shape. The completely balanced hoist is useless where continuous hoisting from more than one level is required. If operated from levels other than for which designed, it must do so out of balance. An occasional trip would not be objectionable. Continuous operation out of balance would prove dangerous to the motor and be very uneconomical.

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[1] PEELE, R. (ed.), "Mining Engineers' Handbook," 3d ed., Sec 16, p. 9, John Wiley & Sons, Inc., New York. (A diagram shows this very nicely.)

Of the various methods available, but three ordinarily prove desirable for metal mines. Two of these have a very limited use. The totally unbalanced system is used almost entirely for shaft-sinking operations or where a very small tonnage per day is required, and to a limited extent on service hoists. The counterweighted system finds its greatest application on service hoists (handling men, supplies, etc.). The method most commonly used is the partly balanced. In this system we provide power for hoisting the contents of the skip or car, part of the weight of the rope, and overcoming friction. It is commonly referred to as the balanced system.

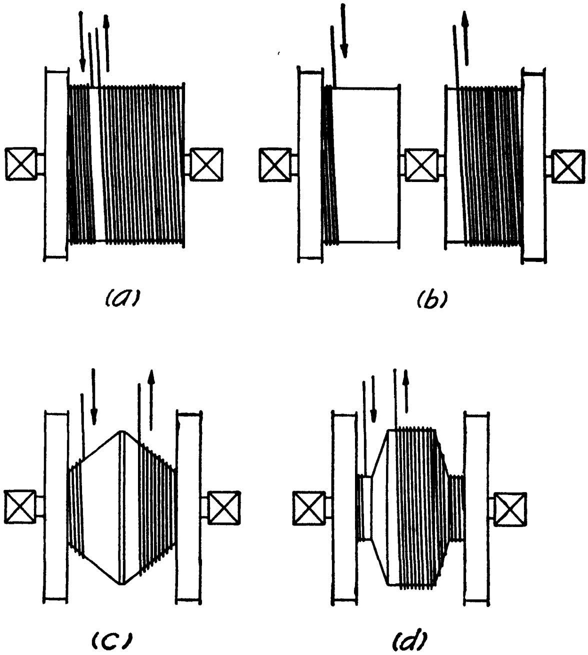

FIG. 132. Common types of hoist drums.

Drums. The usual types of hoist drums are shown in Fig. 132. The single cylindrical drum shown in *a* is used for partly balanced hoisting where but one level is to be served, or when a counterbalance is used. The double cylindrical hoist drum illustrated in *b* is the type ordinarily installed in metal mines. The two drums may be disconnected by means of a clutch or clutches. This allows adjustment to levels and permits partly balanced hoisting from all depths. Figure 132*c* is a conical drum. The two cones are sometimes separated and a bearing placed between them. It is adapted to only one level. This type of drum may be designed so that complete balance is obtained, thus requiring power for only the ore and friction. In *d* is shown a cylindroconical drum. It may or may not have two brakes as shown in the diagram. These drums may be designed for complete balance and may be clutched together.

Probably the chief advantage of the conical and cylindroconical drums is the reduction of the starting torque necessary. The two diameters are so chosen that the moment of the ascending load (which is of course the heavier) is practically balanced by the descending load. Their chief disadvantage is that their use is limited to hoisting in balance from but one level. They are usually satisfactory only for the conditions for which they are designed and cost much more than cylindrical drums.

The usual practice when one side of the hoist is counterweighted is to make the weight of the counterbalance equal to the skip or cage and car, plus one-half the contents. This permits the use of a smaller motor and also increases somewhat the efficiency of the installation. The duty on the motor is practically the same when raising the counterbalance as when raising the load. Advantages of counterbalancing are illustrated later.

When hoisting must be conducted from more than one level, the use of the double drum is necessary if the greatest efficiency is to be obtained. The maximum economy does not necessarily result from the highest efficiency; power consumption and equipment costs must be considered. A tail rope for balancing purposes cannot be used here. Nevertheless the size of the power plant required to drive the hoist is very materially reduced from that required for a single drum or a counterbalanced hoist, the tonnage requirements in each case being the same. As will be illustrated later, the high starting peak for this type of installation can be effectively prevented from showing on the power mains by means of a flywheel motor-generator set; or this peak may be "eased" onto the line by using a synchronous motor-generator set. The initial cost of such equipment is high; therefore this must be considered when selecting a motor-generator set.

Clutches. The clutched drums on large hoists are usually driven by one of two methods: positive multiple-tooth clutch or some form of double-disk multiple-arm friction clutch. The multiple-tooth clutch is commonly used on hoists operating from one level or at the most from but a very few levels. The members are usually made of steel. The driven member is fastened to the drum spider, while the driving member slides on the shaft and revolves with it.

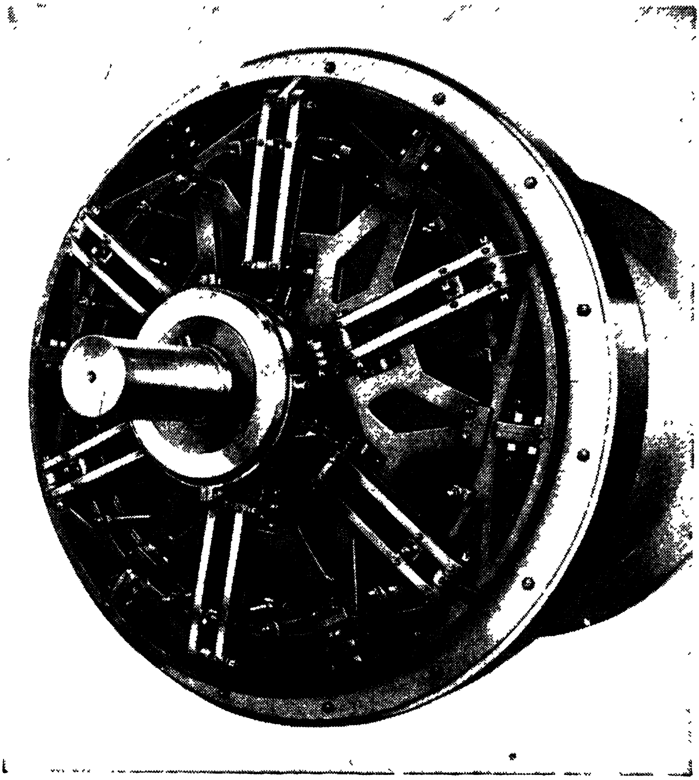

FIG. 133. Double-disk multiple-arm friction clutch. (*Allis-Chalmers Manufacturing Company.*)

One type of the double-disk multiple-arm friction clutch is illustrated by Fig. 133. It consists of the following principal parts: a driver, keyed to the drum shaft; two ribbed disks to which the friction blocks are fastened and provided with special keys to prevent shear of the blocks; toggle levers for actuating the disks; an engaging clutch ring for fastening to the brake wheel or drum; and a sliding sleeve for operating the clutch. The driver is made with arms designed for driving direct at their outer ends instead of through pins. Such a design ensures a uniform distribution of pressure, thus reducing wear

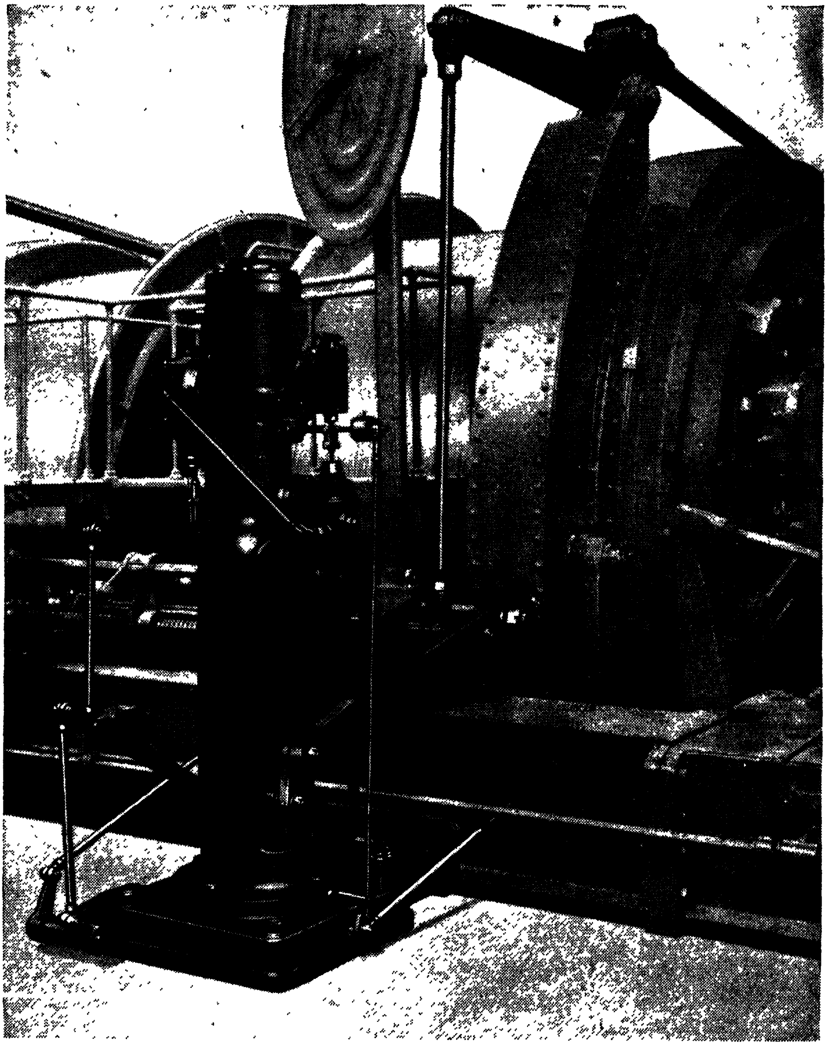

FIG. 134. Oil-operated brake engine. (*Allis-Chalmers Manufacturing Company.*)

and danger of cutting. The ribbed disks are actuated through the toggle levers, the motion being imparted by means of the sleeve that slides on the driven hub. The rings release equally on both sides and do not drag at any point when the clutch is disengaged. As shown in the figure, all moving parts are lubricated by means of alemite attachments. The clutches on large hoists are usually operated by oil-clutch engines.

Brakes. For all large hoists, and especially for those handling men, the parallel-motion-post brake is recommended. A brake post of built-up structural steel is to be preferred. The brake should be so designed that equal pressure and release of the brake blocks at all points are obtained when the brakes are operated.

On brakes 8 ft or less in diameter, hand-operated devices (levers or wheels) are satisfactory. For large brakes, a brake engine actuated by oil or air under pressure is desirable. The oil-operated engine receives the preference. Such an attachment is shown in Fig. 134.

POWER SYSTEMS

There are four methods whereby hoists may be driven.

1. Electric power
2. Steam
3. Compressed air
4. Internal-combustion engines

Of these, we shall concern ourselves with only the first. Steam and compressed air, while used in certain special cases, are largely being replaced by electricity. The electric hoist motor, especially the d-c type, provides much better control, and in most instances is cheaper and more efficient. Internal-combustion engines are used almost entirely for prospects or small-scale development work, or where small intermittent tonnages are required.

Electric Motors. The choice will lie between the various a-c motors and a d-c motor. There are a number of points to be considered when making this selection. The nature of the power available (alternating or direct current) is sometimes, but by no means always, the deciding factor. Where accurate and sensitive control is desired, a d-c motor in conjunction with some type of motor-generator set is usually preferred.[1]

If the installation is influenced by the cost of the motor, the a-c type is usually used, as it has a lower first cost. For duty cycles requiring motors of about 1000 rated hp or more the d-c motor is generally used. Such requirements usually mean high-speed hoisting (rope speed in excess of 1800 fpm), which in turn calls for rapid deceleration. Dynamic braking offers the most economical and safe means of retarding. It is best accomplished by the d-c motor. Dynamic braking is obtained by moving the control lever toward the neutral position. When this is done, the motor becomes a generator and is driven by the energy stored in the moving parts. The power is fed back to the motor-generator set. By controlling the field current, the motor is brought almost to rest. The final operation is by means of the mechanical brakes. As a d-c motor does not have a synchronous speed (as do the a-c motors), it will act as a generator at any speed. The a-c motors will regenerate only when operated above synchronous speed. This practically eliminates ordinary methods of dynamic braking so far as they are concerned. A system[1] has been devised for obtaining dynamic braking with a-c motors. Direct-current excitation of constant value is supplied the stator winding and the speed controlled by a secondary rheostat. Such control is economical but very coarse. It would be justified where only occasional lowering of heavy loads out of balance was necessary. The initial investment is larger than for the straight-induction motor. A larger motor is required because of the greater excitation with the direct current; also d-c generating equipment must be provided.

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[1] SAGE, R. S., Electric Drive for Mine Hoists, GET-198, General Electric Company, 1930.

BRIGHT, G.: Determination of Electrical Equipment for a Mine Hoist, *Trans. AIMME*, **66** (1921).

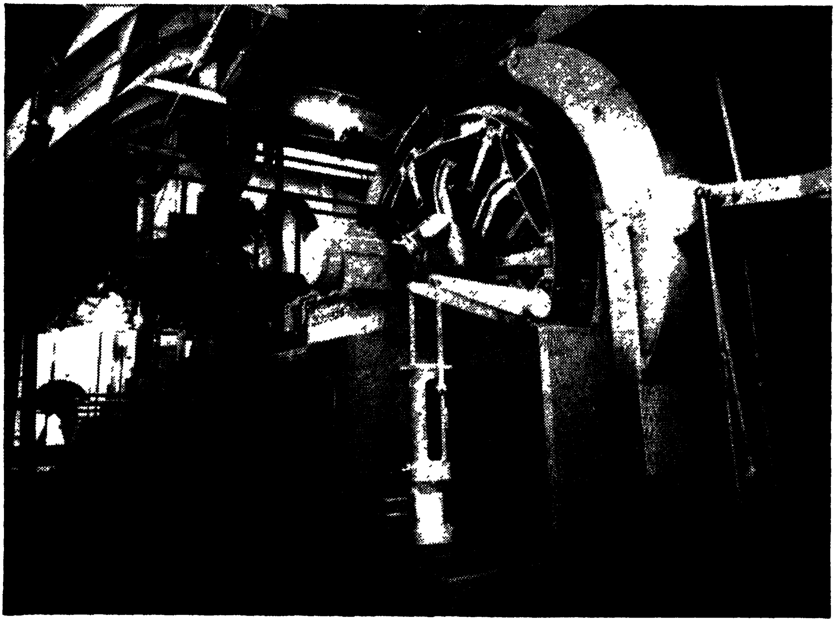

FIG. 135. Clutch and brake for new hoist, Bunker Hill and Sullivan Mining and Concentrating Company.

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[1] SAGE, *op. cit.*, p. 30.

In exceptional cases, the motor must be direct-connected to the hoist. In such instances, because of the slow speed, d-c motors are used. A driving arrangement of this kind might be necessary where the power available was limited. The inertia due to gearing would be eliminated, thus reducing the starting torque. Bright, in the reference quoted (pages 592 and 596), gives an example which illustrates this point. In the case of an a-c motor geared to the drum, a 2350-hp motor, having a no-load torque of 3690 hp, was necessary. Under the same conditions a d-c direct-connected 1500-hp motor with a no-load torque of 2500 hp was satisfactory.

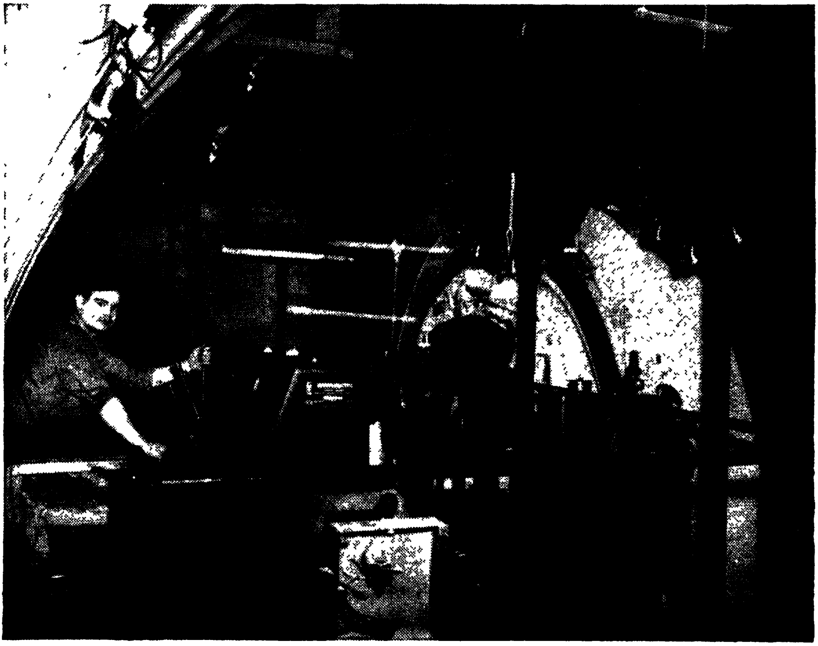

FIG. 136. Hoist controls, Bunker Hill and Sullivan Mining and Concentrating Company.

In some instances the maximum power demand cannot be permitted to reach the line. This may be because of interference with other operations or because of exorbitant charges for excessive peaks. Under such conditions d-c motors driven from flywheel motor-generator sets are used. The foregoing discussion, and in addition that to follow, will form a groundwork on which to base the choice between a-c and d-c equipment.

Hoist motors are ordinarily designed for a maximum temperature rise of 40°C for altitudes less than 3000 ft above sea level

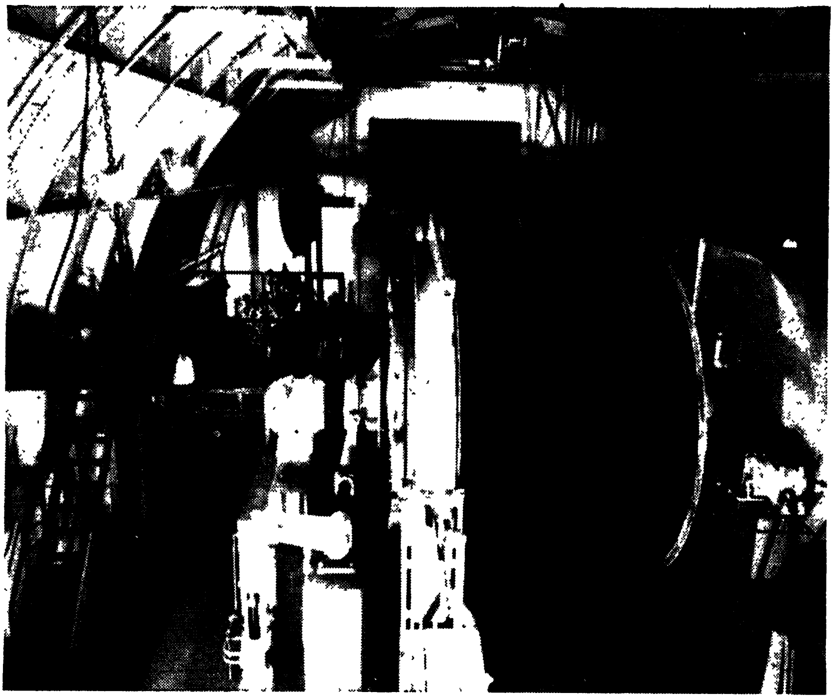

FIG. 137. Large underground hoist, Bunker Hill and Sullivan Mining and Concentrating Company.

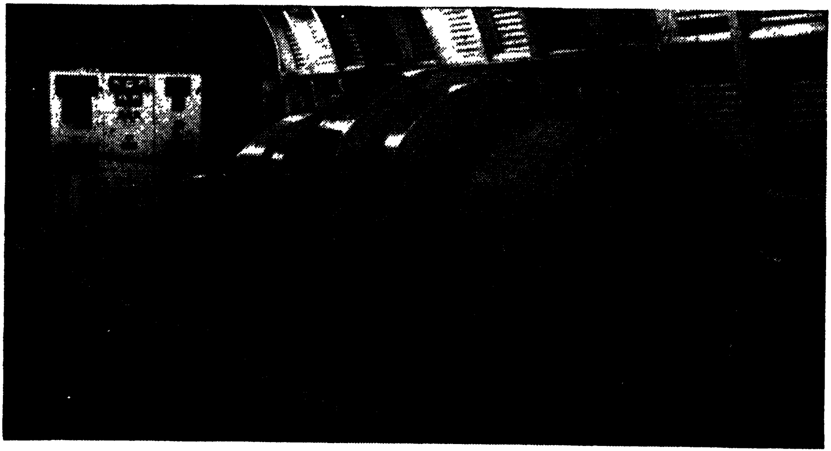

FIG. 138. Motor-generator set, amplidyne control, Bunker Hill and Sullivan Mining and Concentrating Company.

Figures 135 to 138 show some of the features of the new underground hoist at the Bunker Hill and Sullivan mine, Kellogg, Idaho. In Fig. 135, attention is called to the construction of the underground station. Essential data for this installation are as follows:

1. Two d-c 250-volt motors, each 600 hp.
2. Diameter of rope = 1⅝ in.
3. Load
   - *a*. Ore = 20,000 lb
   - *b*. Skip = 11,000 lb
   - *c*. Maximum rope pull = 48,000 lb
4. Inclination of shaft = 50 deg from the horizontal
5. Depth of shaft
   - *a*. Present = 3373 ft on the incline
   - *b*. Ultimate = 6000 ft on the incline
6. Tonnage per 24 hours = 125 tons per hr or 2000 tons per 16 hr of hoisting time from the present lowest level (29th)
7. Rope speed
   - *a*. With men = 1050 ft per min
   - *b*. With ore = 1500 ft per min
8. Control, amplidyne
9. Drums are 12 ft in diameter by 8 ft for each drum; 5,000,000 WR^2 for revolving parts
10. Three layers of rope wound on drum
11. The hoist is 9000 ft from the portal of the Kellogg tunnel

CONTROL OF ELECTRICALLY DRIVEN HOIST MOTORS[1]

The following methods of control are in general use. Their selection depends more or less on the same reasons given in the preceding discussion.

1. Induction motor with the following:
   - *a*. Master controller, contactors, and grid secondary resistance
   - *b*. Primary contactors and liquid secondary resistance
   - *c*. Drum controllers and grid secondary resistance
2. Ilgner-Ward Leonard system:
   Direct-current separately excited motor operated from motor-generator set with flywheel by Ward Leonard control
3. Ward Leonard control:
   Direct-current separately excited motor operated from motor-generator set without flywheel by Ward Leonard control
4. Direct-current motors:
   Rheostatic control, using either drum controller or magnetic control
5. Amplidyne control

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[1] SAGE, *op. cit.*, p. 25.
FOX, G., Electric Operation of Hoists, *Eng. Mining J.*, **126**, 91 (1928).
BRIGHT, *op. cit.*

Alternating-current Motors. For hoist motors up to 1800 hp, wound-rotor induction motors are widely used.[1] They have a low first cost and no idling losses. They do not respond to a very accurate or complete control. Because of their high speed, they usually have a rather high gear ratio. Under these conditions the inertia of the rotating parts becomes very important, with the consequence of a high starting peak. When the rope speed exceeds 1800 fpm and the cycle approaches 3 min, the advantages of using a-c motors becomes doubtful.[2] The inertia effect is high; power losses are great; peak loads on the system are excessive; and dynamic braking is very uncertain.

Several methods of using the motor as a brake, in addition to those previously given, may be used.

1. *Regeneration.*[3] The motor must be driven above synchronous speed. This can usually be done only in the case of deep shafts. The load is stopped by means of the brakes and by reversing (plugging) the motor.
2. *Reversing.* It may be accomplished by reversing (plugging) the motor and using high secondary resistance. This method is adapted to shallow shafts. It is wasteful and inefficient.

The usual operating voltage for induction motors is 440 or 2200 volts. The medium- and large-sized motors operate on 2200 volts.

Although the a-c motor is not inherently well adapted to hoisting service, it is widely used. The reason is the low first cost, wide distribution, and cheapness of a-c power, and the simplicity and reasonableness of the installation.

Direct-current Motors. The motor commonly used for this type of drive is a shunt-wound low-speed direct-connected machine usually operating on 550 volts. The power supply may come from one of two sources: Ward Leonard control or Ilgner-Ward Leonard control. When available, a d-c power system may be capable of carrying the load. In the case of the Ward Leonard system, the motor driving the motor-generator set is usually a synchronous motor, but it may be an induction motor. In the Ilgner type, a flywheel is mounted on the motor-generator shaft. The motor is an induction motor. Figure 139 shows the diagrammatic hookup for the Ilgner system.[4] The excitation of the hoisting motor is kept constant by means of the exciter E, while the speed of the motor is controlled by the resistance r in the generator-field circuit, by means of which the voltage E_t applied to the motor terminals may be varied. As the load on the hoist motor increases, the current to the induction motor tends to increase in the same ratio; but a slight increase in the current I causes the plunger p of the solenoid S to raise the plates of the water rheostat, thereby increasing the resistance in the rotor circuit, so that the rotor current and torque are maintained constant. This torque is not sufficient for the load; therefore the speed drops and causes the flywheel to give up energy.

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[1] FOX, G., *loc. cit.*

[2] BRIGHT, G., *op. cit.*, p. 589.

[3] BROUGHTON, H. H., "Electric Winders," p. 73, Ernest Benn, Ltd., London.

[4] GRAY, and WALLACE, "Principles and Practice of Electrical Engineering," 3d ed., p. 347, McGraw-Hill Book Company, Inc., New York.

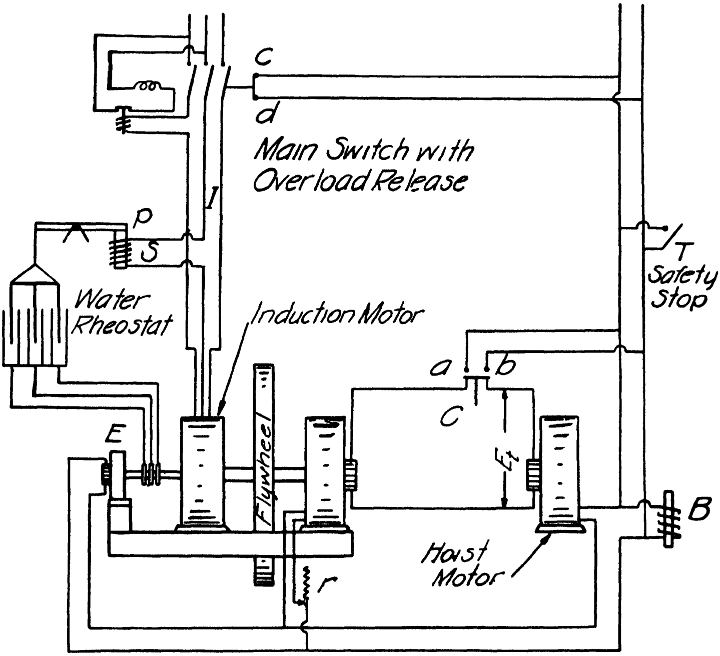

FIG. 139. Diagram of Ilgner-Ward Leonard system for mine hoists. (*Courtesy of Gray and Wallace, "Principles and Practice of Electrical Engineering."*)

If the load on the hoist motor now decreases, the current I tends to decrease, but a slight decrease in this current causes the pull of solenoid S to decrease and the plates of the water rheostat to drop, thereby decreasing the resistance in the rotor circuit, so the rotor current and the rotor torque are maintained. This torque is greater than necessary for the load and so accelerates the flywheel.

The safety brake is controlled by the solenoid *B* and will set and hold the load as soon as current ceases to flow in *B*.

If the cage overtravels, it closes the switch *T* in the hatchway, and this shunts the current from the solenoid *B* and allows the brake to set. The resulting overload then opens the circuit breaker *C*.

The solenoid *B* is also shunted when, owing to an excessive overload, the circuit breaker *C* opens and closes the contacts *ab*. A similar pair of contacts, *cd*, ensure that the brake shall be set when the main switch is open.

The d-c motor with either Ward Leonard or Ilgner control is the ideal arrangement. It is very easily and effectively controlled. The system responds readily to dynamic braking without loss of the regenerated power, since it is returned to the line. The big objections to such installations are the first cost and the large loss of power through the idling of the generator set during rest periods. For high-speed hoisting of heavy loads from deep shafts, some form of the Ward Leonard or Ilgner system is almost universally used. A slip regulator is used to control the speed of the motor for the motor-generator set. If the hoist motor attempts to draw too much power from the set, the induction-motor load is partly removed from the line and the flywheel gives up energy. In giving up its energy, the flywheel slows down. This prevents a high peak on the power-supply line (see Fig. 149).

Amplidyne Control.[1] The amplidyne control is a rotating control equipment applied to the Ward Leonard or Ilgner-Ward Leonard type of control. There are a considerable number of mine hoist motors under amplidyne control in use; for example, the 900-hp d-c hoist motor at the Athens Mine of the Cleveland Cliffs Iron Co., Ishpeming, Michigan, and at the Bunker Hill and Sullivan Mines, Kellogg, Idaho; a 1000-hp motor at a New York iron mine; and many others.

The advantages of using the amplidyne as the generator field exciter are as follows:

1. Positive limitation of motor current during acceleration and retardation
2. Closer regulation of generator voltage on running points of the controller
3. Elimination of all generator field contactors and relays used in the conventional control system
4. Availability of at least twice the usual number of control points, thereby approaching the control afforded by the steam valve on old steam-engine hoists

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[1] Personal communication from R. S. Sage, engineer, Mining Division, General Electric Company, Schenectady, N.Y.

5. Elimination of much wiring by purchaser due to simplification of connections and to use of a factory-connected controller

6. Saving in space for control panels and resistors

Cost of Driving Systems.[1] The following information presents a general idea of the cost of hoist motors with various controls. The prices are for 1928. They probably provide an accurate relative value for the present day.

Wound-rotor Induction Motors for Geared Drive. For 200-hp, 600-rpm, 2200-volt, three-phase, 60-cycle motor with magnetic control and switchboard, $22.50 per horsepower. Same as above, but with liquid-rheostat secondary control, $31.50 per horsepower.

For 600-hp, 450-rpm 2200-volt three-phase 60-cycle induction motor, with magnetic control, $15 per horsepower; with liquid rheostat, $17.50 per horsepower.

For 1300-hp, 360-rpm 2200-volt three-phase 60-cycle induction motor, with magnetic control or liquid rheostat, $14.50 per horsepower.

Direct-current Motor Geared to Hoist Drum, with Motor-generator Set and Ward Leonard Control. For 800-hp 350-rpm 550-volt d-c motor, 700-kw motor generator with 500-kva synchronous motor, $35 per horsepower. For 800-hp 350-rpm 550-volt d-c motor, 700-kw motor generator, with 600-hp induction motor and 18,000-lb flywheel, Ilgner control, $48 per horsepower.

Direct-current Motor, Direct-connected to Hoist Drum, with Motor-generator Set and Ward Leonard Control. For 950-hp, 93-rpm 550-volt d-c motor, 700-kw motor generator, with 500-kva synchronous motor, $43 50 per horsepower.

For 950-hp, 93-rpm, 550-volt d-c motor, 700-kw motor generator, with 600-hp induction motor and 18,000-lb flywheel, $53 50 per horsepower.

Comparison of Systems. The following outline shows the advantages and disadvantages of the alternating (three-phase) and the direct current.[2]

Connection of Motor to Hoist. Hoist drives are usually first or second motion. In the first-motion type the motor (almost always direct current) is direct-connected to the drum shaft. This is either done by pressing the motor armature onto the shaft or by means of a coupling. The electrical characteristics of the induction motor make them unsuitable for direct connection. Slow-speed induction motors are also more expensive than high-speed motors.

Second-motion hoists are those having but one gear reduction. The armature shaft is connected to the pinion (usually by means of a flexible coupling). This gear and its shaft are part of the hoist; their inertia must be considered in power calculations. Practically all hoists are driven by herringbone type of gears. By so doing very high

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[1] Fox, *op. cit.*, p. 94.

[2] *Proceedings* Third Empire Mining and Metallurgical Congress, Part II, Rand Mining, Johannesburg. South Africa. 1930. p. 167.

WARD LEONARD SYSTEM VS. THREE-PHASE SYSTEM

Feature	Three-phase system	Ward Leonard system
First cost	Electrical 100; electrical and mechanical 100	Electrical 230, electrical and mechanical 140
Operating cost	Higher except for intermittent raising with long stops or long balance winding hauls	Lower, especially for lowering duty and when load factor is high
Lowering and/or braking	Reverse power control consumes power	Regenerative control pumps current back into line
Operation	Harder to control and not easy for automatic aids near end of wind	Easy control and can be made semiautomatic near end of wind by usual cam-retarding gear
Standstill losses	Nil	About 4 per cent of winder rating
Size	Limited by controller design	Practically unlimited
Starting	Peak may affect small power-supply systems, large rheostatic control loss	Easier on power system, rheostatic losses small
Overwind	Can occur at high speed, necessitating severe braking with consequent damaging effect	Can occur only at reduced speed, braking can be normal
Housing	Very suitable underground where chamber dimensions must be kept small	Large; it may require excessive chamber dimensions or separate chambers for M-G set and winder
Heating	Large, owing to greater rheostatic losses and may be a serious factor in underground installations with heavy lowering duty	Less than for three-phase, may be a deciding factor for an underground winder
Power failure	Brakes winder and may hang load up in shaft	Energy of M-G set can bring load to collar
Repairs	Small	Probably larger owing to more machines
Breakdown risk	Small	Greater due to more machines
Overspeeding	Can be done with drivers lever in neutral when lowering load, thus overstressing mechanical and electrical parts	Speed cannot become excessive or out of control
Maintenance	From a machinery point of view should be smaller than for Ward Leonard; controller costs usually overbalance machinery spare savings	Small

tooth speeds and high reduction ratios are permitted. The use of herringbone gears practically eliminates the necessity of more than one reduction. With this type of gear, reductions as high as 15:1 may be used.

Low-speed motors, either a-c or d-c, are more costly and less efficient than high-speed motors. On the other hand, as previously cited, the lack of gearing reduces considerably the inertia of the moving parts. An attempt should be made to strike the best balance between first cost and operating costs. This then will dictate the motor speed. If possible more than one gear reduction should be avoided.

Table 39 gives information on hoist motors at various properties.[1]

Inertia of Moving Parts. The WR^2 or effective weight of the drums, gears, sheaves, and motor rotor or armature at a given radius must be known to determine the horsepower necessary to accelerate these members of the hoisting system. There is a scarcity of available data on hoists in operation. Fairly close information can be utilized by selecting a drum whose dimensions are similar to that required and

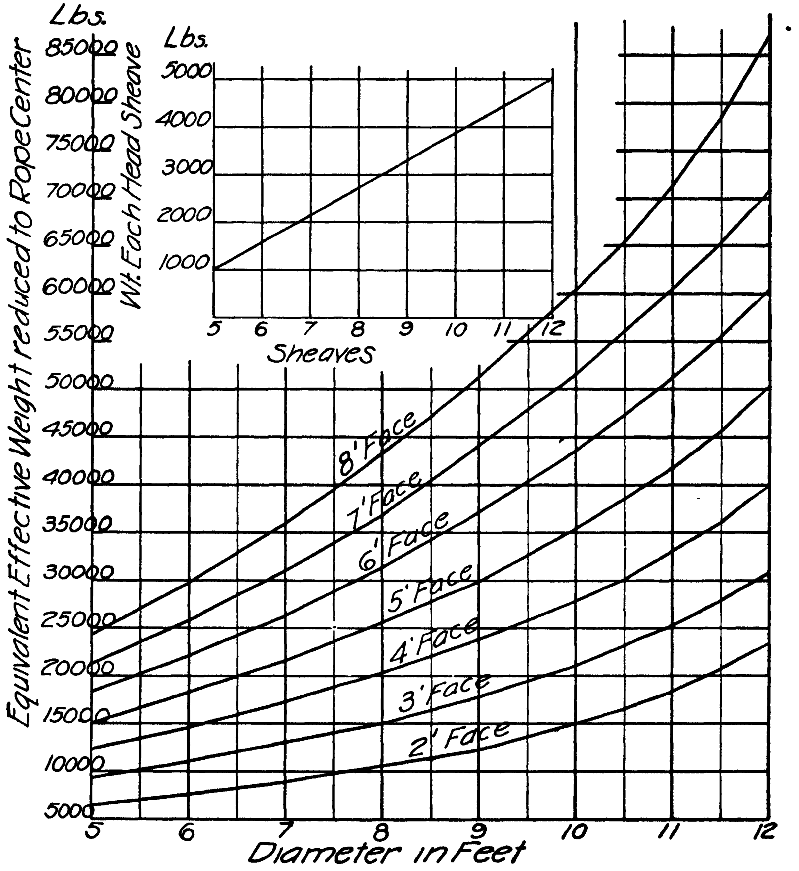

FIG. 140. Effective weights of single cylindrical drums and head sheaves (for estimating purposes only). For gears add 10 per cent. For double drums add 100 per cent. (*Courtesy of Sage, "Electric Drive for Mine Hoists."*)

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[1] Fundamental Hoisting Problems, *Eng. Mining J.*, **122**, 529 (1926).

obtaining the desired data from the manufacturers. In lieu of this a close approximation may be had from one of the following methods:

Figure 140 shows a series of curves in which diameter of cylindrical drum is plotted against equivalent effective weight at the rope center.[1]

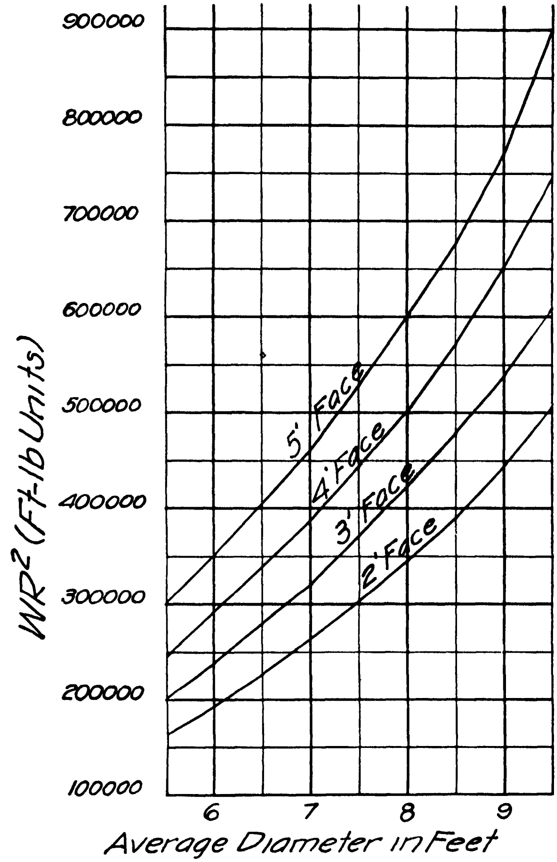

FIG. 141. Effective weights of single conical drums (for estimating purposes only). Add 10 per cent for gears. For double drums add 100 per cent. (*Courtesy of Sage, "Electric Drive for Mine Hoists."*)

These curves are for various widths of drum faces.[2] Included in the figure are data for various sizes of sheaves. Figure 141 gives similar information for conical drums.[1]

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[1] SAGE, *op. cit.*, p. 14.

[2] The results derived from Figs. 140 and 141 do not apparently include an allowance for brakes, clutches, and other moving parts. As the size of these parts depends on varying conditions (rope speed, rope pull, etc.), it is difficult to give information for determining their weight.

Table 39. Motor Specifications of Electric Hoists

Motor			Supply		Duty			System	Mine
R.m.s.	Max. b.hp.	R.p.m.	Volts	Period	Depth of shaft	Speed, ft./min.	Net load, lb.		
1500	3700	33	3000	25	4500	2200	16000	Ward Leonard	East Rand Proprietary Mines, S. Africa.
1850	. . .	71	550	60	4000	3000	14000	Ilgner	North Butte Mining Co. Montana.
1530	3000	96	2100	50	3800	3500	10000	Ward Leonard	Brakpan Mines, Ltd., S. Africa.
875	2000	292	2000	50	4500	2000	14000	Geared a.c.	Robinson Deep, Ltd., S. Africa.
600	1000	60	2100	50	3000	2000	6000	Ward Leonard	West Rand Consolidated Mines, Ltd., S. Africa.
670	1000	360	2100	50	3000	2000	8000	Geared a.c.	Witwatersrand Deep, Ltd., S. Africa.
550	875	365	3300	50	1450	2460	2912	Geared a.c.	Bengal Coal Co., India.
350	600	360	2000	50	2200	1600	3600	Ward Leonard	Randfontein Central C.M. Co., S. Africa.
400	581	277	2200	50	494	1020	8800	Geared a.c.	Tharsis Sulphur and Copper Co., Spain.
275	425	230	550	50	1500	1440	6000	Ward Leonard	Cam and Motor Mines, S. Africa.
2400	3850	350	500	D.c.	3000	3150	16800	Stubbs Perry	Harworth Main Colliery, England.
2200	. . . .	575	2200	60	607	4000	22000	Ilgner	New Orient Mine, Illinois.
700	. . . .	695	. . . .	. .	1700	1000	1600	Ilgner	United Verde, Arizona.

Another method, and one which checks closely the above curves, is as follows[1]: "Inertia of a cylindrical drum, with head sheaves and gears, is obtained by allowing 200 lb per sq ft of drum surface with a radius of gyration 3 in. inside the drum surface."

Either of the above methods may be used. They give results sufficiently close for estimating purposes.

A third source is Table 40 in which one of the leading hoist manufacturers has tabulated the data required according to the drum size.[2]

TABLE 40. WR^2 OF HOIST DRUMS

Drums, steel	Brake diameter	Weight, lb.	WR^2, ft.-lb.^{2}
48 × 30 in.	68 in.	7,800	30,200
60 × 48 in.	80 in.	15,700	170,400
6 × 5 ft.	80 in.	19,000	185,800
7 × 5½ ft.	8 ft.	24,500	264,700
8 × 6 ft.	10 ft.	50,250	760,000
9 × 8½ ft.	10 ft.	51,000	850,500
10 × 6 ft.	12 ft.	64,500	1,574,000
10 × 10 ft.	11 ft.	63,000	1,422,000
12 × 6 ft.	14 ft.	82,500	2,326,000
12 × 7 ft.	15 ft.	101,500	2,650,000
12 × 10 ft.	15 ft.	109,600	2,915,000

TABLE 41. WR^2 OF GEARS

Drum size	Weight of gear, lb.	WR^2, ft.-lb.^{2}
48 × 30 in.	2,800	11,200
60 × 48 in.	4,200	67,100
6 × 5 ft.	4,000	77,400
7 × 5½ ft.	4,200	54,400
8 × 6 ft.	16,000	368,000
9 × 8½ ft.	11,000	213,000
10 × 6 ft.	21,000	660,000
10 × 10 ft.	14,000	470,000
12 × 6 ft.	20,500	576,000
12 × 7 ft.	27,500	450,000
12 × 10 ft.	27,500	450,000

The data in Table 40 are for single drums. The values given include the weight of one drum shell, one brake wheel, two spiders, and one main gear. For a double-drum hoist it is necessary that the above weight, or WR^2, be doubled and that the weight, or WR^2, of one gear be deducted. Table 41 presents the necessary data for the gears that go with the above drums.[1]

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[1] BRIGHT, *op. cit.*, p. 583.

[2] Reproduced by permission of Allis-Chalmers Manufacturing Company

The WR^2 of the gears depends on the gear-reduction ratio.

Inertia of the Rotor or Armature. If the motor is geared to the hoist, the inertia of the armature must be multiplied by the square of the gear ratio in order that it be referred to the drum radius. This presumes, of course, that data are available on either the weight or WR^2 of motor armatures. Such information, even of a speculative nature, is hard to obtain, for the reason that very few if any of the large hoist motors are sufficiently alike. If the weight can be approximated, the inertia can be calculated by assuming the radius of gyration to be 75 to 80 per cent of the radius of the rotor, depending on the details of the construction.[2] This procedure often means that the calculations must be repeated one or more times.

If information on the weight or WR^2 is not available, the following approximation may be used for estimating purposes.[3]

Maximum horsepower required to accelerate motor rotor in 1 sec:

Induction motors and geared d-c motors = 150 to 180 *per cent of normal rating.*

Direct-connected d-c motors = 80 to 125 *per cent of normal rating.*

The lower values apply to the lower speed motors and the higher values to the higher speed motors. Cases may arise in which the values vary considerably from those given above.

When the WR^2 of the armature is known, the horsepower to accelerate is given[3] by

$$\text{Maximum hp} = \frac{WR^2 \times \text{rpm}^2 \times 0.62}{1{,}000{,}000 \times t}$$

where t = time of acceleration, sec

Friction of Hoist Parts. There is a great deal of controversy on this subject. Some engineers assume an efficiency for the hoist (friction in the hoist parts; friction between skip or cage shoes and guides; bending of rope; and windage in the shaft) at 75 to 85 per cent.[4]

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[1] Reproduced by permission of Allis-Chalmers Manufacturing Company.

[2] BRIGHT, *op. cit.*, p. 584.

[3] SAGE, *op. cit.*, p. 16.

[4] STONE, F. L., Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating, *Proc. AIEE*, October, 1918.

STONE, F. L., Calculation of Horsepower for Hoists in Shallow Shafts; *Coal Age*, Oct. 12 and Oct. 19, 1922.

BRIGHT, *op. cit.*, p. 585.

Bright[1] states that Westinghouse engineers assume that the total friction, expressed in pounds, of a hoist is based on a rope pull of 7.5 per cent of the total suspended weight on the drums for single-reduction gearing; and 5.5 per cent for direct-connected hoists. This value, when multiplied by the drum radius, gives the friction moment. It is assumed to represent the friction throughout the entire cycle. A considerable variation in the frictional resistance will have but a small effect on the capacity of the motor.

Sage[2] gives the following formulas for determining the efficiency of the hoist, and thus the frictional losses.

PERCENTAGE EFFICIENCY OF HOIST MECHANISM

$$\begin{array}{cccc} & \text{1st motion} & \text{2d motion} & \text{3d motion} \\ E = & \dfrac{w}{w + kW} & \dfrac{0.95w}{w + kW} & \dfrac{0.90w}{w + kW} \end{array}$$

where $W = (w + 2w_s + w_r) \sin \theta$ (for balanced vertical and inclined hoists)

$= (w + w_s + 0.5w_r) \sin \theta$ (for unbalanced vertical and inclined hoists)[3]

$= (w + w_s + w_r) \sin \theta + 0.02 \cos \theta + 0.10w_r \cos \theta$ (for balanced slope hoist)

$= (w + w_s + 0.5w_r) \sin \theta + (w + w_s)0.02 \cos \theta + 0.05w_r \cos \theta$ (for unbalanced slope hoist)[3]

$k = 0.04$ for vertical and inclined shafts

$= 0.03$ for slopes

w = weight of material, lb

w_s = weight of one skip (or cage and car), lb

w_r = weight of rope per side, lb

θ = angle of inclination with the horizontal

Having E, the frictional moment F in foot-pounds is given by

$$F = wr\left(\frac{1 - E}{E}\right) \text{ (for cylindrical drums, balanced hoisting, vertical shafts)}$$

$$= w\left(\frac{r_1 + r_2}{2}\right)\left(\frac{1 - E}{E}\right) \text{ (for conical drums, balanced hoisting, vertical shafts)}$$

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[1] BRIGHT, *loc. cit.*

[2] SAGE, *op. cit.*, p. 15.

[3] For unbalanced operation of a normally balanced hoist it is suggested that 70 per cent of the friction loss for balanced operation be used.

where r = diameter of drum, ft, for cylindrical drums
r_1 = small diameter of drum, ft, for conical drums
r_2 = large diameter of drum, ft, for conical drums

Motor Capacity. The motor capacity is usually determined by the "root mean square" (rms) method. It is based on the heating of the motor. The heating of the motor is proportional to the square of the current and the current is proportional to the torque. The rms value depends, therefore, on the torque (proper adjustment being made for accelerating, constant speed, retarding, and rest periods). The torque is given by horsepower values determined at various stages in the duty cycle. The rms value is the square root of the average of the squared ordinates of the entire torque curve (area) representing the duty cycle.

A hoist motor is of the intermittent type. It has periods of maximum heating and periods of maximum cooling. The formula generally used for determining the rms value is of the following type:

$$\text{Rms-hp} = \sqrt{\frac{(A^2 \times t_a) + \left(\frac{B^2 + C^2 + (BC)}{3} \times t_s\right) + (D^2 \times t_r)}{kt_a + t_s + kt_r + lt_0}}$$

If the difference[1] between B and C is not greater than 15 per cent, $\left(\frac{B+C}{2}\right)^2 \times t_s$ may be substituted for $\frac{B^2 + C^2 + BC}{3} \times t_s$.

Values for k and l are[2]

Direct-current motors, $k = \frac{3}{4}$, $l = \frac{1}{2}$

Alternating-current motors, $k = \frac{1}{2}$, $l = \frac{1}{4}$

Bright[3] makes no adjustment of the time during actual operation ($k = l$), and uses $l = \frac{1}{3}$ for geared motors and $l = \frac{1}{2}$ for large direct-connected motors. If forced (auxiliary) ventilation is used on the motor, the full time for the rest period is used.

Figure 142 shows the location of the values in the rms formula on the duty-cycle diagram.

In many hoisting cycles it will be found that, during retardation or unbalanced lowering, the power demand is negative. This means that the motor acts as a generator and supplies electricity to the line. In such cases the motor or the brakes must absorb energy during retardation. If D (see Fig. 142) is positive, the motor must deliver power during retardation. When induction motors are used, power will be delivered to the line only when the motor operates above synchronous speed. This happens very seldom, so that the motor must be brought to rest either by applying the brakes or by reversing (plugging) the motor. In either case, whether D is negative or positive, it is considered as producing heating, and it must be considered in the rms calculations.

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[1] Stone, *loc. cit.*
Bright, *loc. cit.*

[2] Sage, *op. cit.*, p. 16.

[3] Bright, *op. cit.*, p. 587.

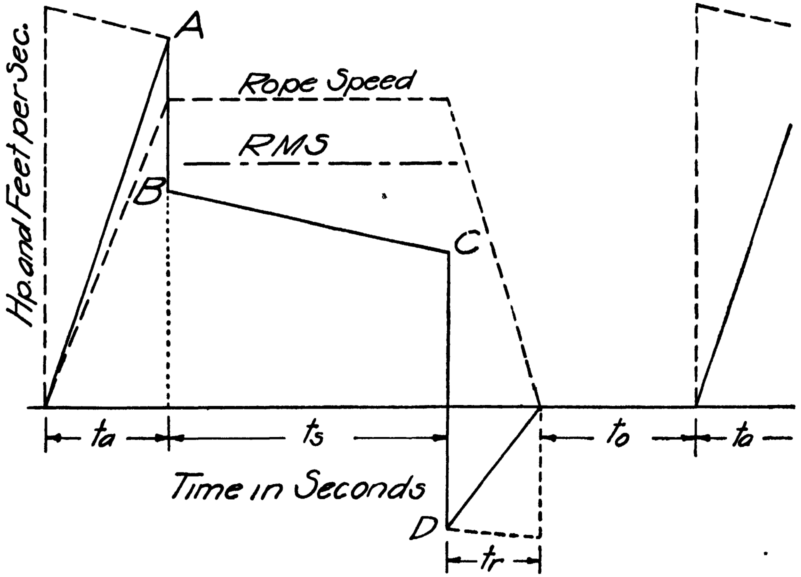

FIG. 142. Duty-cycle diagram.

Duty Cycle. The duty cycle is illustrated by a series of diagrams showing power consumption, torque, amperes, moments, rope speed, etc., at various stages of the hoisting operation. Calculation of values for the above items is made for the beginning of the cycle (zero time); end of acceleration; start of constant speed; end of constant speed; start of retardation; end of retardation; and, finally, the end of the cycle (zero time). The rest period between operations is included in the calculations for determining the rms value of the cycle.

The selection of time values for acceleration, constant speed, retardation, and rest periods, depends on various factors. Table 20 gives information on acceleration data.

Duty-cycle diagrams will be illustrated later when calculations for a hoist motor are made. A later section presents information more or less necessary for selecting the duty cycle (see Data Required for Calculating Size of Hoist Motor).

Rest Period. The rest period is the time required to load the skip or cage at the level, and at the same time to unload at the surface. The time required for these operations depends on the method of handling the material. When skips are used, a minimum time of 3 sec[1] may be approached for the rest period. When cars are caged by hand, the time required is from 6 to 10 sec.

Acceleration and Retardation. The time selected depends to some extent on the total time available for the trip. For most metal mines where the cycle is rather long (relatively deep shafts as compared with coal mines), a few seconds one way or the other makes little difference. For fast coal-mine hoists, the accelerating time may be between 4 and 8 sec. For deep metal mines (or coal mines), the time is 10 to 20 sec. The retarding time is usually several seconds shorter than the accelerating time. The friction of the hoist parts partly aids in slowing down the load. Generally the retarding time is selected so that there is a negative input required during retardation. This permits the use of dynamic braking, thus saving wear on the mechanical brakes.

Acceleration should be done in as short a time as possible, within the limits of the starting capacity of the motor. The maximum rope speed will be lowered by so doing, and in general the power consumption will be less. Too brief an acceleration period will, with the exception of deep shafts (long ropes), cause excessive stress in the rope and hoist parts. Table 42 gives information on some operating mines.[2]

Out-of-balance Hoisting. Hoisting out of balance at some time or other is required of all balanced hoists. When changing levels, one side must be shifted without balance from the other; men, supplies, or equipment may be handled with one side at rest. Such operations, as a rule, would impose no excessive strain on the equipment. When the maximum capacity (for one side) must be approached with half the hoist not operating, the effect on the motor and power available should be investigated. Operations of this sort might arise from rope changes; repairs to the shaft; repairs to the hoist (relining brakes, etc.); or in the selection of flywheel-motor-generator sets for load-equalization purposes. In general the motor could be designed for unbalanced hoisting in one of two ways: continuous operation by limiting the load or speed, or both; and limiting the number of consecutive full-load trips. The rest period may be lengthened in either instance to permit further cooling of the motor. In emergency cases the motor could be cooled by means of a fan.

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[1] SAGE, *op. cit.*, p. 8.

[2] Fundamental Hoisting Problems, *Eng. Mining J.*, **122** (1926).

Selection of Drum Size. There is no accepted rule limiting the number of layers of rope that may be wound on a cylindrical drum. Experience seems to indicate a longer life for drums and ropes when only one layer is wound. For shafts about 1000 ft deep or less, no difficulty is ordinarily experienced in confining the rope to one layer. Cases do, of course, arise, where the location of the hoist causes a large fleet angle if the rope is wound in one layer. In such instances two or more layers may have to be wound.

There are, in general, two important factors to be considered in selecting the drum: (1) the initial cost and (2) the operating cost. By operating cost is not meant repairs and maintenance, but the power charges. The power consumption is directly responsible to the duty cycle. About 25 per cent (depending on method of hoisting) of the total power consumption is used to manipulate the drum mass.

An inspection of Fig. 140 will show that a number of drums of varying diameters and width of face, having the same effective weight, are available. In many instances we may find that it becomes desirable to sacrifice first cost in favor of lower power costs.

As was discussed under headframes, the minimum diameter of the drum depends on the rope diameter. This will, to a certain extent, control the drum size. In addition to the information previously given the following is added at this point as a guide toward choosing the minimum drum diameter.[1] Sheaves should have the same diameter as the drum; or at least approximately so.

Diam. of Rope, In.	Minimum Diam. of Drum, Ft
1⅛	6
1¼	7
1⅜	8
1½	9
1⅝	10
1¾	12

When drums other than cylindrical are to be used, a careful investigation of the diameter of the small and large cylindrical portion and the slope of the conical section should be made. The minimum size of the smaller diameter will usually be governed by the diameter of the rope.

If hoisting is to be done from but one level, conical or cylindroconical drums give a much better capacity factor than do the cylindrical.

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[1] BRIGHT, *op. cit.*, p. 583.

TABLE 42. HOISTING SPEED AT VARIOUS MINES

Mine	Depth, ft.	Max. speed, ft. per min.	Accel., ft. per sec.	Distance to reach max. speed, ft.	Time to reach max. speed, sec.	Loading, sec.	Complete hoist, sec.
New Jersey Zinc Co	1610	3000	3.33	375	15	5	50
International Nickel Co	1600	1100	2.33	260	15		
Wetherbee, Sherman Co	1000	1200	1.33	150	15		
Copper Range	1500	2360	2.48	130	10		
Sulphide Corporation	825	3650	5.06	300	15		
United Verde	850	2000	4.7	115	10		
United Verde	1700	1000	.....	...	10		
Great Boulder	2000	2000	5.83	95	6		
Boston and Montana	3500	3500	.....	...	...	8	
Newport Mining Co	2700	3000	.....	...	...	5	60
Harworth Main Colliery, Eng	3000	3170	.....	232	15.6	10	89.2
New Orient, Illinois	607	4000	13.5	...	5	9	26

They cannot be used for levels other than those for which designed except out of balance. Also, the first cost of these types of drums is greater than that of an equivalent cylindrical drum.

As a guide toward the investigation of conical and cylindroconical drums, the reader is referred to an article by Stone.[1]

DATA REQUIRED FOR CALCULATING THE SIZE OF THE HOIST MOTOR

A considerable amount of information is necessary before calculations can be made for selecting the motor. The exact amount will depend a great deal upon the motor size, and the operating conditions. The following tabulation includes that recommended by Sage,[2] Allis-Chalmers Manufacturing Company,[3] and the Nordberg Manufacturing Company.[4]

1. Weight of ore per trip and condition (wet, dry, sticky, etc.)
2. Weight of skip (plus cage if used; or if skip is not used, cage and number of cars per cage)
3. Diameter and weight of rope
4. Maximum tonnage per shift
    a. Number of hours per hoisting shift
    b. Number of hoisting shifts per day
    c. Distribution of tonnage between levels

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[1] STONE, F. L., Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating, *Proc. AIEE*, October, 1918.

[2] SAGE, *op. cit.*, p. 7.

[3] Allis-Chalmers Manufacturing Company, *Bull.* 1830.

[4] Nordberg Manufacturing Company, *Bull.* 43.

5. If manufacturer is to select the rope speed, give:
    a. Maximum number of trips per hour from stated level
    b. Time for loading and dumping (rest)
    c. If a cage hoist, state whether the cage is landed at the collar and caged off or whether the car is dumped by means of self-dumping platform cage
6. If manufacturer is not to select rope speed, give:
    a. Rope speed at full speed of motor
    b. Time for loading and dumping (rest)
    c. Accelerating and retarding time (or let manufacturer select them)
7. Is the "slack-rope system" of hoisting used? If so, give length of slack used and details of operation
8. Depth of shaft (length of travel from loading to dumping position)
    a. Ultimate depth
    b. Present depth
    c. Estimate time before ultimate depth is reached
9. Shallowest and deepest level; *i.e.*, length of travel from which hoist may be required to operate at full load for an hour or more consecutively
10. Tonnage to be hoisted from each level (as in 9) and time required
11. If incline, give angle of inclination with horizontal or grade in per cent; if it varies, give inclination and length of each stretch (per cent grade is generally taken as $\frac{\text{vertical rise}}{\text{length along incline}}$)
12. Balanced or unbalanced hoist?
    a. If normally balanced, when will operation out of balance be necessary?
    b. Can rope speed or load or both be reduced for unbalanced operation?
    c. Number of unbalanced trips in succession
13. To what extent and for what reasons will partial-speed operations be necessary? At what loads and speeds? Duration and frequency of such operations?
14. Will men be handled?
    a. At what speed?
    b. In balance or out of balance?
15. Drums
    a. Double or single drums
    b. Fixed or clutched
    c. Diameter
    d. Width of face or number of layers of rope
    e. If other than a cylindrical drum, furnish a sketch of drums with working diameters and number of active turns on each part of drum
    f. If a reel is used, give minimum working diameter, and thickness of rope
    g. If an existing hoist, give WR^2 of drums or equivalent weight at a stated radius
    h. If a new hoist, advise what hoist manufacturers give as WR^2
16. Motor geared or direct-connected
    a. Number of reductions
17. Power data
    a. Alternating current
        (1) Voltage

        (2) Frequency
        (3) Phase
    b. Direct current
        (1) Voltage

18. Capacity and character of generating station or system
19. Is flywheel equalization required?
    a. If so, for what reason?
    b. If power is purchased, furnish if possible, a copy of all clauses of proposed power contract covering reservation and kilowatt-hour charges
20. If hoist is to be installed underground, give dimensions and weight of largest piece that can be taken in through shaft and drifts
21. If to be exported, give means of transportation and largest section that can be handled
22. Is location dry, damp, or wet?
23. If above 3000 ft give altitude of hoist house above sea level
24. If to electrify old hoist, give full details, also drawings or sketches of hoist, and photographs if possible
25. Is overwind protection contemplated? If so, advise maximum rope travel beyond upper landing or dump before damage can occur
26. Give full information on local conditions and unusual requirements or details not covered by the foregoing

EFFECT OF BALANCED HOISTING OR OF COUNTERWEIGHT ON THE SIZE OF MOTOR REQUIRED

The advantage of operating in balance or of using a counterbalance over unbalanced hoisting is best illustrated by an example. For this purpose, the following problem will be solved. The conditions are the same for all three cases and are as follows:

1. Total lift = 500 ft
2. Weight of cage = 5000 lb
3. Weight of car = 2500 lb
4. Weight of ore = 4000 lb
5. Capacity per day = 600 tons
6. Net hoisting time = 6 hr
7. Rest period at top and bottom = 8 sec
8. Rope 1⅛-in. at 2 lb per ft
9. Drums, cylindrical, 6 ft diam., 4-ft face. Equivalent effective weight at the rope center for one drum is 15,000 lb; the gear is 10 per cent of this, or 1500 lb (see Fig. 140)
10. Sheaves, each = 2000 lb
11. Acceleration and retardation, each = 6 sec

UNBALANCED HOIST

The following procedure is one of a number that may be followed.

$$\text{Trips per min} = \frac{T}{H \times w \times 60}$$

where T = total tons to be hoisted
H = net operating time
w = tons hoisted each trip

$$\text{Trips per min} = \frac{600}{6 \times 2 \times 60} = \frac{5}{6}$$

$$\text{Time per trip} = t = \frac{60}{5/6} = 72 \text{ sec}$$

The rest period t_o is 8 sec. In unbalanced hoisting we have this period at the bottom and the top at separate times.

The actual running time for a round trip is

$$t - 2t_o = 72 - 2 \times 8 = 56 \text{ sec}$$

The running time for one way = $X = 56/2 = 28$ sec.

The maximum drum speed, in rps = v = the number of active turns divided by the equivalent full-speed hoisting time.

The number of active turns is found by dividing the total lift in feet by the circumference of the drum.

$$500 \div 6\pi = 26.5$$

Equivalent full-speed hoisting time = $e = \left(X - \frac{t_a + t_r}{2}\right)$, for unbalanced hoist, where t_a and t_r are the accelerating and retarding time, respectively.

$$e = \left(28 - \frac{6+6}{2}\right) = 22 \text{ sec}$$

$$v = 26\ 5 \div 22 = 1.204 \text{ rps, or } 72.2 \text{ rpm}$$

$$\text{Time for full speed} = t_s = e - \frac{t_a + t_r}{2} = 22 - \frac{6+6}{2} = 16 \text{ sec}$$

$$\text{Turns of drum during acceleration} = \frac{vt_a}{2} = \frac{1.204 \times 6}{2} = 3.61$$

$$\text{Turns during retardation} = \frac{vt_r}{2} = \frac{1.204 \times 6}{2} = 3.61$$

$$\text{Turns during constant speed} = vt_s = 1.204 \times 16 = 19.26$$

The rope wound during each of the above periods is now found.

Rope wound during acceleration = $3.61 \times 6\pi$ = 68 2 ft
Rope wound during retardation = $3.61 \times 6\pi$ = 68 2 ft
Rope wound during const. speed = $19.26 \times 6\pi$ = 363.6 ft

With these data we are prepared to calculate the moments and hence the horsepower required of the motor under the various conditions of the hoisting cycle.

Moments are calculated for the up load and up rope; for accelerating the load, drum, and sheave; and friction (shaft guides, gears, and windage) at each of the time changes in the hoisting cycle. For the loss due to friction we shall consider the average of the maximum and minimum moments as being 80 per cent efficient. The moments are calculated for four stages of the hoisting operation. This is done so that proper allowance may be made for winding up the rope on the loaded (ascending) side and the addition of rope to the unloaded (descending) side. The two ropes are only in counterbalance where the skips pass each other. Before this takes place, the rope on the descending side does not balance the ascending rope. The above statement does not apply to the present case, as we are considering an unbalanced hoist; in the other two cases, however, it is of importance.

Moments. The up load consists, exclusive of the rope, of the cage, car, and contents. This equals

$$5000 + 2500 + 4000 = 11{,}500 \text{ lb}$$

The moment, in foot-pounds, is equal to the load times the radius of the drum. Actually the radius to the rope center should be used. The slight difference in using the radius of the drum is more than offset by the convenience of avoiding the small fractions of a foot which would otherwise be necessary. The error incurred is in no way serious.

UP LOAD

0 turns 11,500 lb × 3 ft = 34,500 ft-lb
3.61 turns 11,500 lb × 3 ft = 34,500 ft-lb
22.87 turns 11,500 lb × 3 ft = 34,500 ft-lb
26.5 turns 11,500 lb × 3 ft = 34,500 ft-lb

UP ROPE

0 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb
3.61 turns (431.8 ft × 2 lb) × 3 ft = 2591 ft-lb
22.87 turns (68.2 ft × 2 lb) × 3 ft = 409 ft-lb
26.5 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb

TOTAL MOMENTS

Turns	0	3.61	22.87	26.5
Load	34,500	34,500	34,500	34,500
Rope	3,000	2,591	409	0
Total	37,500	37,091	34,909	34,500

Friction Moment. A sufficiently close approximation of the friction moment is obtained as follows:

$$\frac{37{,}500 + 34{,}500}{2} \div 80\% = 45{,}000 \text{ ft-lb}$$

$$M_F = 45{,}000 - \frac{37{,}500 + 34{,}500}{2} = 9000 \text{ ft-lb}$$

Acceleration and Retardation Moments

The load to be accelerated and retarded is as follows:

Cage	5,000 lb
Car	2,500 lb
Load	4,000 lb
Rope[*] [(500 + 150) × 2]	1,300 lb
Drum and gear (15,000 + 1500)	16,500 lb
Sheave	2,000 lb
Total	31,300 lb

[*] 150 ft of rope assumed from unloading position to hoist

The velocity in feet per second at full speed (V) is equal to the rps (v) multiplied by the circumference of the drum.

$$V = v \times \pi d = 1.204 \times 6\pi = 22.7 \text{ ft per sec.}$$

Acceleration

$$a = \frac{V}{t_a} = \frac{22.7}{6} = 3.78 \text{ ft per sec.}^2$$

Accelerating force, F_a,

$$F_a = \frac{W}{g} a$$

where W = total weight to be accelerated or retarded
g = acceleration due to gravity = 32.2 ft per sec²

$$F_a = \frac{31{,}300}{32.2} \times 3.78 = 3674 \text{ lb}$$

Accelerating moment, M_a,

$$M_a = F_a \times r = 3674 \times 3 = 11{,}022 \text{ ft-lb}$$

Retardation. In the present problem the retarding time is the same as the accelerating time, so that its numerical value is the same as M_a. It is, however, negative, so that $M_r = -11{,}022$ ft-lb.

The moments are now summed up and the horsepower calculated for the various points in the cycle. The horsepower is obtained from

$$\text{Hp} = \frac{2\pi(\text{rps}) \times M}{550}$$
$$= \frac{2\pi \times 1.204}{550} \times M = 0.0141M$$

Summation of Moments—Unbalanced Hoist

Turns	0	3.61	3.61	22.87	22.87	26.5
Net moment	37,500	37,091	37,091	34,909	34,500	34,500
Friction moment	9,000	9,000	9,000	9,000	9,000	9,000
Accel. and ret. moment	11,022	11,022	......	.....	−11,022	−11,022
Total moment = M	57,522	57,113	46,091	43,909	32,887	32,478
Time, sec	0	6	6	22	22	28
Horsepower	810	805	650	619	464	458

From the above table we find that a motor capable of delivering 810 no-load torque horsepower is necessary. For the purpose of comparison it is not necessary that the rms value for these cases be determined. Nor, for the same reason, is it necessary that we include calculations for the power required to accelerate the motor rotor. This will be done in later examples.

Counterweighed Hoist

As previously stated the counterweight giving the best motor rating is one having a weight equal to the weights of the cage, car and one-half the load. This gives

$$5000 + 2500 + {}^{4000}\!/_{2} = 9500 \text{ lb}$$

By following such a procedure, the cycle for hoisting the load closely approximates that for hoisting the counterweight, which is the desired objective.

The values for t, t_o, t_a, t_r, t_s, e, and v are the same as for the unbalanced hoist.

Two sets of calculations are made: (1) raising the load and lowering the counterweight; and (2) raising the counterweight and lowering the empty cage with car.

Moments (Up Load and Down Counterweight)

0 turns 11,500 lb × 3 ft = 34,500 ft-lb
3.61 turns 11,500 lb × 3 ft = 34,500 ft-lb
22.87 turns 11,500 lb × 3 ft = 34,500 ft-lb
26.5 turns 11,500 lb × 3 ft = 34,500 ft-lb

Up Rope

0 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb
3.61 turns (431.8 ft × 2 lb) × 3 ft = 2591 ft-lb
22.87 turns (68.2 ft × 2 lb) × 3 ft = 409 ft-lb
26.5 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb

Down Counterweight

0 turns 9500 lb × 3 ft = 28,500 ft-lb
3.61 turns 9500 lb × 3 ft = 28,500 ft-lb
22.87 turns 9500 lb × 3 ft = 28,500 ft-lb
26.5 turns 9500 lb × 3 ft = 28,500 ft-lb

Down Rope

0 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb
3.61 turns (68.2 ft × 2 lb) × 3 ft = 409 ft-lb
22.87 turns (431.8 ft × 2 lb) × 3 ft = 2591 ft-lb
26.5 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb

TOTAL MOMENTS. UP LOAD

Turns	0	3.61	22.87	26.5
Load	34,500	34,500	34,500	34,500
Rope	3,000	2,591	409	0
Total	37,500	37,091	34,909	34,500

DOWN COUNTERWEIGHT

Turns	0	3.61	22.87	26.5
Counterweight	28,500	28,500	28,500	28,500
Rope	0	409	2,591	3,000
Total	28,500	28,909	31,091	31,500

NET MOMENTS

Turns	0	3.61	22.87	26.5
Up load	37,500	37,091	34,909	34,500
Down counterweight	28,500	28,909	31,091	31,500
Net	9,000	8,082	3,818	3,000

Friction Moment

$$\frac{9000 + 3000}{2} \div 80\% = 7500 \text{ ft-lb}$$

$$7500 - \frac{9000 + 3000}{2} = 1500 \text{ ft-lb}$$

ACCELERATION AND RETARDATION MOMENTS

The load to be accelerated or retarded is

Counterweight	9,500 lb
Cage	5,000 lb
Car	2,500 lb
Load	4,000 lb
Rope* [(500 + 150) × 2 × 2]	2,600 lb
Drum and gear	16,500 lb
Sheaves* (2000 × 2)	4,000 lb
Total	44,100 lb

* There are two ropes and two sheaves.

$$V = 1.204 \times 6\pi = 22.7 \text{ ft per sec}$$

Acceleration

$$a = \frac{22.7}{6} = 3.78 \text{ ft per sec}^2$$

$$F_a = \frac{44,100}{32.2} \times 3.78 = 5177 \text{ lb}$$

$$M_a = 5177 \times 3 = 15,531 \text{ ft-lb}$$

Retardation

$$M_r = -15{,}531 \text{ ft-lb}$$

Summation of Moments—Counterweighed Hoist (Up Load)

Turns	0	3.61	3.61	22.87	22.87	26.5
Net moment.............	9,000	8,082	8082	3818	3,818	3,000
Friction moment.	1,500	1,500	1500	1500	1,500	1,500
Accel. and ret. moment...	15,531	15,531	......	......	−15,531	−15,531
Total moment = *M*......	26,031	25,113	9582	5312	−10,213	−11,031
Time, sec	0	6	6	22	22	28
Horsepower ...	367	354	135	75	−144	−156

$$\text{Hp} = \frac{2\pi \times 1\ 204}{550} \times M = 0.0141M$$

Up Counterweight and Down Empty Car and Cage

Up Counterweight

0 turns 9500 lb × 3 ft = 28,500 ft-lb
3.61 turns 9500 lb × 3 ft = 28,500 ft-lb
22.87 turns 9500 lb × 3 ft = 28,500 ft-lb
26.5 turns 9500 lb × 3 ft = 28,500 ft-lb

Up Rope

0 turns (500 ft × 2 lb) × 3 = 3000 ft-lb
3.61 turns (431.8 ft × 2 lb) × 3 = 2591 ft-lb
22.87 turns (68.2 ft × 2 lb) × 3 = 409 ft-lb
26.5 turns (0 ft × 2 lb) × 3 = 0 ft-lb

Down Cage and Car

0 turns 7500 lb × 3 ft = 22,500 ft-lb
3.61 turns 7500 lb × 3 ft = 22,500 ft-lb
22.87 turns 7500 lb × 3 ft = 22,500 ft-lb
26.5 turns 7500 lb × 3 ft = 22,500 ft-lb

Down Rope

0 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb
3.61 turns (68.2 ft × 2 lb) × 3 ft = 409 ft-lb
22 87 turns (431.8 ft × 2 lb) × 3 ft = 2591 ft-lb
26.5 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb

Total Moments. Up Counterweight

Turns	0	3.61	22.87	26.5
Counterweight..	28,500	28,500	28,500	28,500
Rope..	3,000	2,591	409	0
Total.	31,500	31,091	28,909	28,500

DOWN CAGE AND CAR

Turns	0	3.61	22.87	26.5
Cage and car	22,500	22,500	22,500	22,500
Rope	0	409	2,591	3,000
Total	22,500	22,909	25,091	25,500

NET MOMENTS

Turns	0	3.61	22.87	26.5
Up counterweight	31,500	31,091	28,909	28,500
Down cage and car	22,500	22,909	25,091	25,500
Net	9,000	8,182	3,818	3,000

Friction Moment

$$\frac{9000 + 3000}{2} \div 80\% = 7500 \text{ ft-lb}$$

$$7500 - \frac{9000 + 3000}{2} = 1500 \text{ ft-lb}$$

Acceleration and Retardation Moment. The load to be accelerated or retarded is

Counterweight	9,500 lb
Cage	5,000 lb
Car	2,500 lb
Ropes	2,600 lb
Drum and gear	16,500 lb
Sheaves	4,000 lb
Total	40,100 lb

$$V = 1.204 \times 6\pi = 22.7 \text{ ft per sec}$$

$$a = \frac{22.7}{6} = 3.78 \text{ ft per sec}^2$$

$$F_a = \frac{40{,}100}{32.2} \times 3.78 = 4707 \text{ lb}$$

$$M_a = 4707 \times 3 = 14{,}121 \text{ ft-lb}$$

$$M_r = -14{,}121 \text{ ft-lb}$$

SUMMATION OF MOMENTS—COUNTERWEIGHTED HOIST (UP COUNTERWEIGHT)

Turns	0	3.61	3.61	22.87	22.87	26.5
Net moment	9,000	8,182	8,182	3,818	3,818	3,000
Friction moment	1,500	1,500	1,500	1,500	1,500	1,500
Accel. and ret. moment	14,121	14,121	......	......	−14,121	−14,121
Total moment = *M*	24,621	23,803	9,682	5,318	− 8,803	− 9,621
Time, sec	0	6	6	22	22	28
Horsepower	347	336	137	75	−124	−135

$$\text{Hp} = 0.0141M$$

BALANCED HOIST

$$\text{Trips per min} = \frac{T}{H \times w \times 60} = \frac{600}{6 \times 2 \times 60} = \frac{5}{6}$$

$$\text{Time per trip} = t = \frac{60}{5/6} = 72 \text{ sec}$$

$$\text{Equivalent full-speed time} = e = \left(t - t_o - \frac{t_a + t_r}{2}\right)$$

where t = time per trip, sec
t_o = time of rest
t_a = accelerating time
t_r = retarding time

$$e = \left(72 - 8 - \frac{6+6}{2}\right) = 58 \text{ sec}$$

$$v = 26.5 \div 58 = 0.457 \text{ rps or } 27.4 \text{ rpm}$$

$$\text{Time for full speed} = t_s = e - \frac{t_a + t_r}{2} = 58 - \frac{6+6}{2} = 52 \text{ sec}$$

$$\text{Turns passed over during acceleration} = \frac{vt_a}{2} = \frac{0.457 \times 6}{2} = 1.371$$

$$\text{Turns during retardation} = \frac{vt_r}{2} = \frac{0.457 \times 6}{2} = 1.371$$

Turns during constant speed $= vt_s = 0.457 \times 52 = 23.75$
Rope wound during acceleration $= 1.371 \times 6\pi = 25.9$ ft
Rope wound during retardation $= 1.371 \times 6\pi = 25.9$ ft
Rope wound during constant speed $= 23.75 \times 6\pi = 448.2$ ft

MOMENTS. UP LOAD

0 turns 11,500 lb × 3 ft = 34,500 ft-lb
1.371 turns 11,500 lb × 3 ft = 34,500 ft-lb
25.121 turns 11,500 lb × 3 ft = 34,500 ft-lb
26.5 turns 11,500 lb × 3 ft = 34,500 ft-lb

UP ROPE

0 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb
1.371 turns (474.1 ft × 2 lb) × 3 ft = 2845 ft-lb
25.121 turns (25.9 ft × 2 lb) × 3 ft = 155 ft-lb
26.5 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb

DOWN LOAD

0 turns 7500 lb × 3 ft = 22,500 ft-lb
1.371 turns 7500 lb × 3 ft = 22,500 ft-lb
25.121 turns 7500 lb × 3 ft = 22,500 ft-lb
26.5 turns 7500 lb × 3 ft = 22,500 ft-lb

DOWN ROPE

0 turns (0 ft × 2 lb) × 3 ft = 0 ft-lb
1.371 turns (25.9 ft × 2 lb) × 3 ft = 155 ft-lb
25.121 turns (474.1 ft × 2 lb) × 3 ft = 2845 ft-lb
26.5 turns (500 ft × 2 lb) × 3 ft = 3000 ft-lb

TOTAL MOMENTS. UP LOAD

Turns	0	1 371	25.121	26 5
Load	34,500	34,500	34,500	34,500
Rope	3,000	2,845	155	0
Total	37,500	37,345	34,655	34,500

DOWN LOAD

Load	22,500	22,500	22,500	22,500
Rope	0	155	2,845	3,000
Total	22,500	22,655	25,345	25,500

NET MOMENT

Net	15,000	14,690	9,310	9,000

Friction Moment

$$\frac{15{,}000 + 9000}{2} = 80\% = 15{,}000 \text{ ft-lb}$$

$$15{,}000 - \frac{15{,}000 + 9000}{2} = 3000 \text{ ft-lb}$$

ACCELERATION AND RETARDATION MOMENTS

The total load to be accelerated or retarded is

Cages, 2 at 5000 lb.	10,000 lb
Cars, 2 at 2500 lb	5,000 lb
Ore	4,000 lb
Rope	2,600 lb
Drums and gear*	31,500 lb
Sheaves	4,000 lb
Total	57,100 lb

* As we are hoisting in balance, we need a double drum. If hoisting had been from only one level, one drum would be sufficient. Maximum possibilities are assumed.

$$V = 0.457 \times 6\pi = 8.62 \text{ ft per sec}$$

$$a = \frac{8.62}{6} = 1.437 \text{ ft per sec}^2$$

$$F_a = \frac{57{,}100}{32.2} \times 1.437 = 2550 \text{ lb}$$

$$M_a = 2550 \times 3 = 7650 \text{ ft-lb}$$

$$M_r = -7650 \text{ ft-lb}$$

SUMMATION OF MOMENTS—BALANCED HOISTING

Turns	0	1.371	1.371	25.121	25.121	26.5
Net moment	15,000	14,690	14,690	9,310	9,310	9,000
Friction moment	3,000	3,000	3,000	3,000	3,000	3,000
Accel. and ret. moment	7,650	7,650	......	......	−7,650	−7,650
Total moment = *M*	25,650	25,340	17,690	12,310	4,660	4,350
Time, sec	0	6	6	58	58	64
Horsepower	134	132	92	64	24	23

$$\text{Hp} = \frac{2\pi \times 0.457}{550} \times M = 0.00522M$$

The following table summarizes the results obtained by the preceding calculations.

Method of hoisting	Horsepower						Rms*	Motor needed†
	Start of accel.	End of accel.	Start of const. speed	End of const. speed	Start of ret.	End of ret.		
Unbalanced	810	805	650	619	464	458	695	700
Counterweighted	367	354	135	75	−144	−156	210	250
Balanced	134	132	92	64	24	23	87	100

* Using a-c motor.

† For nearest frame size, see page 355.

From the above table it is readily seen that the balanced hoist is by far the most economical. The extra initial cost of two cages, cars, etc., is greatly offset by the cheaper hoist motor and power costs.

ESTIMATION OF MOTOR AND DRUM SIZE FOR A GIVEN SET OF CONDITIONS

The principles discussed on previous pages will now be illustrated by a number of concrete examples. Example 1 is for a moderately sized metal mine, hoisting from a minimum and maximum depth with occasional periods out of balance. The object is to select a motor which will prove satisfactory under all the conditions to which it will be subjected. In Example 2 the calculations for a conical-drum, coal-mine hoist motor are made. Example 3 gives the calculations for a cylindroconical drum hoist under the same conditions as Example 2. Example 4 shows the procedure for calculating a d-c hoist motor with voltage control by either a flywheel or synchronous motor-generator set.

The method of moments is used in all examples, although different procedures for so doing are followed. The final results obtained would be the same regardless of the method.

EXAMPLE 1

Statement of Conditions

Weight of ore per trip = 10,000 lb
Weight of skip = 5000 lb
Weight of sheave (8 ft in diameter) = 2700 lb
Rope $1\frac{1}{4}$-in. in diameter at $2\frac{1}{2}$ lb per ft.
Hours per hoisting shift (actual) = 6
Number of hoisting shifts per day = 1
Distribution of tonnage between levels:
1. Minimum depth, 400-ft level, 2 hr hoisting = 600 tons
2. Maximum depth, 1000-ft level, 4 hr hoisting = 1400 tons
3. Minimum depth, 400-ft level, 4 hr hoisting = 1400 tons
4. Out of balance from 1000-ft level, 6 hr = 1000 tons

Tons per hoisting shift = 2000 tons
Time for accelerating = 8 sec
Time for retarding = 6 sec
Time for rest = 8 sec
Maximum hoisting depth (1000-ft level skip pocket) = 1030 ft
Minimum hoisting depth (400-ft level skip pocket) = 430 ft
Inclination of shaft = 90 deg
Type of hoist = Balanced
Speed at which hoist operates out of balance = 27.96 ft per sec
Type of drums = Double cylindrical
Size of drums:
1. Diameter = 8 ft
2. Width of face = 6 ft
3. Equivalent effective weight of two drums with gear, at rope center = 67,200 lb
4. Number of layers of rope = 1

Type of motor
1. Alternating current, wound rotor:
   a. Connection to drum = By gear
   b. Volts = 2200
   c. Cycles = 60
   d. Phase = 3
   e. Rpm of motor = 450
   f. Gear ratio = 6.2:1
   g. Horsepower = 1100

2. Direct-current motor, shunt wound:
   - a. Connection to drum = Direct-connected
   - b. Volts = 550
   - c. Rpm of motor = 73
   - d. Horsepower = 900

The rms-hp will be determined for the following conditions:

Case 1
   - a. Six hundred tons to be handled from the 400-ft level in 2 hr
   - b. Fourteen hundred tons to be hoisted from the 1000-ft level in 4 hr

Case 2
   - a. Fourteen hundred tons to be hoisted from the 400-ft level in 4 hr
   - b. Six hundred tons from the 1000-ft level in 2 hr

Case 3
   - a. Out of balance, 1000 tons from the 1000-ft level in 6 hr

Selection of Drum Size. A condition of the problem is that all the rope must be wound in one layer. We must also know the total length or weight of rope in order to determine the acceleration moment for the rope. The total length of rope in service is

Distance from 1000-ft level skip pocket to collar = 1030 ft
Distance from collar to top of sheave wheel = 100 ft
Distance from sheave wheel to center of drum = 160 ft
Wound permanently on the drum = 3 turns

The amount of rope wound on the drum under maximum conditions consists of 1030 ft + 60 ft (collar to dumping roller) = 1090 ft. In addition it is customary[1] to have at least 3 turns on the drum. As a trial we shall assume a drum 9 ft in diameter. This drum will, in one revolution, wind up 9π, or 28.44 ft of rope. The total number of turns necessary to wind up 1090 ft of rope is

$$1090 \div 28.44 = 38.3$$

Taking three turns of inactive rope on the drum, we must have sufficient width of face to wind up 38.3 + 3 = 41.3, or 42, turns of 1¼-in. rope. If the drum is grooved and each turn of rope is separated by ¼-in., the length of the drum face becomes

$$42 \times (1\tfrac{1}{4} + \tfrac{1}{4}) = 63 \text{ in., or } 5\tfrac{1}{4} \text{ ft}$$

The effective weight of a 9- by 5¼-ft drum is about 32,000 lb. Probably a 9 by 6 drum would be the nearest available commercial size. Its equivalent weight is 37,000 lb.

An 8-ft drum will wind up 25.12 ft of rope per revolution. The number of turns are

$$\frac{1090}{25.12} + 3 = 46.4, \text{ or } 47 \text{ turns}$$

$$\text{Width of face} = 47 \times 1\tfrac{1}{2} = 70.5 \text{ in., or } 6 \text{ ft}$$

We shall use an 8- by 6-ft drum.

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[1] In some states by law.

Effective Weight of Drums. The WR^2 or the effective weight of the drums and gear at the radius of the rope center (for all practical purposes the radius of the drum) must be known. From Fig. 140 we find the effective weight of an 8 by 6 drum to be about 32,000 lb. The weight of the sheave, from the same figure, is 2700 lb. The problem is to be solved for both a-c and d-c motors. As previously stated under Connection of Motor to Hoist, a-c motors do not have desirable characteristics for low speeds (direct connection to drum). From Fig. 140 we find that 10 per cent of the drum weight must be added for the gear. The total effective weight of the drums and gear for an a-c motor is

$$(2 \times 32{,}000) + (10\% \times 32{,}000) = 67{,}200 \text{ lb}$$

For the d-c motor the weight is 64,000 lb.

From an inspection of the three cases under which hoisting may occur, it is seen that case 1*b* will probably be the influencing factor. The motor for this case will be determined first.

Case 1*b*. The known data are

Ore to be hoisted = 1400 tons
Total lift to dumping roller (1030 + 60) = 1090 ft
Weight of skip = 5000 lb
Weight of ore per skip = 10,000 lb
Size of rope = 1¼ in.
Weight of rope per side (1090 × 2.5) = 2725 lb
Drums = 8 by 6 ft
Effective weight of two drums and gear = 67,200 lb
Effective weight of each sheave = 2700 lb
Number of active turns $1090 \div 8\pi = 43.4$
Accelerating time (t_a) = 8 sec
Retarding time (t_r) = 6 sec
Rest (t_o) = 8 sec
Net operating time = 4 hr

$$\text{Trips per min} = \frac{T}{H \times w \times 60} = \frac{1400}{4 \times 5 \times 60} = \frac{7}{6}$$

$$\text{Time per trip} = t = \frac{60}{7/6} = 51\frac{3}{7}, \text{ or } 51 \text{ sec}$$

$$\text{Equivalent full-speed hoisting time} = e = \left(51 - 8 - \frac{8+6}{2}\right) = 36 \text{ sec}$$

$$\text{Maximum drum speed} = v = 43.4 \div 36 = 1.205 \text{ rps, or } 72.3 \text{ rpm}$$

$$\text{Time for full speed} = t_s = 36 - \frac{8+6}{2} = 29 \text{ sec}$$

$$\text{Turns during acceleration} = \frac{vt_a}{2} = \frac{1.205 \times 8}{2} = 4.82$$

$$\text{Turns during retardation} = \frac{vt_r}{2} = \frac{1.205 \times 6}{2} = 3.62$$

$$\text{Turns during constant speed} = vt_s = 1.205 \times 29 = 34.94$$

$$\text{Rope wound during acceleration} = 4.82 \times 8\pi = 122 \text{ ft}$$

$$\text{Rope wound during retardation} = 3.62 \times 8\pi = 91 \text{ ft}$$

$$\text{Rope wound during constant speed} = 34.94 \times 8\pi = 877 \text{ ft}$$

MOMENTS. UP LOAD

0 turns 15,000 lb × 4 ft = 60,000 ft-lb
4.82 turns 15,000 lb × 4 ft = 60,000 ft-lb
39.76 turns 15,000 lb × 4 ft = 60,000 ft-lb
43.4 turns 15,000 lb × 4 ft = 60,000 ft-lb

UP ROPE

0 turns (1090 ft × 2.5 lb) × 4 ft = 10,900 ft-lb
4.82 turns (968 ft × 2.5 lb) × 4 ft = 9,680 ft-lb
39.76 turns (91 ft × 2.5 lb) × 4 ft = 910 ft-lb
43.4 turns (0 ft × 2.5 lb) × 4 ft = 0 ft-lb

DOWN LOAD

0 turns 5000 lb × 4 ft = 20,000 ft-lb
4.82 turns 5000 lb × 4 ft = 20,000 ft-lb
39.76 turns 5000 lb × 4 ft = 20,000 ft-lb
43.4 turns 5000 lb × 4 ft = 20,000 ft-lb

DOWN ROPE

0 turns (0 ft × 2.5 lb) × 4 ft = 0 ft-lb
4.82 turns (122 ft × 2.5 lb) × 4 ft = 1,220 ft-lb
39.76 turns (999 ft × 2.5 lb) × 4 ft = 9,990 ft-lb
43.4 turns (1090 ft × 2.5 lb) × 4 ft = 10,900 ft-lb

NET MOMENT

Up

Turns	0	4.82	39.76	43.4
Load	60,000	60,000	60,000	60,000
Rope	10,900	9,680	910	0
Total	70,900	69,680	60,910	60,000

Down

Load	20,000	20,000	20,000	20,000
Rope	0	1,220	9,990	10,900
Total	20,000	21,220	29,990	30,900
Net	50,900	48,460	30,920	29,100

Friction Moment. The average of the maximum and minimum net moments will be taken as 80 per cent of the total moment. The remainder will be the loss due to friction, windage, etc.

$$\frac{50{,}900 + 29{,}100}{2} \div 80\% = 50{,}000 \text{ ft-lb}$$

$$50{,}000 - \frac{50{,}900 + 29{,}100}{2} = 10{,}000 \text{ ft-lb}$$

ACCELERATION AND RETARDATION MOMENTS

The load to be accelerated or retarded is as follows:*

Skips (2 at 5000 lb)	10,000 lb
Ore	10,000 lb
Rope*	6,825 lb
Drums and gear	67,200 lb
Sheaves (2 at 2700 lb)	5,400 lb
Total	99,425 lb

* The total weight comes from the following:

1000-ft level skip pocket to collar	1030 ft
Collar to top of sheave	100 ft
Sheave to drum	160 ft
Wound on drum (3 × 25.12)	75 ft
Total	1365 ft

This must be doubled because the hoist is in balance.

$$1365 \times 2 \times 2.5 \text{ lb} = 6825 \text{ lb}$$

Velocity at full speed $= V = 1.205 \times 8\pi = 30.3$ ft per se

Acceleration

$$a = \frac{V}{t_a} = \frac{30.3}{8} = 3.79 \text{ ft per sec}^2$$

$$F_a = \frac{W}{g}a = \frac{99{,}425}{32.2} \times 3.79 = 11{,}700 \text{ lb}$$

$$M_a = 11{,}700 \times 4 = 46{,}800 \text{ ft-lb}$$

Retardation

$$-a = \frac{30.3}{6} = 5.05 \text{ ft per sec}^2$$

$$F_r = \frac{99{,}425}{32.2} \times (-5.05) = -15{,}570 \text{ lb}$$

$$M_r = -15{,}570 \times 4 = -62{,}280 \text{ ft-lb}$$

SUMMATION OF MOMENTS—CASE 1*b* (ALTERNATING-CURRENT MOTOR)

Turns	0	A	B	C	D	43.4
		4.82	4.82	39.76	39.76	
Net moment	50,900	48,460	48,460	30,920	30,920	29,100
Friction moment	10,000	10,000	10,000	10,000	10,000	10,000
Accel. and ret. moment	46,800	46,800	......	......	−62,280	−62,280
Total moment = *M*	107,700	105,260	58,460	40,920	−21,360	−23,180
Time, sec	0	8	8	37	37	43
Horsepower	1,480	1,450	804	563	−294	−319

$$\text{Hp} = \frac{2\pi \times 1.205}{550} \times M = 0.01375M$$

A check on the calculations may be made by finding the total horsepower-seconds of the duty cycle. This should check the net work done in the shaft, expressed in horsepower-seconds.

$$\text{Net work in shaft} = \frac{10{,}000 \times 1090}{550} = 19{,}818 \text{ hp-sec}$$

Area of duty cycle expressed in horsepower-seconds:

Acceleration $(1450 \times \frac{8}{2})$	5,800 hp-sec
Full speed $\left(\frac{804 + 563}{2} \times 29\right)$	19,821 hp-sec
Retardation $(-294 \times \frac{6}{2})$	−882 hp-sec
Total	24,739 hp-sec

The operations were considered as being 80 per cent efficient. The total energy expended in the shaft is 19,818 ÷ 80% = 24,772 hp-sec, which practically checks the duty cycle.

Motor Capacity. *Alternating-current Motor.* The motor capacity is determined by the rms method. Before this can be done, part of the horsepower data for case 1*b* must be corrected for the horsepower necessary to accelerate and retard the motor rotor. The speed of the motor for the present cycle will not be especially high or low, so that for estimating purposes we can assume 160 per cent of its normal rating to accelerate the rotor in 1 sec (see page 297). Estimating (roughly) the rms value to be equal to

$$\frac{1480 + 563}{2} = 1021, \text{ or } 1000 \text{ hp}$$

in order to get a normal rating on which to base the power needed to accelerate the armature, and taking 160 per cent of it, we get

$$1000 \times 1.6 = 1600 \text{ hp}$$

if the acceleration or retardation takes place in 1 sec. The accelerating time t_a is 8 sec and the retarding time t_r is 6 sec. Then

$$\begin{aligned} {}^{1600}\!/_{8} &= 200 \text{ hp to accelerate rotor} \\ {}^{1600}\!/_{6} &= 266, \text{ or } 270 \text{ hp to retard rotor} \end{aligned}$$

The horsepower at points A and D in the table for case 1*b* now becomes

$$\begin{aligned} A &= 1450 + 200 = 1650 \text{ hp} \\ D &= -294 - 270 = -564 \text{ hp} \end{aligned}$$

The rms-hp may now be determined.

$$\text{Rms-hp} = \sqrt{\frac{(A^2 \times t_a) + \left[\frac{B^2 + C^2 + (BC)}{3} \times t_s\right] + (D^2 \times t_r)}{kt_a + t_s + kt_r + lt_o}}$$

where (for a-c motors) $k = \frac{1}{2}$

$l = \frac{1}{4}$

Substituting values from case 1*b*

$$\text{Rms-hp} = \sqrt{\frac{(1650^2 \times 8) + \left[\dfrac{804^2 + 563^2 + (804 \times 563)}{3} \times 29\right] + (564^2 \times 6)}{(\frac{1}{2} \times 8) + 29 + (\frac{1}{2} \times 6) + (\frac{1}{4} \times 8)}}$$

$$\begin{aligned}
1650^2 \times 8 &= 2{,}722{,}500 \times 8 &&= 21{,}780{,}000 \text{ hp}^2\text{-sec} \\
\frac{804^2 + 563^2 + 804 \times 563}{3} \times 29 &= 472{,}012 \times 29 &&= 13{,}688{,}346 \text{ hp}^2\text{-sec} \\
564^2 \times 6 &= 318{,}096 \times 6 &&= \underline{1{,}908{,}576} \text{ hp}^2\text{-sec} \\
& & \text{Total} &= 37{,}376{,}922 \text{ hp}^2\text{-sec}
\end{aligned}$$

$$\tfrac{1}{2} \times 8 + 29 + \tfrac{1}{2} \times 6 + \tfrac{1}{4} \times 8 = 38 \text{ sec}$$

$$\text{Rms-hp} = \sqrt{\frac{37{,}376{,}922}{38}} = 992$$

Most induction motors have a pull-out torque of twice the full-load torque,[1] so that a motor with a normal full-load rating of 1000 hp would appear to prove satisfactory. The pull-out torque should be investigated for the motor selected.

If power mains are subjected to frequent drops in line voltage, either a motor larger than required for normal operations should be recommended, or the pull-out torque should be advanced to 2.5 times full load. Bright recommends that an induction motor having a value of 25 to 30 per cent greater than the maximum value on the load cycle be used to provide for a drop of at least 10 per cent in the line voltage.[2] The 1000-hp motor selected for the above problem, with a pull-out of 2, would not prove large enough with a 10 per cent drop in voltage. The maximum value on the load cycle is at zero turns and is 1480 + 200 = 1680 hp. Thirty per cent of 1680 = 504 hp. A motor to stand a 10 per cent line drop must have a pull-out torque of 1680 + 504 = 2184 hp. An 1100-hp motor would be sufficient with a pull-out of 2; otherwise the 1000-hp motor should have a pull-out of 2.5.

An 1100-hp 2200-volt 3-phase 60-cycle 450 rpm wound-rotor induction motor would be recommended.

Gear Reduction. The maximum speed of the hoist drum is 72.3 rpm. The gear reduction, using a 450-rpm motor, would be 450/72.3 = 6.22.

Rope Speed. The maximum rope speed is $72.3 \times 8\pi = 1816$ fpm. Or it may be found by

$$S = \frac{D}{R - \dfrac{t_a + t_r}{2}} \quad \text{ft per sec}$$

where D = total distance hoisted
R = running time per trip, sec

$$S = \frac{1090}{43 - \dfrac{8 + 6}{2}} = 30.28 \text{ ft per sec}$$

$$30.28 \times 60 = 1817 \text{ fpm maximum rope speed}$$

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[1] BRIGHT, G., *op. cit.*, p. 587.

[2] BRIGHT, *op. cit.*, p. 587.

Direct-current Motor. To select a d-c motor we must change part of the previous calculations for the hoisting cycle. The d-c motor is direct-connected to the drum shaft. This eliminates the effect of the gear included in the a-c motor determination. The load to be accelerated and retarded is

Skips	10,000 lb
Ore	10,000 lb
Rope	6,825 lb
Drums	64,000 lb
Sheaves	5,400 lb
Total	96,225 lb

a = 3.49 ft per sec², the same as before, and $-a$ = 5.05.

$$F_a = \frac{96{,}225}{32.2} \times 3.79 = 11{,}420 \text{ lb}$$
$$M_a = 11{,}420 \times 4 = 45{,}680 \text{ ft-lb}$$
$$F_r = \frac{96{,}225}{32.2} \times (-5.05) = -15{,}100 \text{ lb}$$
$$M_r = -15{,}100 \times 4 = -60{,}400 \text{ ft-lb}$$

SUMMATION OF MOMENTS—CASE 1*b* (DIRECT-CURRENT MOTOR)

Turns	0	*A*	*B*	*C*	*D*	43.4
		4.82	4.82	39.76	39.76	
Net moment	50,900	48,460	48,460	30,920	30,920	29,100
Friction moment	10,000	10,000	10,000	10,000	10,000	10,000
Accel. and ret. moment	45,680	45,680	......	......	−60,400	−60,400
Total moment = *M*	106,580	104,140	58,460	40,920	−19,480	−21,300
Time, sec	0	8	8	37	37	43
Horsepower	1,465	1,431	804	563	− 268	− 293

$$\text{Hp} = 0.01375 \times M$$

The drum speed is 72.3 rpm. The motor selected will have a speed of 73 rpm. For estimating the horsepower necessary to accelerate and retard the armature, we shall use the minimum value of 80 per cent of the normal rating. The estimated rms value is

$$\frac{1465 + 563}{2} = 1014, \text{ or } 1000 \text{ rms-hp}$$
$$1000 \times 0.8 = 800 \text{ hp, if accelerated in 1 sec}$$
$$^{800}\!/_{8} = 100 \text{ hp for accelerating armature}$$
$$\frac{800}{-6} = -133 \text{ or } 130 \text{ hp for retarding}$$
$$A = 1431 + 100 = 1531 \text{ hp}$$
$$D = -268 - 130 = -398 \text{ hp}$$

The formula for calculating the rms value for the d-c motor duty cycle is the same as for the a-c motor, except that the values for k and l must be changed. For the d-c motor these are $k = \frac{3}{4}$ and $l = \frac{1}{2}$.

$$\text{Rms-hp} = \sqrt{\frac{(1531^2 \times 8) + \left[\frac{804^2 + 563^2 + (804 \times 563)}{3} \times 29\right] + (398^2 \times 6)}{(\frac{3}{4} \times 8) + 29 + (\frac{3}{4} \times 6) + (\frac{1}{2} \times 8)}}$$

$$1531^2 \times 8 = 2{,}343{,}961 \times 8 = 18{,}751{,}688 \text{ hp}^2\text{-sec}$$

$$\frac{804^2 + 563^2 + 804 \times 563}{3} \times 29 = 472{,}012 \times 29 = 13{,}688{,}346 \text{ hp}^2\text{-sec}$$

$$398^2 \times 6 = 158{,}404 \times 6 = 950{,}424 \text{ hp}^2\text{-sec}$$

$$\text{Total} = 33{,}390{,}458 \text{ hp}^2\text{-sec}$$

$$\tfrac{3}{4} \times 8 + 29 + \tfrac{3}{4} \times 6 + \tfrac{1}{2} \times 8 = 43.5 \text{ sec}$$

$$\text{Rms-hp} = \sqrt{\frac{33{,}390{,}458}{43.5}} = 876$$

A d-c motor will stand two times the rated full-load torque. This means that a 875-hp motor would be necessary to meet the demands of the maximum point in the cycle, if there should be no voltage drop. Unless peculiar operating conditions requiring excessive braking must be met, or direct current were the only power available, there would be very little advantage in using a d-c motor. If it were used, it would have 73 rpm and would operate at 550 volts.

Cases 1*a*, 2*a*, *and* 2*b*. The calculations will not be given. The procedure is identical with that for case 1*b*. A summary of the data will be given later.

Case 3 (*Hoist Out of Balance*). The conditions are the same as before except as follows:

Ore to be hoisted = 1000 tons
Total lift = 1090 ft
Number of active turns = 43.4
Net operating time = 6 hr
Equivalent weight of one drum and gear = 35,200 lb
Equivalent of one sheave = 2700 lb

$$\text{Trips per min} = \frac{1000}{6 \times 5 \times 60} = \frac{5}{9}$$

$$\text{Time per trip} = \frac{60}{5/9} = 108 \text{ sec}$$

Actual running time of hoist for a round trip = 108 − 2 × 8 = 92 sec
Running time for one way = $^{92}\!/_{2}$ = 46 sec

$$\text{Equivalent full-speed hoisting time} = 46 - \frac{8 + 6}{2} = 39 \text{ sec}$$

$$\text{Full speed} = t_s = 39 - \frac{8 + 6}{2} = 32 \text{ sec}$$

$$v = 43.4 \div 39 = 1.113 \text{ rps, or } 66.9 \text{ rpm}$$

$$\text{Turns during acceleration} = \frac{1.113 \times 8}{2} = 4.45$$

$$\text{Turns during retardation} = \frac{1.113 \times 6}{2} = 3.34$$

Turns during constant speed = 1.113 × 32 = 35.61
Rope wound during acceleration = 4.45 × 8π = 112
Rope wound during retardation = 3.34 × 8π = 84
Rope wound during constant speed = 35.61 × 8π = 894

MOMENTS. UP LOAD

Same as for case 1*b* (60,000 ft-lb for all turns).

UP ROPE

0 turns (1090 ft × 2.5 lb) × 4 ft = 10,900 ft-lb
4.45 turns (978 ft × 2.5 lb) × 4 ft = 9,780 ft-lb
40.06 turns (84 ft × 2.5 lb) × 4 ft = 840 ft-lb
43.4 turns (0 ft × 2.5 lb) × 4 ft = 0 ft-lb

TOTAL MOMENTS

Turns	0	4.45	40.06	43.4
Load..................	60,000	60,000	60,000	60,000
Rope	10,900	9,780	840	0
Total.	70,900	69,780	60,840	60,000

Friction Moment. Sage[1] suggests that for a normally balanced hoist operating out of balance, 70 per cent of the friction for the balanced hoist should be used. Under case 1*b*, we found the friction moment to be 10,000 ft-lb. The friction moment for case 3 would be, approximately,

$$70\% \times 10{,}000 = 7000 \text{ ft-lb}$$

This gives the efficiency of the hoist as follows:

$$\frac{70{,}900 + 60{,}000}{2} = 65{,}450 \text{ ft-lb}$$

$$65{,}450 + 7000 = 72{,}450 \text{ ft-lb}$$

$$\text{Efficiency} = \frac{65{,}450}{72{,}450} \times 100 = 90.3 \text{ per cent}$$

ACCELERATION AND RETARDATION MOMENTS

The load to be handled is

Skip	5,000 lb
Ore....	10,000 lb
Rope (6825 ÷ 2)..	3,412 lb
Drum and gear	35,200 lb
Sheave	2,700 lb
Total	56,312 lb

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[1] SAGE, *op. cit.*, p. 10.

$$V = 1.113 \times 8\pi = 27.96 \text{ ft per sec}$$

$$a = \frac{27.96}{8} = 3.49 \text{ ft per sec}^2$$

$$F_a = \frac{56{,}312}{32.2} \times 3.49 = 6100 \text{ lb}$$

$$M_a = 6100 \times 4 = 24{,}400 \text{ ft-lb}$$

$$-a = \frac{27.96}{6} = 4.66 \text{ ft per sec}^2$$

$$F_r = \frac{56{,}312}{32.2} \times (-4.66) = -8150 \text{ lb}$$

$$M_r = -8150 \times 4 = -32{,}600 \text{ ft-lb}$$

SUMMATION OF MOMENTS—CASE 3

Turns	0	*A*	*B*	*C*	*D*	43.4
		4.45	4.45	40.06	40.06	
Net moment.	70,900	69,780	69,780	60,840	60,840	60,000
Friction moment.	7,000	7,000	7,000	7,000	7,000	7,000
Accel. and ret. moment	24,400	24,400	. . . . . .	. . . . . .	−32,600	−32,600
Total moment = M. . . .	102,300	101,180	76,780	67,840	35,240	34,400
Time, sec.	0	8	8	40	40	46
Horsepower.	1302	1288	978	864	448	438

$$\text{Hp} = \frac{2\pi \times 1.113}{550} \times M = 0.01271 \times M$$

A check on the calculations is

$$\text{Net work} = \frac{(10{,}000 + 5000 + 1362^*) \times 1090 \text{ ft}}{550} = 32{,}400 \text{ hp-sec}$$

$$\text{Total work} = 32{,}400 \div 90.3\% = 35{,}900 \text{ hp-sec}$$

Acceleration $(1288 \times \frac{8}{2})$.	5,152 hp-sec
Full speed $\left(\frac{977 + 864}{2} \times 32\right)$	29,456 hp-sec
Retardation $(448 \times \frac{6}{2})$	1,344 hp-sec
Total	35,952 hp-sec

$$\text{Estimating, rms-hp} = \frac{1302 + 864}{2} = 1084, \text{ or } 1100$$

To accelerate in 1 sec = 1100 × 1.6 = 1760 hp

To accelerate rotor = $\frac{1760}{8}$ = 220 hp

To retard rotor = $\frac{1760}{6}$ = 293 hp

A = 1288 + 220 = 1508 hp

D = 448 + 293 = 741 hp

---

* 1362 lb = one-half the weight of the rope raised.

$$\text{Rms-hp} = \sqrt{\frac{(1508^2 \times 8) + \left[\frac{978^2 + 864^2 + (978 \times 864)}{3} \times 32\right] + (741^2 \times 6)}{2[(\frac{1}{2} \times 8) + 32 + (\frac{1}{2} \times 6) + (\frac{1}{4} \times 8)]}}$$

$$\begin{aligned}
1508^2 \times 8 &= 2{,}274{,}064 \times 8 = 18{,}192{,}512 \text{ hp}^2\text{-sec} \\
\frac{978^2 + 864^2 + 978 \times 864}{3} \times 32 &= 848{,}650 \times 32 = 27{,}156{,}800 \text{ hp}^2\text{-sec} \\
741^2 \times 6 &= 549{,}081 \times 6 = \underline{3{,}294{,}486} \text{ hp}^2\text{-sec} \\
\text{Total} &= 48{,}643{,}798 \text{ hp}^2\text{-sec} \\
2(\tfrac{1}{2} \times 8 + 32 + \tfrac{1}{2} \times 6 + \tfrac{1}{4} \times 8) &= 82 \text{ sec}^{1} \\
\text{Rms-hp} = \sqrt{\frac{48{,}643{,}798}{82}} &= 770
\end{aligned}$$

If the cooling time equals 43 sec (no allowance being made for lowering, and a rest period of 16 sec used), the rms = 1064 hp. This (the 1064 hp) is but 72 hp greater than that found under case 1*b*. The motor selected there would prove satisfactory.

The following table sums up the data from the three cases.

Case	Maximum drum speed, ft. per sec.	Horsepower						
			*A**	*B*	*C*	*D**		
		Start of accel.	End of accel.	Start of const. speed	End of const. speed	Start of retard.	End of retard.	R.m.s. value of cycle
1*a*	10.9	355	411	267	226	196	113	288
1*b*	30.3	1480	1650	804	563	−564	−319	992
2*a*	13.6	469	543	333	284	218	106	377
2*b*	24.25	1084	1207	650	447	−296	−119	711
3	27.96	1302	1508	978	864	741	438	770

* Includes estimated power to accelerate and retard motor rotor.

A wound-rotor induction motor, to work satisfactorily under the conditions imposed by the three cases, should have the following qualifications:

Voltage = 2200
Cycles = 60
Phase = 3
Horsepower = 1100*
Accelerating time = 8 sec
Retarding time = 6 sec
Rest period = 8 sec

---

[1] The corrected time for hoisting is $\frac{1}{2} \times 8 + 32 + \frac{1}{2} \times 6 + \frac{1}{4} \times 8 = 41$ sec. The time for lowering the empty skip is also taken as 41 sec. The rms value of the motor should be based on both operations as heating and cooling occur during both hoisting and lowering. (SAGE, R. S., personal communication.)

Gear ratio = 6.2:1[1]
Rpm of motor = 450[1]
Maximum temperature rise = 40°C
Pull-out torque = 2

Figure 143 shows the moment diagram for case 1*b*. Figure 144 shows the duty cycle for case 1*b*.

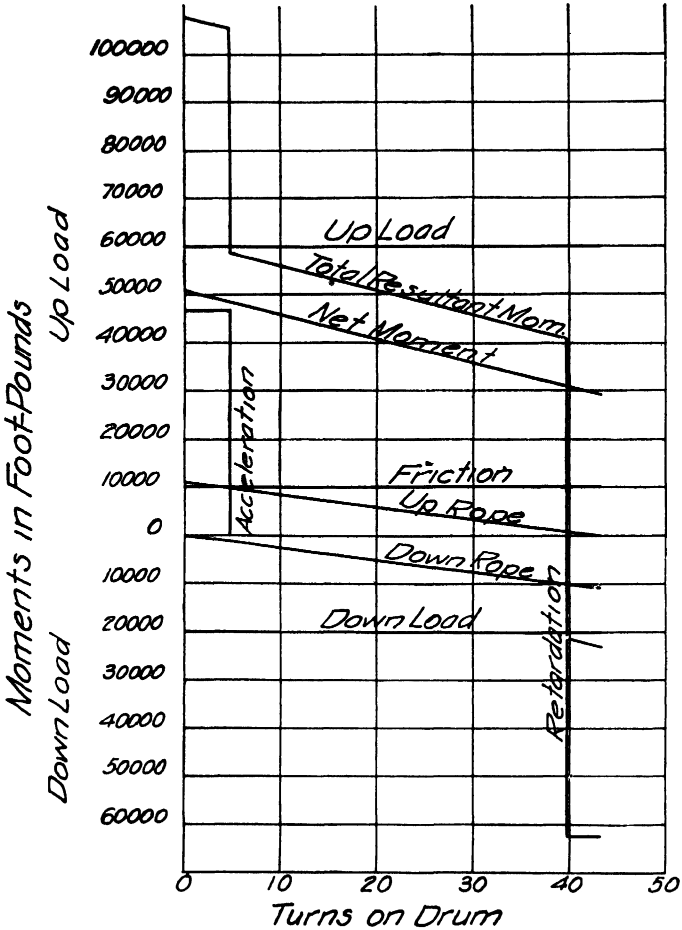

FIG. 143. Moment diagram for case 1*b*.

In the selection of the above motor no allowance was specifically made for the momentarily unbalanced condition when hoisting starts; *i.e.*, the skip in the dumping position exerts only about one-half its effective weight toward balancing the ascending load. This means that the loaded skip for a small distance and slight period of time must be lifted out of balance. If the motor is sufficiently large to make one complete trip out of balance, the above condition will not prove serious. (See article by Bright, page 606.) The motor selected meets this requirement.

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[1] The nearest frame size is 1250 hp at 350 rpm. This would give a gear ratio of 4.84:1 (see p. 355).

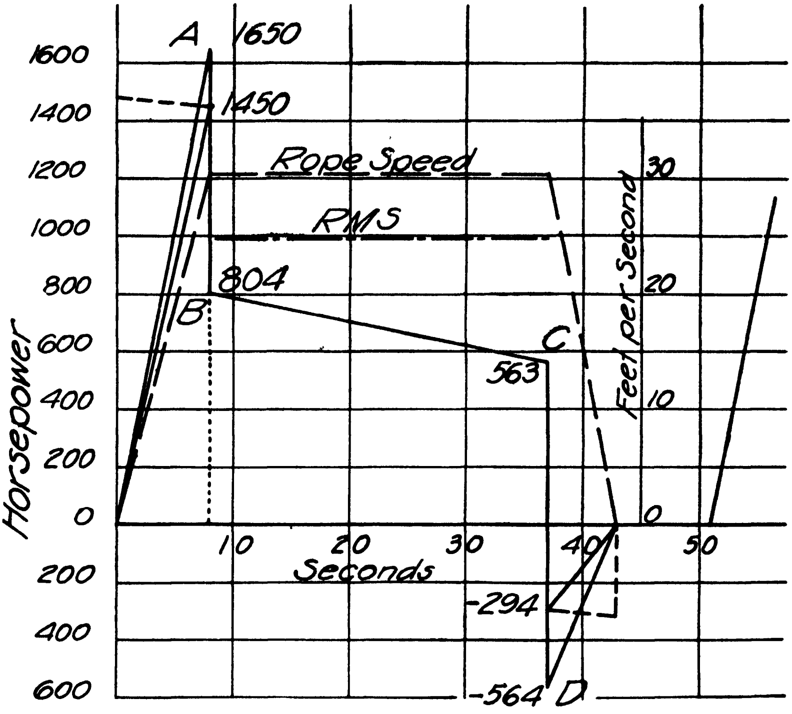

FIG. 144. Duty-cycle diagram for case 1*b*.

EXAMPLE 2[1]

The following example illustrates a somewhat different procedure from the previous example. The hoisting conditions are

Coal-mine hoist—vertical shaft:
Wanted—2500 short tons per 8-hr day
Total lift to dumping roller = 410 ft
Weight of self-dumping cage, each = 11,000 lb
Weight of car = 3000 lb
One car per cage
Weight of coal per car = 6000 lb
Size of rope, diameter = 1.5 in.
Weight of rope per side (3.55 × 410) = 1420 lb

---

[1] SAGE, *op. cit.*, p. 17.

Drums, conical, working radii:

$r_1 = 4$ ft
$r_2 = 5$ ft

WR^2 of drums = 700,000 ft-lb units
Weight of each head sheave = 3300 lb
Number of active turns ($T = 410 \div 9\pi$) = 14.5
Select accelerating time (t_a) = 6 sec
Select retarding time (t_b) = 5 sec
Assume actual net operating time per shift = 7 hr
Dumping and loading time per trip (rest period), t_o = 6 sec
The method of procedure is then as follows:

$$\text{Trips per min} = \frac{2500}{7 \times 3 \times 60} = 2$$

$$\text{Time per trip} = \frac{60}{2} = 30 \text{ sec}$$

$$\text{Equivalent full-speed hoisting time} = \left(30 - 6 - \frac{6+5}{2}\right) = 18.5 \text{ sec}$$

Maximum drum speed $14.5 \div 18.5 = 0.784$ rps, or 47.0 rpm

$$t_s = 18.5 - \frac{6+5}{2} = 13 \text{ sec} = \text{time of full speed}$$

$$p = \text{radial pitch} = \frac{r_2 - r_1}{T} = \frac{5-4}{14.5} = 0.069 \text{ ft}$$

$$T_a = \frac{6 \times 0.784}{2} = 2.35 \text{ turns} = \text{turns during acceleration}$$

$$T_b = \frac{5 \times 0.784}{2} = 1.96 \text{ turns} = \text{turns during retardation}$$

$T_s = T - (T_a + T_b)$ turns $= 14.5 - (2.35 + 1.96) = 10.19$
= turns during full speed
r_{ua} = radius of up side at end of acceleration $= r_1 + pT_a$
$= 4 + (0.069 \times 2.35) = 4.162$ ft
r_{da} = radius down side at end of acceleration $= r_2 - pT_a$
$= 5 - (0.069 \times 2.35) = 4.838$ ft
r_{ub} = radius of up side at beginning of retardation $= r_2 - pT_b$
$= 5 - (0.069 \times 1.96) = 4.865$ ft
r_{db} = radius down side at beginning of retardation $= r_1 + pT_b$
$= 4 + (0.069 \times 1.96) = 4.135$ ft
W_{ua} = weight of rope wound on during acceleration
$= \pi T_a(r_1 + r_{ua}) \times$ weight per foot
$= \pi \times 2.35(4 + 4.162) \times 3.55 = 214$ lb
W_{da} = weight of rope wound off during acceleration
$= \pi T_a(r_2 + r_{da}) \times$ weight per foot
$= \pi \times 2.35(5 + 4.838) \times 3.55 = 258$ lb
W_{ub} = weight of rope wound on during retardation
$= \pi T_b(r_2 + r_{ub}) \times$ weight per foot
$= \pi \times 1.96(5 + 4.865) \times 3.55 = 216$ lb

W_{db} = weight of rope wound off during retardation
$= \pi T_b(r_1 + r_{db}) \times$ weight per foot
$= \pi \times 1.96(4 + 4.135) \times 3.55 = 178$ lb
E = percentage efficiency of 2nd-motion hoist (see page 298)

$$= \frac{0.95w}{w + kW}$$

$$= \frac{0.95 \times 6000}{6000 + 0.04(6000 + 2 \times 14{,}000 + 1420)} = 76.0\%$$

Calculation of Moments

Beginning Acceleration

$U = (6000 + 3000 + 11{,}000) \times 4 = 80{,}000$ ft-lb (up load)
$R_{u1} = 1420 \times 4 = 5680$ ft-lb (up rope)
$D = (3000 + 11{,}000) \times 5 = 70{,}000$ ft-lb (down load)

End of Acceleration

$U_a = (6000 + 3000 + 11{,}000) \times 4.162 = 83{,}240$ ft-lb (up load)
$R_{ua} = (1420 - 214) \times 4.162 = 5019$ ft-lb (up rope)
$D_a = (3000 + 11{,}000) \times 4.838 = 67{,}732$ ft-lb (down load)
$R_{da} = 258 \times 4.838 = 1248$ ft-lb (down rope)

Beginning of Retardation

$U_b = (6000 + 3000 + 11{,}000) \times 4.865 = 97{,}300$ ft-lb (up load)
$R_{ub} = 216 \times 4.865 = 1051$ ft-lb (up rope)
$D_b = (3000 + 11{,}000) \times 4.135 = 57{,}890$ ft-lb (down load)
$R_{db} = (1420 - 178) \times 4.135 = 5136$ ft-lb (down rope)

End of Retardation

$U_2 = (6000 + 3000 + 11{,}000) \times 5 = 100{,}000$ ft-lb (up load)
$D_2 = (3000 + 11{,}000) \times 4 = 56{,}000$ ft-lb (down load)
$R_{d2} = 1420 \times 4 = 5680$ ft-lb (down rope)

Friction

$$F = \frac{w(r_1 + r_2)}{2} \times \left(\frac{1 - E}{E}\right)$$

$$= \frac{6000(4 + 5)}{2} \times \left(\frac{1 - 0.76}{0.76}\right) = 8500 \text{ ft-lb}$$

Acceleration Moment A_M

Up load, up rope, and head sheave is

$$\frac{(w + w_s + w_r + w_h)2\pi r_{ua}^2 \times \text{rps}}{32.2 \times t_a}$$

$$= \frac{(6000 + 3000 + 11{,}000 + 1420 + 3300) \times 2\pi \times 4.162^2 \times 0.784}{32.2 \times 6}$$

$= 10{,}950$ ft-lb

$$\text{Down rope} = \frac{w\pi(r_1^2 + r_2^2) \times \text{rps}}{32.2 \times t_a}$$

$$= \frac{1420 \times \pi(4^2 + 5^2) \times 0.784}{32.2 \times 6} = 743 \text{ ft-lb}$$

$$\text{Down load and sheave} = \frac{(w_s + w_h) \times 2\pi \times r_2^2 \times \text{rps}}{32.2 \times t_a}$$

$$= \frac{(11{,}000 + 3000 + 3300) \times 2\pi \times 5^2 \times 0.784}{32.2 \times 6} = 11{,}050 \text{ ft-lb}$$

$$\text{Drums and gear} = \frac{WR^2 \times 2\pi \times \text{rps}}{32.2 \times t_a} = \frac{700{,}000 \times 2\pi \times 0.784}{32.2 \times 6}$$

$$= 17{,}900 \text{ ft-lb}$$

$$\text{Total } A_M = 10{,}950 + 743 + 11{,}050 + 17{,}900 = 40{,}643 \text{ ft-lb}$$

Retardation Moment B_M

$$\text{Up load and head sheave} = \frac{(w + w_s + w_h)2\pi r_2^2 \times \text{rps}}{32.2 \times t_b}$$

$$= \frac{(6000 + 3000 + 11{,}000 + 3300) \times 2\pi \times 5^2 \times 0.784}{32.2 \times 5}$$

$$= 17{,}850 \text{ ft-lb}$$

$$\text{Up rope} = \frac{w_r \times \pi(r_1^2 + r_2^2) \times \text{rps}}{32.2 \times t_b}$$

$$= \frac{1420 \times \pi(4^2 + 5^2) \times 0.784}{32.2 \times 5} = 892 \text{ ft-lb}$$

$$\text{Down load, rope, and sheave} = \frac{(w_s + w_r + w_h)2\pi r_{db}^2 \times \text{rps}}{32.2 \times t_b}$$

$$= \frac{(11{,}000 + 3000 + 1420 + 3300) \times 2\pi \times 4.135^2 \times 0.784}{32.2 \times 5}$$

$$= 9800 \text{ ft-lb}$$

$$\text{Drums and gear} = \frac{WR^2 \times 2\pi \times \text{rps}}{32.2 \times t_b} = \frac{700{,}000 \times 2\pi \times 0.784}{32.2 \times 5}$$

$$= 21{,}500 \text{ ft-lb}$$

$$\text{Total } B_M = 17{,}850 + 892 + 9800 + 21{,}500 = 50{,}042 \text{ ft-lb}$$

Resultant Moment

Acceleration (A_M) = 40,643
Retardation (B_M) = 50,042
Beginning accel. = 80,000 + 5680 − 70,000 = 15,680
End of accel. = 83,240 + 5019 − 67,732 − 1248 = 19,279
Beginning ret. = 97,300 + 1051 − 57,890 − 5136 = 35,325
End of ret. = 100,000 − 56,000 − 5680 = 38,320
Friction = 8,500

The horsepower (see Fig. 142) at A, B, C, and D can now be found.

$$\text{Hp at } A = 40{,}643 + 8500 + \frac{19{,}279 + (2 \times 15{,}680)}{3} \times \frac{2\pi \times 0.784}{550}$$

$$= 592 \text{ hp}$$

$$\text{Hp at } B = 19{,}279 + 8500 \times \frac{2\pi \times 0.784}{550} = 249 \text{ hp}$$

$$\text{Hp at } C = 35{,}325 + 8500 \times \frac{2\pi \times 0.784}{550} = 393 \text{ hp}$$

$$\text{Hp at } D = -50{,}042 + 8500 + \frac{35{,}325 + (2 \times 38{,}320)}{3} \times \frac{2\pi \times 0.784}{550}$$
$$= -38 \text{ hp}$$

Estimating the rms value of this cycle by inspection to be 500 hp for induction-motor drive and assuming the maximum power to accelerate the armature in 1 sec to be 1.6 times the assumed rating, we have

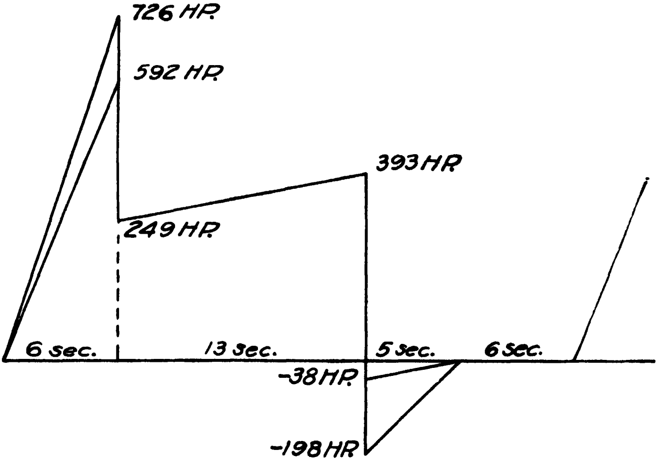

FIG. 145. Power diagram for conical-drum duty cycle. (*Sage, "Electric Drive for Mine Hoists."*)

$$\text{Acceleration peak} = 592 + 134 = 726 \text{ hp}$$
$$\text{Retardation peak} = -38 - 160 = -198 \text{ hp}$$

$$\text{Rms-hp} = \sqrt{\frac{(726^2 \times 6) + \left[\dfrac{249^2 + 393^2 + (249 \times 393)}{3} \times 13\right] + (198^2 \times 5)}{6/2 + 13 + 5/2 + 6/4}}$$
$$= 486 \text{ hp}$$

Selecting a standard 500-hp 450-rpm motor and checking back, using the actual WR^2 of its rotor, the horsepower to accelerate is 127 and to retard 152, and the rms-hp is 482. The motor selected should be capable of operating continuously at the rms output given above with a temperature rise not exceeding 40°C. As the full speed of the drum is 47 rpm, it will be satisfactory to use a single reduction of gearing with the motor of the speed selected.

The calculations are checked as follows:

$$\text{Net work done in shaft} = \frac{6000 \times 410}{550} = 4473 \text{ hp-sec}$$

Area under the load diagram is

Acceleration ($^{592}/_{2} \times 6$)	1776 hp-sec
Constant speed $\left(\frac{249 + 393}{2} \times 13\right)$	4173 hp-sec
Retardation $\left(\frac{-38}{2} \times 5\right)$	−95 hp-sec
Total...	5854 hp-sec

$E = {}^{4473}/_{5854} = 0.76$, which checks the mechanical efficiency previously found.

Figure 145 shows the power diagram for the conical-drum duty cycle.

EXAMPLE 3

The following example will illustrate the determination of the duty cycle for a cylindroconical drum.[1] The conditions are the same as for Example 2, with the following changes:

Small diameter = 8 ft
Large diameter = 10 ft
Active turns on conical portion = 4
WR^2 of drum = 800,000 lb-ft^2
Time for acceleration = 6 sec
Time for retardation = 5 sec
Time for full speed = 13 sec

The first step is to determine the distribution of rope turns upon the drum. Let Z = rps of drum when at full speed.

$$\text{Rope wound on during acceleration} = \frac{6Z}{2} \times 8\pi$$

$$\text{Rope wound on during retardation} = \frac{5Z}{2} \times 10\pi$$

$$\text{Rope wound on cone} = \frac{10 + 8}{2} \times 4\pi$$

Rope wound on large cylinder of drum, full speed = $(13Z - 4) \times 10\pi$

$$\left(\frac{6Z}{2} \times 8\pi\right) + \left(\frac{5Z}{2} + 10\pi\right) + \left(\frac{10 + 8}{2} \times 4\pi\right) + (13Z - 4) \times 10\pi = 410$$

$$179Z = \frac{410}{\pi} + 4$$

$$Z = 0.75 \text{ rps}$$

$$\text{Turns on small cylinder} = T_a = \frac{6 \times 0.75}{2} = 2.25$$

$$\text{Turns during retarding} = T_b = \frac{5 \times 0.75}{2} = 1.88$$

Turns on large cylinder = $T_b + 13Z - 4 = 7.63$

Total turns on drum = $T_a + T_b + 13Z = 2.25 + 1.88 + 9.75 = 13.88$

[1] SAGE, *op. cit.*, p. 19.

Rope hanging from the drum:

Revolutions of drum	Up side		Down side	
	Feet	Pounds	Feet	Pounds
0	410	1420	0	0
2.25	353	1220	71	245
6.25	240	830	197	680
7.63	197	680	240	830
11.63	71	245	353	1220
12 01	58	200	364	1260
13 88	0	0	410	1420

MOMENTS (FT.-LB.)

Revolution of drum	Rope		Load	
	Up side	Down side	Up side	Down side
0	1420 × 4 = 5680	0 × 5 = 0	20,000 × 4 = 80,000	14,000 × 5 = 70,000
2.25	1220 × 4 = 4880	245 × 5 = 1225	20,000 × 4 = 80,000	14,000 × 5 = 70,000
6 25	830 × 5 = 4150	680 × 5 = 3400	20,000 × 5 = 100,000	14,000 × 5 = 70,000
7 63	680 × 5 = 3400	830 × 5 = 4150	20,000 × 5 = 100,000	14,000 × 5 = 70,000
11 63	245 × 5 = 1225	1220 × 4 = 4880	20,000 × 5 = 100,000	14,000 × 4 = 56,000
12 01	200 × 5 = 1000	1260 × 4 = 5040	20,000 × 5 = 100,000	14,000 × 4 = 56,000
13.88	0 × 5 = 0	1420 × 4 = 5680	20,000 × 5 = 100,000	14,000 × 4 = 56,000

Acceleration Moment A_M

$$\text{Up load, up rope, and head sheave} = \frac{(20{,}000 + 1420 + 3000) \times 2\pi \times 4^2 \times 0.75}{32.2 \times 6}$$

$$= 9550 \text{ ft-lb}$$

$$\text{Down rope and sheave} = \frac{(14{,}000 + 3000) \times 2\pi \times 5^2 \times 0.75}{32.2 \times 6}$$

$$= 10{,}400 \text{ ft-lb}$$

$$\text{Drum}^1 \text{ and down rope} = \frac{(50{,}000 + 1770) \times 2\pi \times 4^2 \times 0.75}{32.2 \times 6}$$

$$= 20{,}000 \text{ ft-lb}$$

$$\text{Total } A_M = 9550 + 10{,}400 + 20{,}000 = 39{,}950 \text{ ft-lb}$$

Retardation Moment B_M

$$\text{Up load and sheave} = \frac{(20{,}000 + 3000) \times 2\pi \times 5^2 \times 0.75}{32.2 \times 5} = 16{,}800 \text{ ft-lb}$$

---

[1] The WR^2 of the drum = 800,000. At a 4-ft radius the equivalent weight = $800{,}000/4^2$ = 50,000 lb.

$$\text{Down load, down rope, and sheave} = \frac{(14{,}000 + 1420 + 3000) \times 2\pi \times 4^2 \times 0.75}{32.2 \times 5}$$

$$= 8630 \text{ ft-lb}$$

$$\text{Drum and up rope} = \frac{(50{,}000 + 1770) \times 2\pi \times 4^2 \times 0.75}{32.2 \times 5}$$

$$= 24{,}300 \text{ ft-lb}$$

$$\text{Total } B_M = 16{,}800 + 8630 + 24{,}300 = 49{,}730 \text{ ft-lb}$$

Acceleration Due to Cone[1]—Moments

$$\text{At revolution 2.25} = \frac{(20{,}000 + 1220 + 3000) \times 2\pi \times 0.75(5 - 4) \times 4}{32.2 \times 4 \div 0.75}$$

$$= 2670 \text{ ft-lb}$$

$$\text{At revolution 6.25} = \frac{(20{,}000 + 830 + 3000) \times 2\pi \times 0.75(5 - 4) \times 5}{32.2 \times 4 \div 0.75}$$

$$= 3180 \text{ ft-lb}$$

Retardation Due to Cone—Moments

$$\text{At revolution 7.63} = \frac{(14{,}000 + 830 + 3000) \times 2\pi \times 0.75(5 - 4) \times 5}{32.2 \times 4 \div 0.75}$$

$$= 3180 \text{ ft-lb}$$

$$\text{At revolution 11.63} = \frac{(14{,}000 + 1220 + 3000) \times 2\pi \times 0.75(5 - 4) \times 4}{32.2 \times 4 \div 0.75}$$

$$= 2000 \text{ ft-lb}$$

Friction Load—Moment.[2] Assume mechanical efficiency to be 76 per cent.

$$\text{Total work done} = \frac{6000 \text{ lb} \times 410 \text{ ft}}{550} = 4475 \text{ hp-sec}$$

$$\text{Energy lost in friction} = 4475 \times \left(\frac{1 - 0.76}{0.76}\right) = 1415 \text{ hp-sec}$$

$$\text{Average hp friction} = \frac{1415}{18.5 \text{ sec}} = 76.5 \text{ hp}$$

$$\text{Average hp-friction torque} = \frac{76.5}{2\pi \times 0.75 \div 550} = 8900 \text{ ft-lb}$$

$$\text{Hp load} = \text{moment} \times \frac{2\pi \times 0.75}{550} = 0.00856 \times M$$

---

[1] The acceleration on the cone is obtained from $a = (v_1 - v_2) \div t$, where v_1 = velocity due to maximum diameter = $2 \times \pi \times r_1 \times$ rps
v_2 = velocity due to minimum diameter = $2 \times \pi \times r_2 \times$ rps
t = time of travel on cone = $\dfrac{\text{number of turns on cone}}{\text{rps}}$.
$F = \dfrac{w}{g}a$, the same as in previous examples
$M = F \times r$

[2] The friction moment may also be determined in the following way. This does not check the calculations used, because the assumed efficiency of 80 per cent is used. The difference is not serious.

The total area in moment turns is found, from the net up and down moments,

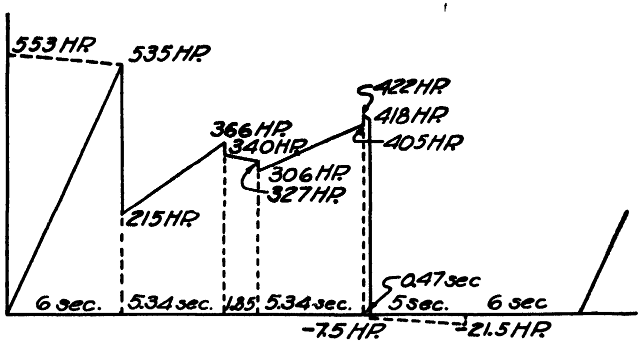

FIG. 146. Power diagram for cylindroconical-drum duty cycle. (*Sage*, "*Electric Drive for Mine Hoists.*")

The data are tabulated in the table entitled Cylindroconical Drum Figure 146 shows the duty cycle for the cylindroconical drum.

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by finding the average moment turn for each change in the duty cycle.

Turns	0	2.25	6.25	7.63	11.63	12.01	13.88
Up load	80,000	80,000	100,000	100,000	100,000	100,000	100,000
Up rope	5,680	4,880	4,150	3,400	1,225	1,000	0
Total	85,680	84,880	104,150	103,400	101,225	101,000	100,000
Down load	70,000	70,000	70,000	70,000	56,000	56,000	56,000
Down rope	0	1,225	3,400	4,150	4,880	5,040	5,680
Total	70,000	71,225	73,400	74,150	60,880	61,040	61,680
Net	15,680	13,655	30,750	29,250	41,345	39,960	38,320

$$\frac{15{,}680 + 13{,}655}{2} \times 2.25 = 33{,}000$$

$$\frac{13{,}655 + 30{,}750}{2} \times 4 = 88{,}808$$

$$\frac{30{,}750 + 29{,}250}{2} \times 1.38 = 41{,}400$$

$$\frac{29{,}250 + 41{,}345}{2} \times 4 = 141{,}188$$

$$\frac{41{,}345 + 39{,}960}{2} \times 0.38 = 15{,}448$$

$$\frac{39{,}960 + 38{,}320}{2} \times 1.87 = 73{,}192$$

$$\text{Area in moment turns} = 393{,}036$$

$$\text{Average ordinate} = \frac{393{,}036}{13.88} = 28{,}315 \text{ ft-lb}$$

$$28{,}315 \div 80\% = 35{,}394 \text{ ft-lb}$$

$$\text{Friction moment} = 35{,}394 - 28{,}315 = 7079 \text{ ft-lb}$$

SUMMATION OF MOMENTS—CYLINDROCONICAL DRUM

Turns	0	2.25	2.25	6.25	6.25	7.63	7.63	11.63	11.63	12.01	12.01	13.88
Up load	80,000	80,000	80,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Up rope	5,680	4,880	4,880	4,150	4,150	3,400	3,400	1,225	1,225	1,000	1,000	0
Friction	8,900	8,900	8,900	8,900	8,900	8,900	8,900	8,900	8,900	8,900	8,900	8,900
Acceleration	39,950	39,950	2,670	3,180	......	......	......	......	......	......	......	......
Total (+) moment	134,530	133,730	96,450	116,230	113,050	112,300	112,300	110,125	110,125	109,900	109,900	108,900
Down load	70,000	70,000	70,000	70,000	70,000	70,000	70,000	56,000	56,000	56,000	56,000	56,000
Down rope	0	1,225	1,225	3,400	3,400	4,150	4,150	4,880	4,880	5,040	5,040	5,680
Retardation	......	......	......	......	......	......	2,450	2,000	......	......	49,730	49,730
Total (−) moment	70,000	71,225	71,225	73,400	73,400	74,150	76,600	62,880	60,880	61,040	110,770	111,410
Net moment	64,530	62,505	25,225	42,830	39,650	38,150	35,700	47,245	49,245	48,860	−870	−2,510
Horsepower	553	535	216	366	340	327	306	405	422	418	−7.5	−21.5
Time, sec.	0	6	6	11.34	11.34	13.19	13.19	18.53	18.53	19	19	24

EXAMPLE 4

The present example will illustrate the choice between a large a-c motor, and a d-c motor with voltage control by either a motor-generator-flywheel set or a synchronous motor-generator set.[1]

The conditions are such that on one hand the choice lies between two a-c motors, one a 1500-hp motor with 1875 hp rating at 50°C; the other a 2000-hp motor with 2350 hp rating at 50°C. The margin of safety for the smaller is insufficient, even with a pull-out of 2, and a 50-deg rise, if the assumed conditions are exceeded or strictly adhered to. On the other hand, the 2000-hp motor required a very considerable amount of power to accelerate and retard its rotor. The power calculations for the 2000-hp motor show that the rms value of the cycle is practically equal to the capacity of the motor at 50° rise (2350 hp). With either motor the power required during the retardation period is too great to be absorbed by mechanical brakes. If dynamic braking is accomplished with the alternating-current motors the energy is all lost in the rheostat.

The calculations for the 1500-hp motor are given in Example 4*a* and for the 2000-hp motor in example 4*b*. Examples 4*c* and 4*d* give the calculations for the d-c motor and control equipment. The selection of a synchronous motor-generator set is illustrated in Example 4*e*.

EXAMPLE 4a

Power: 2200 volts; three-phase; 60 cycles.

General data:

Depth of shaft = 500 ft
Angle of slope = 90 deg

$$\text{Maximum rope speed} = \frac{500}{16 - 5} = 45.5 \text{ ft per sec}$$
$$= 2730 \text{ ft per min}$$

Net weight of load = 5000 lb
Maximum drum speed = 108.5 rpm = 1.81 rps
Weight of cage, skip = 12,000 lb
Size of rope = $1\frac{3}{8}$-in.
Weight = 3.65 lb per ft
Weight of car = 2500 lb
Size and shape of drums = 8-ft-diameter cylindrical
Balanced, unbalanced
WR^2 drums, gears, sheaves = 400,000 lb-ft^2
Maximum capacity per day = 3000 tons
Angular velocity = $2\pi \times 1.81 = 11.38$
Trips per hour = 171
Time for caging = 5 sec
Running time per trip = 16 sec
Total time per trip = 21 sec

Time

Acceleration = 5 sec
Retardation = 5 sec

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[1] BRIGHT, G., *op. cit.*, pp. 590–605.

Constant speed = 6 sec
Distance during acceleration = 113.8 ft
Distance during retardation = 113.8 ft
Distance during constant speed = 272.4 ft
Angular velocity = 11.38
Angular acceleration = 2.275
Angular retardation = 2.275

Inertia

Drums, gears, sheaves	12,500
Motor ($48,500 \div 32 \times 3.22^2$)	15,600
Load ($38,750 \div 32 \times 4^2$)	19,375
Total inertia	47,475

Angular-acceleration moment = 47,475 × 2.275 = 108,000
Angular-retardation moment = 47,475 × 2.275 = 108,000
Friction assumed at 7.5% of 38,750 = 2900 lb
Moment due to friction = 2900 × 4 = 11,600

The inertia is obtained as follows:

Load	5,000 lb
2 cages	24,000 lb
2 cars	5,000 lb
Ropes	4,750 lb
Total	38,750 lb

The WR^2 of drum, gears, sheaves $= 400,000 \text{ lb-ft}^2$

$$\text{The inertia} = \frac{400,000}{32} = 12,500$$

Load Diagram

		Ft-lb	Hp
1.	$(5000 + 1825) \times 4 + 11,600 + 108,000 =$	146,900	3030
2.	$(5000 + 995) \times 4 + 11,600 + 108,000 =$	143,580	2965
3.	$(5000 + 995) \times 4 + 11,600 =$	35,580	735
4.	$(5000 - 995) \times 4 + 11,600 =$	27,620	572
5.	$(5000 - 995) \times 4 + 11,600 - 108,000 =$	−80,380	−1660
6.	$(5000 - 1825) \times 4 + 11,600 - 108,000 =$	−83,700	−1730

Capacity of Hoist Motor

$$\begin{array}{r} 44,800,000 \\ 2,595,000 \\ \underline{14,320,000} \\ 17\frac{2}{3})\overline{61,715,000} \\ 3,495,000 \end{array} \quad \text{Rms} = \sqrt{\frac{\dfrac{3030^2 + 2965^2}{2} \times 5 + \dfrac{735^2 + 572^2}{2} \times 6 + \dfrac{660^2 + 1730^2}{2} \times 5}{16 + \frac{5}{3}}} = 1870 \text{ hp}$$

Input with Rheostat Control

Acceleration = [(3030 + 2965) ÷ 2] × 5 = 15,000 hp-sec
Constant speed = [(735 + 572) ÷ 2] × 6 = 3,921 hp-sec
Retardation = [(1660 + 1730) ÷ 2] × 5 = 8,475 hp-sec
Total input to hoist = 27,396 hp-sec

Average efficiency of motor = 88 % Input to motor = 31,100 hp-sec
Input to motor = 6.5 kw-hr Kw-hr per ton = 2.6
Shaft hp-sec = 5000 × 500 ÷ 550 = 4550

$$\text{Over-all efficiency} = \frac{\text{shaft hp-sec}}{\text{total hp-sec}} = \frac{4550}{31{,}100} = 14.8\%$$

The input to the motor in kilowatt-hours is obtained by converting horsepower-seconds into kilowatt-hours.

$$\frac{31{,}100}{3600} \times 0.75 = 6.5 \text{ kw-hr}$$

The kilowatt-hours per ton is found from

$$\frac{6.5 \text{ kw-hr}}{2.5 \text{ tons per trip}} = 2.6 \text{ kw-hr}$$

The rms of the above cycle is 1870 hp. The 50° rating of a 1500-hp motor is 1875 hp. As previously stated the margin of safety is not sufficient for excessive operating conditions.

EXAMPLE 4b

The data for Example 4*b* down to, and including, the item *Time* is the same as for Example 4*a* and will not be repeated. The variation between the two calculations is caused by the difference in the WR^2 of the two rotors.

Inertia

Drums, gears, sheaves	12,500
Motor ($91{,}000 \div 32 \times 3.22^2$)	29,500
Load ($38{,}750 \div 32 \times 4^2$)	19,375
Total inertia	61,375

Angular-acceleration moment = 61,375 × 2.275 = 139,500
Angular-retardation moment = 61,375 × 2.275 = 139,500
Friction assumed at 7.5 per cent of 38,750 = 2900 lb
Moment due to friction = 2900 × 4 = 11,600

Load Diagram

	Ft-lb	Hp
1. $(5000 + 1825) \times 4 + 11{,}600 + 139{,}500 =$	178,400	3690
2. $(5000 + 995) \times 4 + 11{,}600 + 139{,}500 =$	175,080	3620
3. $(5000 + 995) \times 4 + 11{,}600 =$	35,580	735
4. $(5000 - 995) \times 4 + 11{,}600 =$	27,620	572
5. $(5000 - 995) \times 4 + 11{,}600 - 139{,}500 =$	−111,880	−2310
6. $(5000 - 1825) \times 4 + 11{,}600 - 139{,}500 =$	−115,200	−2380

Capacity of Hoist Motor

$$\text{Rms} = \sqrt{\frac{\frac{3960^2 + 3620^2}{2} \times 5 + \frac{735^2 + 572^2}{2} \times 6 + \frac{2310^2 + 2380^2}{2} \times 5}{16 + 5/3}}$$

$$\begin{array}{r} 66{,}500{,}000 \\ 2{,}595{,}000 \\ \underline{27{,}500{,}000} \\ 17\tfrac{2}{3}\overline{)96{,}595{,}000} \\ 5{,}470{,}000 \end{array} \quad = 2340 \text{ hp}$$

Input with Rheostatic Control

Acceleration [(3690 + 3620) ÷ 2 × 5]	18,250 hp-sec
Constant speed [(735 + 572) ÷ 2 × 6]	3,921 hp-sec
Retardation [(2310 + 2380) ÷ 2 × 5]	11,725 hp-sec
Total input to hoist	33,896 hp-sec

Average efficiency of motor = 88% Input to motor = 38,500 hp-sec.
Input to motor = 7.98 kw-hr Kw-hr per ton = 3.19
Shaft hp-sec, 5000 × 500 ÷ 550 = 4550

$$\text{Over-all efficiency} = \frac{4550}{38{,}500} = 11.8\%$$

EXAMPLE 4c

The same problem is used here as for the a-c motors. The WR^2 of the drum and sheaves is only 300,000 because the motor is direct-connected, thus eliminating the inertia of the gears. The WR^2, or weight, of the d-c motor armature is far greater than the a-c rotors. However, as the motor is direct-connected, the equivalent inertia at the drum is much less. The friction is less, being taken at 5.5 per cent of the suspended weight instead of 7.5 per cent for the geared hoist.

The data up to and including the item *Time*, with the exception of the WR^2 of the drum, etc., are the same as in the previous solution.

Inertia

Drums and sheaves	9,400
Motor	8,600
Load	19,375
Total inertia	37,375

Angular-acceleration moment = 37,375 × 2.275 = 85,000
Angular-retardation moment = 37,375 × 2.275 = 85,000
Friction assumed at 5.5 per cent of 38,750 = 2130 lb
Moment due to friction = 2130 × 4 = 8520

Load Diagram

	Ft-lb	Hp
1. (5000 + 1825) × 4 + 8520 + 85,000 =	120,820	2500
2. (5000 + 995) × 4 + 8520 + 85,000 =	117,500	2435
3. (5000 + 995) × 4 + 8520 =	32,500	673
4. (5000 − 995) × 4 + 8520 =	24,540	508
5. (5000 − 995) × 4 + 8520 − 85,000 =	−60,460	−1250
6. (5000 − 1825) × 4 + 8520 − 85,000 =	−63,800	−1320

Capacity of Hoist Motor

30,300,000
2,136,000
8,250,000
18½)40,686,000
2,200,000

$$\text{Rms} = \sqrt{\frac{\dfrac{2500^2 + 2433^2}{2} \times 5 + \dfrac{673^2 + 508^2}{2} \times 6 + \dfrac{1250^2 + 1320^2}{2} \times 5}{16 + \frac{5}{2}}} = 1485 \text{ hp}$$

Figure 147 shows the duty cycle of the three motors. In discussing this figure, Bright states,[1] with respect to the d-c motor:

> With field control, there are no rheostatic losses in the main armature circuits, so that the power follows the diagonal line from the origin instead of the vertical line, as in the case of the a-c motor. The light dotted line starting at 2500 hp indicates the torque that must be delivered by the hoist motor.

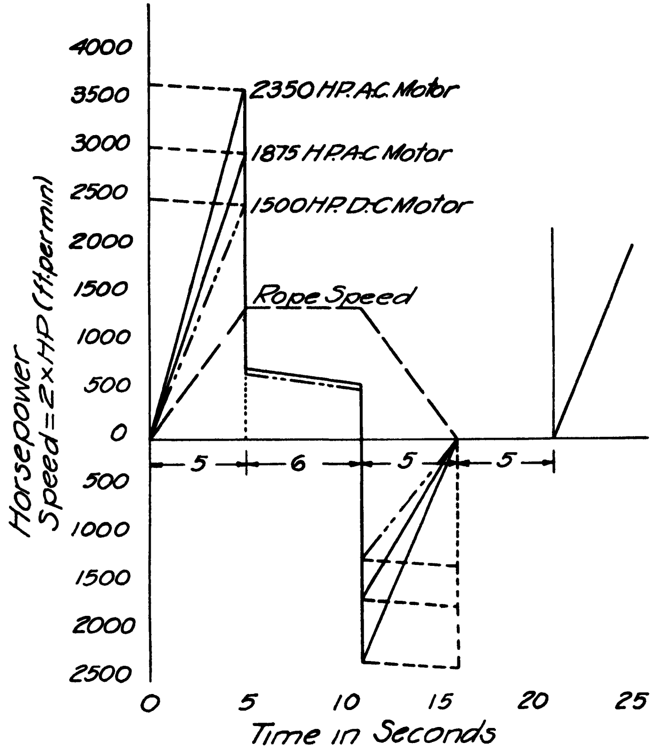

FIG. 147. Duty cycle for a-c and d-c hoisting systems. (*Bright*, "*Determination of Electrical Equipment for a Mine Hoist.*")

> In other words the hoist motor delivers a high torque at the start with a low expenditure of power, for there are no rheostatic losses. As the current in the hoist motor is proportional to the torque, this line also represents the current from which the heating can be calculated. The power represented in the upper triangle during the accelerating period is only apparent and not real when using a full control system. The power represented in the upper triangle of the retarding part of the cycle is returned to the system instead of being drawn from the line as with the a-c motor.

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[1] BRIGHT, *op. cit.*, p. 594.

The power taken from the line, when using a flywheel set, passes through the induction motor (see Fig. 139). Since the flywheel cannot change its speed quickly, the load on the induction motor cannot change quickly. As the load comes on the hoist motor, it is transferred through the generator to the induction motor. When the load on the a-c motor reaches a predetermined value, the slip regulator operates, causing a resistance to be inserted in the secondary of the induction motor. The speed of the induction motor drops, thus causing the flywheel to give up part of its energy. By this arrangement the load on the induction motor can be fixed at a desired value. When the load on the hoist motor exceeds the predetermined value, the slip regulator functions, and the flywheel supplies the surplus energy required. When the load is below the maximum for which the regulator is set, the induction motor speeds up and the flywheel stores energy. The result is practically a constant load on the induction motor.

Voltage Control with Motor-generator Flywheel Set. The determination of the capacity of the generator, the weight of the flywheel, the capacity of the induction motor, and the over-all efficiency are given in Example 4*d*.

EXAMPLE 4d

	Hoist-motor Output	Hoist-motor Input	Generator Input			Hp-sec Input to Generator
			Appar.	Real		
1.	2500 + 210 =	2710 + 170 =	2880	335	÷ 2 × 5 =	7837
2.	2435 + 200 =	2635 + 163 =	2798	2798		
3.	673 + 36 =	709 + 32 =	741	741	÷ 2 × 6 =	3921
4.	508 + 30 =	538 + 28 =	566	566		
						11,758
5.	−1250 + 70 =	−1180 + 50 =	−1130	−1130	÷ 2 × 5 =	−2,618
6.	−1320 + 75 =	−1245 + 53 =	−1192	+83		
			Total hp-sec input to generator		=	9,140

$$\text{Average input to generator} = \frac{\text{total input}}{\text{total time of cycle}} = \frac{9140}{21} = 435 \text{ hp}$$

Size of Flywheel

$$\text{Input to flywheel during accel.} = \frac{2798 - 435}{2} \times 4.8 \text{ sec} = 5670 \text{ hp-sec}$$

$$\text{Input to flywheel during const. speed} = \frac{741 - 435 + 566 - 435}{2} \times 6 = 1311 \text{ hp-sec}$$

Total input = 6981 hp-sec

Normal speed of set = 685 rpm

Normal speed at radius of gyration = V_1 = 254 ft per sec

Diameter of flywheel = 10 ft

Speed at 10 per cent reduction = V_2 = 228.6 ft per sec

Peripheral speed = 21,600 ft per min

$$\text{Weight of flywheel} = \frac{\text{hp-sec} \times 550 \times 2g}{v_1^2 - v_2^2} = \frac{6981 \times 550 \times 64.4}{254^2 - 228.6^2} = 20{,}100 \text{ lb}$$

Use a 22,000-lb wheel; thickness, 7 in.

Size of driving motor	Full-load loss, hp.	No-load loss, hp.
Average input to generator	435	
Motor and generator excitation	25	10
Exciter loss	2	2
Windage and friction of generator	15	15
Flywheel loss	20	20
Slip regulator loss	35	
Iron loss, a.-c. motor		10
Friction and windage, a.-c. motor		10
No-load copper loss, a.-c. motor	...	1
Total output, a.-c. motor	532	
Total no-load loss	...	68

$$\text{Total input, a-c motor} = \frac{\text{output}}{\text{efficiency}} = \frac{532}{0.92} = 578 \text{ hp}$$

Input, a-c motor, hp-sec = 578 × 21 = 12,138 = 2.51 kw-hr

$$\text{Kw-hr per ton} = 1 \qquad \text{Shaft hp-sec} = \frac{5000 \times 500}{550} = 4550$$

$$\text{Over-all efficiency} = \frac{\text{shaft hp-sec}}{\text{input hp-sec}} = \frac{4550}{12{,}138} = 37.5\%$$

Capacity of Generator

$$\begin{array}{r} 35{,}550{,}000 \\ 2{,}367{,}000 \\ \underline{7{,}350{,}000} \\ 21\overline{)45{,}267{,}000} \\ 2{,}150{,}000 \end{array} \quad \text{Rms} = \sqrt{\frac{\dfrac{2710^2 + 2635^2}{2} \times 5 + \dfrac{709^2 + 538^2}{2} \times 6 + \dfrac{1180^2 + 1245^2}{2} \times 5}{21}} = 1470 \text{ hp} = 1097 \text{ kw}$$

Recommendations

For hoist motor = 1500 hp. For generator = 1100 kw.
For a-c motor = 600 hp. For flywheel = 22,000 lb.

Figure 148 gives the variable losses for the equipment under consideration. In the calculations for Example 4*d*, the column marked Hoist-motor Output is obtained by adding to the load-cycle values obtained in Example 4*c*, the variable losses from the hoist-motor curve in Fig. 148. The data under Hoist-motor Input are found by adding the variable losses of the generator to the output. These values are obtained from the generator curve, Fig. 148. Before the variable losses can be determined, a tentative selection of the hoist motor and generator must be made. From tests on the motor and generator, curves similar to those in Fig. 148 are drawn. During the retarding portion of the cycle the hoist motor is acting as a generator and returns power to the flywheel. The losses must be subtracted and not added. One is the apparent input to the generator; the other is the actual input. On lines (1) and (6) the torque value is represented, and not the actual horsepower. The real values are substituted and consist of the copper losses of the hoist motor and the generator. The other losses are practically zero at the moment of starting and stopping. The column Hp-sec Input to Generator is the average power input to the generator during the three stages of the cycle.

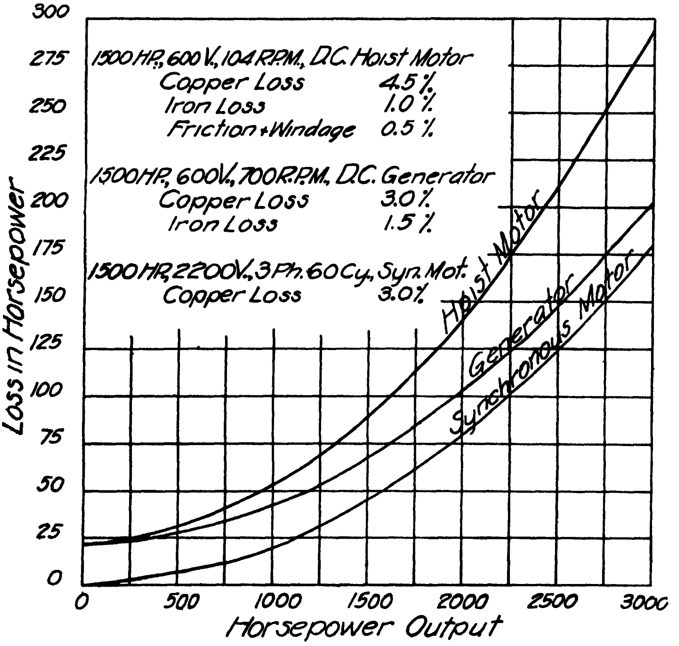

FIG. 148. Motor and generator variable losses. (*Bright, "Determination of Electrical Equipment for a Mine Hoist."*)

As the generator must run for the entire cycle, the time is 21 sec and is used with its full value. The average horsepower input to the generator is found as shown in the calculations.

Size of Flywheel. Before the weight of the flywheel can be determined, the speed of the motor-generator set must be decided upon. Flywheels generally used are those made of steel plates riveted together, making a solid disk. This type of wheel can be safely operated at 20,000 to 24,000 ft per min rim speed.[1] The rim speed ordinarily used is about 350 ft per sec (21,000 ft per min).[2] With the diameter of the wheel known, the rpm of the set can be calculated; or, if the rpm of the set is known, the diameter of the wheel can be calculated.

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[1] BRIGHT, *op. cit.*, p. 598.

[2] SAGE, *op. cit.*, p. 39.

The capacity of the generator is roughly $\left(\frac{2710 + 538}{2} \times 0.75\right) = 1000$ kw. A good speed for such a capacity is 720 rpm synchronous when driven from a 60-cycle motor.[1] At full load the speed of the set with the regulator plates closed will be about 685 rpm. (There is a loss of about 3 per cent due to normal slip in the secondary of the induction motor; and about 2 per cent in the motor windings and leads.

$$720 \times 95\% = \text{about } 685 \text{ rpm}$$

The peripheral speed of the wheel (using a 10-ft-diameter wheel) is

$$\begin{aligned}\text{Peripheral speed} &= 2\pi \times r \times \text{rpm} = 2\pi \times {}^{10}\!/_{2} \times 685 \\ &= 21{,}600 \text{ ft per min}\end{aligned}$$

For a solid-disk wheel the radius of gyration is about 0.707 times the radius of the rim.

The speed, in feet per second, at the radius of gyration is

$$\text{Radius of gyration} = 0.707 \times 5 = 3.535 \text{ ft}$$

$$\text{Speed at radius of gyration} = \frac{2\pi \times 3.535 \times 685}{60} = 254 \text{ ft per sec} = V_1$$

In the present problem a 10 per cent drop in speed will give partial equalization of the high peaks from the power system.

The speed at 10 per cent reduction is

$$90\% \times 254 = 228.6 \text{ ft per sec} = V_2$$

The weight of the flywheel is obtained from $E = \frac{1}{2}MV^2$, where E = hp-sec × 550.

$$M = \frac{W}{g}$$

$$V^2 = (V_1^2 - V_2^2)$$

$$\text{Hp-sec} \times 550 = \frac{W}{2g}(V_1^2 - V_2^2)$$

$$W = \frac{\text{hp-sec} \times 550 \times 2g}{(V_1^2 - V_2^2)}$$

The horsepower-second input to the flywheel is found by plotting Fig. 149, from the values obtained under Real Generator Input, against time. A horizontal line representing the average input of 435 hp is also drawn. All power under the duty cycle and above the 435-hp line must be supplied by the flywheel. The total is obtained as shown in the calculations.

Substituting values in the above formula,

$$W = \frac{6981 \times 550 \times 2 \times 32.2}{(254^2 - 228.6^2)} = 20{,}200 \text{ lb}$$

As a margin of safety this is increased to 22,000 lb. The thickness of the flywheel is

$$\frac{22{,}000}{480 \times \pi \times 5^2} \times 12 = 7 \text{ in.}$$

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[1] Bright, *op. cit.*, p. 598.

Size of Driving Motor. The average input to the generator was found to be 435 hp. To this must be added the losses listed in the calculations. Their estimated values are as shown.

Capacity of Generator. The generator capacity is found in a manner similar to that of the hoist motor. The values in the column Hoist-motor Input are used.

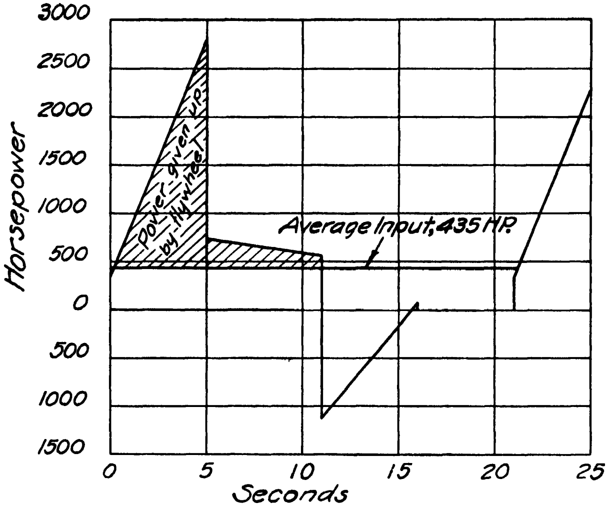

FIG. 149. Input to generator (flywheel). (*Bright*, "*Determination of Electrical Equipment for a Mine Hoist.*")

The full time of 21 sec is used because the generator runs at practically full speed all the time.

FLYWHEELS

The rim speed of the flywheel is ordinarily taken at approximately 350 ft per sec. With this as a basis, the rpm of the set with any given diameter flywheel may be found.

The weight is based on an initial speed of 94 per cent synchronous.[1] As previously stated, the driving motor loses about 6 per cent of its synchronous speed because of motor losses. The maximum peripheral speed of the flywheel should not be greater than the following: Steel = 24,000 ft per min; cast iron = 16,000 ft per min. The friction loss, bearings and windage, is 1.25 to 1.5 hp per ton weight for steel wheels without covers; and for cast-iron wheels is 15 per cent greater than that of the plate wheels.[1] If close-fitting covers are used, the loss may be reduced by 10 to 15 per cent.

The speed reduction of the flywheel will depend on the extent to which complete equalization must be approached. This depends on the power charges for high peaks, the nature of the power-generating equipment, and the interference by high peaks with other operations outside the plant. Ordinarily a speed reduction of 15 per cent for balanced hoisting and 35 per cent for out-of-balance hoisting will prove satisfactory.[1] It is not considered desirable to operate the set with a reduction greater than 35 per cent, as the wheel has at this reduction given up about 57 per cent of its total energy.

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[1] SAGE, *op. cit.*, p. 39.

Approximate Data for Determining Flywheel Motor-generator Set. In the preceding solution for the motor-generator-flywheel set, knowledge of losses and characteristics of the d-c hoist motor; a-c driving motor; and d-c generator were necessary and known. It is quite likely that such information will not always be immediately available when desired. An approximate method,[2] which does not require data on motor and generator losses, may be used. This method gives results that fall well within the limits of standard frame sizes.[2] The average commercial efficiencies, of the equipment for the motor-generator set, are used.

Generator. To find the kilowatt output of the generator, determine its rms-hp from the total horsepower²-seconds required by the hoist motor. This is found from

$$\sqrt{\frac{\text{Hp}^2\text{-sec}}{\text{Total time of duty cycle}}}$$

The value thus obtained is considered as representing 92 per cent of the generator capacity required. The actual generator rating, for service where high peaks frequently reoccur, would be on the basis that 80 per cent of twice the normal rating will be within the limit of the maximum repeated peak. If this rule is followed, good commutation is ensured. The nearest standard frame rating is chosen.

Induction Motor. The rating of the motor for the motor-generator set must be based on the average load requirement of the hoist cycle, plus the losses. The motor and slip-regulator losses result, for all practical purposes, in an over-all efficiency of 78 per cent. The output of the motor is

$$\text{Acceleration} = A \times \frac{t_a}{2} \text{ hp-sec}$$

$$\text{Constant speed} = \frac{B + C}{2} \times t_s \text{ hp-sec}$$

$$\text{Retardation (regeneration)}^3 = D \times \frac{t_r}{2} \text{ hp-sec}$$

$$\text{Total output} = A \times \frac{t_a}{2} + \frac{B + C}{2} \times t_s + D \times \frac{t_r}{2} \text{ hp-sec}$$

$$\text{Average hp} = \frac{\text{total output}}{\text{total time}}$$

$$\text{Apparent rating} = \frac{\text{average hp}}{0.78}$$

To the apparent rating must be added a flywheel windage and friction loss, of 5 to 7.5 per cent of the apparent rating. The nearest standard frame size is then selected.

Flywheel. All the energy required above the average motor horsepower (as found above) must be supplied by the flywheel; and in turn the same amount of energy must be restored to the flywheel before the succeeding cycle begins.

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[1] SAGE, *op. cit.*, p. 38.

[2] Communication from Allis-Chalmers Manufacturing Company.

[3] If D is negative, it is subtracted.

The degree of equalization must be known, that is, the reduction of the flywheel speed. The time, in seconds, that the motor delivers energy to the flywheel is found from (also see Fig. 149)

$$A:t_a::(A - \text{average hp}):y$$

where y = the time, seconds.

The total horsepower-seconds at the motor shaft are

$$\text{During acceleration} = \frac{A - \text{avg hp}}{2} \times y$$

$$\text{During constant speed} = \left[\left(\frac{B+C}{2}\right) - \text{avg hp}\right] \times t_s$$

$$\text{Total} = \frac{A - \text{avg hp}}{2} \times y + \left[\left(\frac{B+C}{2}\right) - \text{avg hp}\right] \times t_s$$

$$\text{The foot-pounds at the flywheel shaft} = \frac{\text{total hp-sec} \times 550}{0.78}$$

$$\text{The flywheel capacity} = \frac{\text{ft-lb at flywheel shaft}}{E}$$

where

$$E = \frac{V_1^2 - V_2^2}{V_1^2} \text{ (see pages 347 and 351)}$$

$$\text{Weight of flywheel} = \frac{\text{flywheel capacity} \times 64.4}{V_1^2}$$

The following example[1] shows the application of the approximate method:

Total horsepower^2-seconds for the duty cycle = 81,812,000
No-load torque horsepower = 2450
Horsepower at A = 2341
Horsepower at B = 1006
Horsepower at C = 542
Horsepower at D = −433
t_a = 11 sec
t_s = 24 sec
t_r = 14 sec
Total time = 55.3 sec

Generator. The rms of cycle using total time equals

$$\sqrt{\frac{81{,}812{,}000}{55.3}} = 1220 \text{ hp}$$

$$\frac{1220 \times 0.746}{0.92} = 990 \text{ kw}$$

The maximum generator peak to be commutated = 2450 hp-torque.

$$\frac{2450 \times 0.746}{0.925} = 1980 \text{ kw for generator to commutate}$$

$$\frac{1980}{0.80} = 2470 \text{ kw}$$

---

[1] Allis-Chalmers Manufacturing Company.

which must be within twice the normal generator rating. The generator rating is 2470 × ½ = 1235 kw. The nearest standard frame rating is 1250 kw.

Induction Motor. **From the duty cycle,**

$$
\begin{aligned}
2341 \times {}^{11}\!/_{2} &= 12{,}900 \text{ hp-sec}\\
\frac{1006 + 542}{2} \times 24 &= 18{,}600 \text{ hp-sec}\\
\text{Total} &= \overline{31{,}500 \text{ hp-sec}}\\
\text{Regeneration} = -433 \times {}^{14}\!/_{2} &= \underline{-3{,}031 \text{ hp-sec}}\\
\text{Net} &= 28{,}469 \text{ hp-sec}
\end{aligned}
$$

$$\frac{28{,}469}{55.3} = 515 \text{ avg hp}$$

$$\frac{515}{0.78} = 660 \text{ hp}$$

To this must be added the flywheel windage and friction losses, which run between 5 and 7.5 per cent of the motor rating.

$$660 \times 1.05 = 693 \text{ hp}$$

The nearest standard frame size is 700 hp.

Flywheel (Full Equalization). The average energy requirement of the induction motor is 515 hp. All energy above this must be delivered by the flywheel; and this energy in turn must be restored to the flywheel before the succeeding cycle begins. The slip speed (reduction of flywheel speed) is 15 per cent. The time through which the motor delivers energy before the flywheel begins to give up its stored energy is

$$2341:11::(2341 - 515):y$$

$$y = 8.56 \text{ sec}$$

$$
\begin{aligned}
\text{Motor-shaft hp-sec} = \frac{2341 - 515}{2} \times 8.56 &= 7{,}810 \text{ hp-sec}\\
\left[\left(\frac{1006 + 542}{2}\right) - 515\right] \times 24 &= 6{,}220 \text{ hp-sec}\\
\text{Total} &= \overline{14{,}030} \text{ hp-sec}
\end{aligned}
$$

$$\frac{14{,}030 \times 550}{0.78} = 9{,}880{,}000 \text{ ft-lb at the shaft of the flywheel}$$

For a 15 per cent reduction in speed, the flywheel capacity[1] is

$$\frac{9{,}880{,}000}{0.2775} = 35{,}600{,}000 \text{ ft-lb}$$

Using for a plate wheel a velocity of 277 ft per sec at the radius of gyration, the weight is

$$\frac{35{,}600{,}000 \times 64.4}{277^2} = 30{,}000 \text{ lb}$$

---

[1] The number, 0.2775, is the reduction in speed expressed in per cent. The operating velocity of the wheel is 277 ft per sec. Therefore, $V_1 = 277$ ft per sec. If this is reduced 15 per cent,

$$V_2 = 277 \times 85\% = 235.45 \text{ ft per sec}$$

(See p. 347.)

$$\frac{277^2 - 235.45^2}{277^2} = 0.2775$$

EXAMPLE 4e

The synchronous motor-generator set is satisfactory where there is no penalty for momentarily high peaks of power demand, provided that the power system is sufficiently large to handle the load.

The calculations are as follows:

	Hoist motor		Generator		Total excitation	Generator frict. and wind.
	Output	Input	Apparent input	Real input		
1	2500	2710	2880	335	36	15
2	2435	2635	2798	2798	43	15
3	673	709	741	741	43	15
4	508	538	566	566	43	15
5	−1250	−1180	−1130	−1130	43	15
6	−1320	−1245	+1192	+ 83	36	15
7	......	......	Rest Period		28	15
8	..	.... ..	Rest Period		28	15

	Synchronous-motor output	Synchronous-motor losses			Synchronous-motor input	Input, hp-sec
		Iron	Copper	Frict. and wind.		
1	386	15	3	15	419	÷ 2 × 5 = 8,672
2	2856	15	162	15	3048	
3	799	15	12	15	841	÷ 2 × 6 = 4,506
4	624	15	7	15	661	
						13,178
5	−1072	15	23	15	−1019	÷ 2 × 5 = −2,135
6	+ 134	15	1	15	+ 165	
						11,043
7	43	15	1	15	74	÷ 2 × 5 = 370
8	43	15	1	15	74	

Total input, hp-sec = 11,413

Total input kw-hr = 2.36 Kw-hr per ton, 0.945

$$\text{Shaft hp-sec} = \frac{5000 \times 500}{550} = 4550$$

$$\text{Over-all efficiency} = \frac{\text{shaft hp-sec}}{\text{total input hp-sec}} = \frac{4550}{11{,}413} = 39.8 \text{ per cent}$$

Capacity of Generator

$$\begin{array}{r} 35{,}550{,}000 \\ 2{,}367{,}000 \\ \underline{7{,}350{,}000} \\ 21)\overline{45{,}267{,}000} \\ 2{,}150{,}000 \end{array} \quad \text{Rms} = \sqrt{\frac{\dfrac{2710^2 + 2635^2}{2} \times 5 + \dfrac{709^2 + 538^2}{2} \times 6 + \dfrac{1180^2 + 1245^2}{2} \times 5}{21}} = 1470 \text{ hp} = 1097 \text{ kw}$$

Capacity of Synchronous Motor

$$\begin{array}{r} 15{,}665{,}000 \\ 3{,}380{,}000 \\ \underline{1{,}700{,}000} \\ 21)\overline{20{,}745{,}000} \\ 988{,}000 \end{array} \quad \text{Rms} = \sqrt{\frac{\dfrac{386^2 + 2856^2 + 386 \times 2856}{3} \times 5 + \dfrac{799^2 + 624^2}{2} \times 6 + \dfrac{1072^2}{3} \times 4.45}{21}} = 994 \text{ hp}$$

Recommendations

For hoist motor: 1500-hp 600-volt 108.5-rpm d-c motor.

For generator: 1100-kw 600-volt 720-rpm d-c generator.

For synchronous motor: 1500-hp 2200-volt three-phase 60-cycle 720-rpm motor.

The first four columns are the same as for the flywheel set. The next two columns give generator losses. The next set of data gives information for the synchronous motor. The first column is the real input to the generator plus the generator losses.

The motor losses appear in the next three columns; when added to the output, we obtain the synchronous-motor input. A little lower power consumption and slightly higher efficiency are obtained with this outfit than with the flywheel set.

The capacity of the generator is the same as in Example 4*d*. It is determined in the same way.

In determining the capacity of the synchronous motor, there is such a large difference between the accelerating and retarding part of the cycle that the method shown in the calculations is used. The small values at the end of the retarding cycle and during the rest period may be neglected since their effect is very small.

A 1000-hp motor would meet the requirements so far as the heating is concerned, but would not have a high enough pull-out torque. Either a 1200-hp motor with a pull-out of 3, or a 1500-hp motor with a pull-out of 2.4 could be used.

Figure 150 shows the input to the hoist from the power system with the three types of hoists under consideration. In the case of the synchronous motor, the maximum acceleration peak load is 3050 hp. This load is not imposed on the line instantly. It starts with 419 hp and requires 5 sec to reach 3050 hp. The power below the zero line is returned to the power system.

INCLINED SHAFTS

The determination of the motor and drum size for a hoist operating through an inclined shaft is almost identical with the treatment for the vertical lift.

The cage or skip is mounted on wheels, which necessitates a change in the friction moment from that used for vertical hoists. The friction is influenced by the following: weight of material and container; wheel-bearing friction; rope friction on the rollers; guide friction (when guides are used, which they occasionally are).

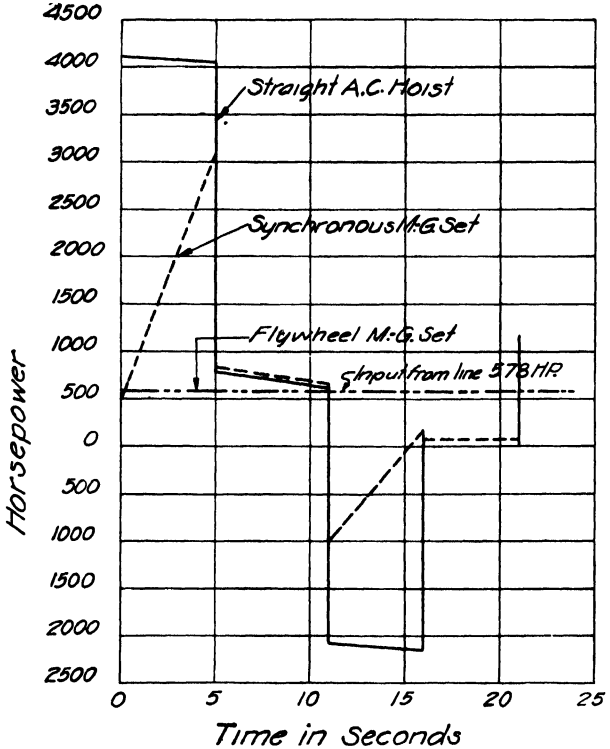

FIG. 150. Input from power station for the three hoists. (*Bright*, "*Determination of Electrical Equipment for a Mine Hoist.*")

Car or Rolling Friction. Sage[1] recommends 40 lb per ton of the load normal to the track. The normal to the track is given by: actual weight of skip and contents times cos θ, where θ is the angle with the horizontal. The "Goodman Mining Handbook" uses 35 lb per ton.

Rope Friction.[2] A rope friction of 200 lb per ton (10 per cent) normal pressure is considered a conservative figure.

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[1] SAGE, *op. cit.*, p. 16.

Goodman Manufacturing Company, "Goodman Mining Handbook," 5th ed., p. 157, 1927.

[2] SAGE, *op. cit.*, p. 16.

Goodman Manufacturing Company, *op. cit.*

Friction in Hoist. The efficiency of the hoist, gears, etc., in terms of friction moment is best determined by the formulas given on page 298. To this may be added the rope friction and the rolling friction.

STANDARD FRAME SIZES FOR HOIST MOTORS AND MOTOR-GENERATOR SETS[1]

The following list gives the standards accepted by manufacturers.

Motors

Horsepower	R.p.m. of Motor
100 125 150 200	450, 400, 350, 300, 250
250 300 350 400 500	900, 720, 600, 500, 450, 400, 350, 300, 250, 225, 200
600 700 800 900 1000	600, 500, 450, 400, 350, 250, 225, 200, 175, 150, 125, 100
1250 1500 1750 2000 2250	350, 300, 250, 225, 200, 175, 150, 125, 100
2500 3000 4000 5000	250, 225, 200, 175, 150, 125, 100

Motor-generator Sets—Synchronous Motor and Flywheel—Two-unit Sets

Kw of Set	Rpm of Set
200	1200
250	1200
300	1200
400	1200
500	900
600	900
750	900
1000	720
1500	514
2000	360

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[1] Allis-Chalmers Manufacturing Company.

TWIN-GENERATOR UNITS

2000	720
2500	720
3000	514
4000	360

If the calculations for a given installation do not correspond to the values given above, the nearest standard frame size larger than that required is selected.

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CHAPTER IX

HANDLING OF WATER FOR MINING PURPOSES

The present chapter will be limited to a discussion of material that is of essential importance to the mine engineer or operator. It would be decidedly out of place to indulge in a theoretical or fundamental treatment of hydraulics. There are, on the market, many excellent books treating that subject. The mining engineer is primarily interested in the handling of water for two reasons: (1) unwatering mine workings; (2) conducting water to or from the property for either domestic or mill consumption.

A mining property may, in general, be unwatered in one of two ways: by some form of pumping or by drainage adits. The latter will receive no consideration in this book. Those especially interested in this important, and it may be added expensive, phase are referred to *Bulletin* 57 of the U.S. Bureau of Mines, "Safety and Efficiency in Mine Tunneling."[1]

It is practically impossible with a subject of this kind to avoid certain fundamental concepts entirely. Some may feel that this side of the subject has been unnecessarily slighted. However, as has been stated elsewhere, the purpose here is to present a working knowledge of mine plant selection, and not a theoretical treatise. In those cases where it is thought necessary, as an aid toward refreshing the memory, elementary principles will be given.

The subject matter might be considered from two viewpoints: (1) the conveyance of water by means of pipes, ditches, conduits, and flumes (pumping being included in this discussion); (2) the measurement of the quantity of water handled. This scheme will be followed, more or less.

In formulas used in hydraulics the quantity of water flowing is expressed in *cubic feet per second;* and the velocity in *feet per second.* Pumps, ditches, and pipes are commonly rated in *gallons per minute.* Before using such ratings in the formulas usually found in textbooks, they must be converted to cubic feet per second.

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[1] See, Mahr Tunnel Encounters 20,000 g.p.m. Flow, *Eng. Mining J.*, **134,** No. 10, 414 (1933).

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At sea level a column of mercury 30 in. high is required to balance the atmospheric pressure (1 atm) of 14.75 lb per sq in. Two atmospheres are

$$2 \times 14.75 = 29.5 \text{ lb per sq in., etc.}$$

About 34 ft of water are necessary to balance 1 atm at sea level. Atmospheric pressure must be considered in many problems involving the flow of water in pipes and tubes. It is this pressure which makes pumping possible when the pump is situated at a point above the water surface.

HEAD AND PRESSURE

If h = the head
w = weight of a cubic unit of water
p = unit pressure under the head h

then

$$p = wh$$

and

$$h = \frac{p}{w}$$

If p is in pounds per square inch,

$$p = \frac{62.4}{144} h = 0.433h$$

and

$$h = 2.31p$$

In other words, a 1-ft head corresponds to 0.434 lb per sq in.; and 1 lb per sq in. equals a 2.3-ft head.

LAW OF FALLING BODIES

Theoretical hydraulics treats of the flow of water when unretarded by friction. The motion of water through orifices and pipes may be produced by the force of gravity. This force is proportional to the acceleration of an object falling freely in a vacuum. It is the change in velocity in 1 sec. Acceleration of gravity is measured in feet per second per second and is denoted by the constant g. This same constant is found in all formulas in mechanics relating to falling bodies.

This is also true of that part of hydraulics considering the flow of water under the influence of gravity. Values for g are not constant for all latitudes and elevations of the earth's surface. The mean value 32.16, or 32.2, may be used without seriously affecting problems of a practical nature.

There are but a few fundamental laws of hydraulics. For the present discussion only the one relating to falling bodies need receive attention. Head or velocity, which enters directly or indirectly into all the formulas involved in the handling or measuring of water, is derived from the law of falling bodies. Because of their importance, the derivation of the law expressing head and velocity will be given.

If a freely falling body has an initial velocity u, its velocity at the end of time t is

$$v = u + gt$$

The velocity v is equal to dh/dt. Substituting this,

$$\frac{dh}{dt} = u + gt \qquad \text{or} \qquad dh = u\,dt + gt\,dt$$

Integrating, we get the distance passed over during the time t

$$\int_0^h dh = u \int_0^t dt + g \int_0^t t\,dt$$
$$h = ut + \tfrac{1}{2}gt^2$$

Eliminating t from the last equation by

$$v = u + gt \qquad \text{or} \qquad t = \frac{v - u}{g}$$

$$h = u\left(\frac{v-u}{g}\right) + \frac{1}{2}g\left(\frac{v-u}{g}\right)^2 = \frac{uv - u^2}{g} + \frac{1}{2}\frac{(v^2 - 2uv + u^2}{g^2}g$$
$$= \frac{v^2 - u^2}{2g}$$

or

$$v = \sqrt{2gh + u^2}$$

When the initial velocity $u = 0$ (which is the case in most hydraulics problems), then

$$v = \sqrt{2gh} \qquad \text{and} \qquad h = \frac{v^2}{2g} \qquad \text{and} \qquad q = av$$

where q = quantity, cu ft per sec
a = area, sq ft
v = velocity, ft per sec

These formulas are the basis for practically every problem met with in hydraulics. Their importance is readily realized.

FLOW OF WATER THROUGH PIPES

The phenomenon of the flow of water through pipes is fairly simple. There is more or less resistance offered at the entrance of the pipe. The amount of friction at the entrance depends on the shape of the connection to the reservoir. Friction and adhesion must be overcome throughout the entire inner surface of the pipe. The velocity of the water in a pipe decreases as the length increases, other things (friction, head, size, material) remaining unchanged. In long pipe the velocity is very small; this shows that most of the head is used in overcoming various resistances. The length of a pipe is measured along its axis, following all the curves.

The value of the velocity used in all practical hydraulics problems is the mean (average) one and may be found by dividing the actual discharge of the pipe by the cross-sectional areas.

The head H that causes the flow is the vertical distance measured from the water surface of the container to the center of the end of the pipe. In case the pipe discharges into the bottom of a lower tank, the head is the vertical distance between the two water surfaces.

It can be shown that there is a certain amount of head lost because of the velocity of flow.[1] This is known as the "velocity head" and is given by $v^2/2g$. The greater the velocity, the larger becomes the lost head. It is desirable, at least in most cases, to keep the velocity down and to obtain the necessary volume of water by means of larger conduits.

Straight Pipe of Uniform Size. With a constant head, the mean velocity at any point along a pipe of uniform size is the same. This is true so far as practical applications are concerned.[2]

The loss in a straight pipe of uniform size consists of two parts:

1. Loss of head, h', due to entrance
2. Loss of head, h'', due to frictional resistance of the interior surface

The loss of head at entrance is always less than the velocity head. It is expressed by

$$h' = m\frac{v^2}{2g}$$

---

[1] Of the total head available, an amount, $h_v = v^2/2g$, is used to create velocity.

[2] BOVEY, H. T., "A Treatise on Hydraulics," 2d ed., p. 202, John Wiley & Sons, Inc., New York. (A theoretical discussion is given.)

From a section of hydraulics, which will not be discussed,

$$m = \left(\frac{1}{c_1}\right)^2 - 1$$

where c_1 is equal to the coefficient of velocity. A mean value for c_1 is 0.98.

The coefficient c_1, and consequently m, depend on the manner in which the pipe is connected at the intake. Figures 151*a*, *b*, and *c* show how pipes are ordinarily connected to a tank. The values for m in these connections are in Fig. 151*a*, $m = 0.93$; in Fig. 151*b*, $m = 0.49$; and in Fig. 151*c*, $m = 0$. The connection most commonly made is that shown in Fig. 151*b*. It is customary to use $m = 0.5$ for this connection. The loss of head in friction is very large in long pipes, compared with the loss of head at entrance.

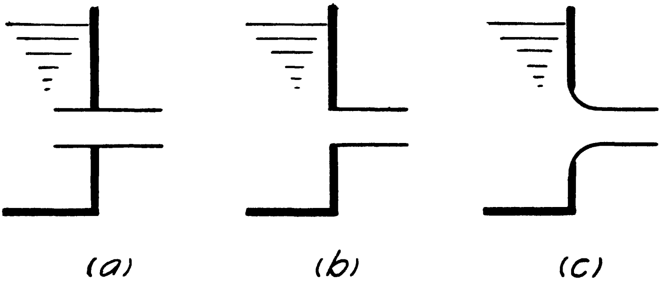

FIG. 151.—Pipe connections to tanks.

Loss of Head Due to Friction. This loss is usually very large, especially for long pipes or old pipes. The laws that govern the flow of water in pipes are only approximate and have been established almost entirely through observation and experimentation. There are five of these laws. They are stated as follows:

1. The loss of head in friction is directly proportional to the length of the pipe.
2. It is inversely proportional to the diameter of the pipe.
3. It is increased nearly as the square of the velocity.
4. It is independent of the pressure of the water.
5. It increases with the roughness of the surface.

The five laws may be expressed by the following formula:

$$h'' = f\frac{l}{d}\frac{v^2}{2g}$$

where h'' = head loss in friction
f = coefficient depending on the degree of roughness of the interior surface and the velocity
l = length of the pipe, ft
d = diameter of the pipe, ft
$v^2/2g$ = velocity head due to mean velocity

TABLE 43. FRICTION FACTORS FOR CLEAN IRON PIPE

Dia., ft.	Velocity, ft. per second						
	1	2	3	4	6	10	15
0 05	0.047	0.041	0.037	0.034	0.031	0.029	0.028
0 1	0.038	0.032	0.030	0.028	0.026	0.024	0 023
0 25	0.032	0.028	0.026	0.025	0.024	0.022	0 021
0 5	0.028	0.026	0.025	0.023	0.022	0.020	0 019
0.75	0.026	0.025	0.024	0.022	0.021	0.019	0 018
1 0	0.025	0.024	0.023	0.022	0.020	0.018	0 017
1 25	0.024	0.023	0.022	0.021	0.019	0.017	0 016
1 5	0.023	0.022	0.021	0.020	0.018	0.016	0.015
1 75	0.022	0.021	0.020	0.018	0.017	0.015	0.014
2.0	0.021	0.020	0.019	0.017	0.016	0.014	0 013
2.5	0.020	0.019	0.018	0.016	0.015	0.013	0.012
3.0	0.019	0.018	0.016	0.015	0.014	0.013	0.012
3.5	0 018	0.017	0.016	0.014	0.013	0 012	
4.0	0 017	0.016	0.015	0.013	0.012	0 011	
5 0	0 016	0.015	0.014	0.013	0.012		
6 0	0.015	0.014	0.013	0 012	0.011		

f is called the "friction factor." Its value ranges from 0.05 to 0.01 for a new clean cast-iron or wrought-iron pipe. A mean value often used as an approximation is $f = 0.02$.

Table 43 gives values of f for new cast-iron pipe.[1]

It will be noted that (1) f decreases as the velocity increases and (2) f decreases as the diameter increases.

Table 44 gives friction loss, in equivalent feet of pipe, for various pipe fittings.[2]

The values in Table 44 are added to the straight pipe to obtain the total length of pipe for finding the friction loss.

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[1] MERRIMAN, M., "Treatise on Hydraulics," 10th ed., p. 217, John Wiley & Sons, Inc., New York.

[2] Crane Company, Chicago, *Tech. Paper* 409, May, 1942. (From Fig. 17.)

EXAMPLE

Let $l = 8000$ ft; $d = 1.25$ ft; $v = 5$ ft per sec.
From Table 43, $f = 0.02$

$$h'' = f\frac{l}{d}\frac{v^2}{2g} = 0.02 \times \frac{8000}{1.25} \times \frac{5^2}{64.4} = 49.7 \text{ ft}$$

Values obtained for h'' in the above manner are accurate to a ± 5 per cent.

Because old pipes have interior surfaces much rougher than new pipes, f must be multiplied by 2, 3, or 4, depending on the degree of roughness. If it is suspected that a pipe, shortly after going into service, will become fouled, a higher value of f than given in the table should be chosen.

Velocity and Discharge in Pipes. If we let h = the total head and v = the mean velocity, then

$$h = h' + h'' + \frac{v^2}{2g}$$

Inserting the mean value for h' and the value of h'' from the friction formula and solving for v,

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}}$$

which is the formula for mean velocity. Because f is a function of (v, d), v cannot be directly determined, but is found through successive trials.

EXAMPLE

Find the velocity of discharge from a clean iron pipe 2000 ft long and 9 in. in diameter, under a head of 20 ft.

Taking for f the mean value of 0.02,

$$v = \sqrt{\frac{2 \times 32.2 \times 20}{1.5 + 0.02 \times \frac{2000}{0.75}}} = 4.8 \text{ ft per sec}$$

From Table 43 we find the factor corresponding to $v = 4.8$ and $d = 0.75$ to be 0.022. Substituting this value, v becomes 4.6. Further reference to Table 43 indicates that $f = 0.022$ is correct. The velocity for a clean iron pipe is then about 4.6 ft per sec.

TABLE 44. FRICTION LOSS OF FITTINGS, FEET OF STRAIGHT PIPE

Inside diam., in.	Type of fitting							
	Globe valve	Angle valve	Check valve	Std. tee	Std. elbow	Long elbow	Gate valve	45° elbow
0.5	17	7	4	3	2	1	0.3	1
0.75	22	11	5	4	2	1	0.4	1
1	28	14	7	6	3	2	0.6	1
1.25	37	18	9	8	4	2	0.8	2
1.5	45	22	12	9	4	3	1	2
2	55	28	14	12	5	4	1	3
2.5	70	34	16	14	7	4	1	3
3	84	43	20	17	8	5	2	4
4	110	57	26	22	11	7	2	5
5	140	72	34	27	13	9	3	6
6	170	85	40	34	16	12	4	7
7	190	100	46	38	18	13	4	9
8	225	115	53	44	21	14	5	10
10	280	140	67	56	25	17	6	12
12	340	170	77	66	32	20	7	15
14	. .	. .	92	78	37	23	8	17
16	.		105	88	42	26	9	19
18		.	120	100	48	30	11	22
20		.	130	110	52	34	12	23
22			142	130	58	38	13	26
24		. .	160	140	62	41	14	30
30		. .	200	160	80	52	17	36

EXAMPLE

Find the discharge in gallons per minute from a clean iron pipe 9 in. in diameter, 2000 ft long, and under a head of 20 ft.

From the preceding example v was found equal to 4.6 ft per sec. The discharge is

$$q = av = \frac{\pi d^2}{4} v = \frac{\pi \times 0.75^2}{4} \times 4.6 = 2.03 \text{ cu ft per sec}$$
$$= 2.03 \times 7.48 \times 60 = 914 \text{ gpm}$$

Long Pipes. A pipe is said to be long when $f(l/d)$, in the formula for mean velocity, is so large that dropping the 1.5 produces an error of less than 1 per cent. This occurs, for the mean value of $f = 0.02$, when l/d is greater than 4000. The formula for the mean velocity becomes

$$v = \sqrt{\frac{2ghd}{fl}} = 8.02\sqrt{\frac{hd}{fl}}$$

The discharge in cubic feet per second is, from $q = av$,

$$q = \pi\frac{d^2}{4}\sqrt{\frac{2ghd}{fl}} = 6.3\sqrt{\frac{hd^5}{fl}}$$

From this formula for q, we can find an expression for d.

$$d^5 = \frac{16flq^2}{2gh\pi^2}$$

or

$$d = 0.479\left(\frac{flq^2}{h}\right)^{1/5}$$

Values for f are taken from the table as previously explained. Successive substitutions must be made. In pipes liable to corrosion or deposition of sediment, $2f$ or more should be used.

EXAMPLE

Determine the diameter of a new cast-iron pipe that is to deliver 5000 gpm. The length is 5000 ft and the head 50 ft.

$$q = \frac{5000}{60 \times 7.48} = 11.14 \text{ cu ft per sec}$$

$$\text{Approximate } d = 0.479 \times \left(\frac{0.02 \times 5000 \times 11.14^2}{50}\right)^{1/5} = 1.45 \text{ ft}$$

From $q = av$,

$$v = \frac{q}{a} = \frac{11.14}{0.7854 \times 1.45^2} = 6.73 \text{ ft per sec}$$

From Table 43, we find that, for $v = 6.73$ and $d = 1.45$, $f = 0.018$. The second trial gives

$$d = 0.479 \times \left(\frac{0.018 \times 5000 \times 11.14^2}{50}\right)^{1/5} = 1.41 \text{ ft}$$

$$v = \frac{11.14}{0.7854 \times 1.41^2} \quad 7.14 \text{ ft per sec}$$

Again consulting Table 43, f is found to be 0.018. The diameter of the pipe required is 1.5 ft. It probably would be best to use a pipe 1 75 ft in diameter. This would allow a certain amount of leeway for fouling.

Relative Discharge Capacity of Two Pipes. When the head and length are the same, the discharge capacity varies approximately as the $5/2$ power of the diameter. For example, two pipes d_1 and d_2, discharging q_1 and q_2 quantities of water, give the following expression:

$$\frac{q_1}{q_2} = \left(\frac{d_1}{d_2}\right)^{\frac{5}{2}}$$

or

$$q_2 = q_1\left(\frac{d_2}{d_1}\right)^{\frac{5}{2}}$$

EXAMPLE

How many 9-in. pipes will be required to carry the same amount of water as one 18-in. pipe?

$$x = \left(\frac{d_2}{d_1}\right)^{\frac{5}{2}} = \left(\frac{18}{9}\right)^{\frac{5}{2}} = 5.65 \qquad \text{9-in. pipes are necessary}$$

Or one 18-in. pipe will carry 5.65 times as much water as one 9-in. pipe.

HYDRAULIC GRADIENT

The hydraulic gradient is illustrated in Fig. 152. It is the line BC that connects the water levels in the piezometer tubes placed in the pipe. Or, in other words, it would pass through the water surfaces if such tubes were placed in the pipe line. For all practical purposes the hydraulic gradient is a straight line so long as the diameter of the pipe remains unchanged. If the lower end of the pipe were closed, the water in the tubes would rise to the elevation of A. The difference in elevation between A and C_1 represents the velocity head plus any losses that occur between the entrance of the pipe and the tube. The value of AC_1 may be found from the expression

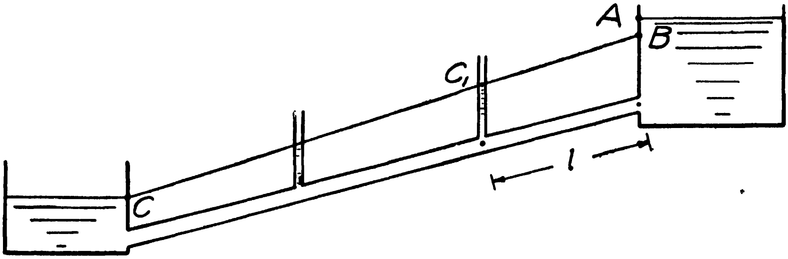

Fig. 152. Hydraulic gradient.

$$AC_1 = H = (1 + m)\frac{v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g}$$

where m is the coefficient at entrance and can be considered as being equal to 0.5. The part of the equation $(1 + m)\frac{v^2}{2g}$ gives the distance AB shown in Fig. 152. The distance l is measured along the axis of the pipe. The formula for H assumes that any curves in the line have a large radius. Otherwise an additional factor is necessary.

A pipe line should not extend above the hydraulic gradient. The gradient line represents the maximum height to which the water will rise under a given set of conditions (head, length of pipe, diameter of pipe, etc.). If parts of the pipe line are appreciably above the gradient, such places trap air and act as buffers. This materially (in some cases completely) reduces the quantity of water that will discharge. If a part of the line must extend above the gradient, it must have absolutely air-tight joints. Even so, the air trapped in the water will escape and gradually form an air pocket.

To determine the hydraulic gradient, a profile of the proposed line is made. The elevations of B and C are determined and connected by a straight line (provided that the diameter of the pipe is constant from start to finish). In this way fill, excavation, etc., are determined. The pipe line can have any number of ups and downs so long as they remain below the gradient. If the pipe discharges into the air instead of a tank, the line is then drawn from B to the center of the end of the pipe.

In the expression $(1 + m)\frac{v^2}{2g}$, v is the important unknown quantity. This equation must be solved to locate the point B from which the hydraulic gradient is started. The mean velocity is determined by the formula previously given (page 363). For this purpose the total length of the pipe line is used. When v is thus determined, its value is sufficiently accurate for most purposes.

EXAMPLE

A pump delivers 1000 gal of water per minute to a tank. The overflow of this tank is 20 ft above the entrance of the discharge pipe from the bottom of the tank. The elevation of the overflow is 3000 ft. The length of the pipes discharging from the tanks is 5000 ft. The elevation of the end of the pipe line is 2500 ft. At a point A, 2000 ft along the pipe from the tank, the pipe has an elevation of 2800 ft. The discharge pipe is to deliver 90 per cent of the pump's capacity.

What diameter should be used for the clean iron pipe, and where is A with respect to the hydraulic gradient?

Solution:

$$q = \frac{90\% \times 1000}{7.5 \times 60} = 2 \text{ cu ft per sec}$$

For a long pipe (one greater than $4000d$),

$$d = 0.479 \left(\frac{flq^2}{h}\right)^{\frac{1}{5}}$$

The head $h = 3000 - 2500 = 500$ ft. Using $f = 0.02$.

$$d = 0.479 \left(\frac{0.02 \times 5000 \times 2^2}{500}\right)^{\frac{1}{5}} = 0.458 \text{ ft, or, say, a 6-in. pipe}$$

The approximate velocity is

$$v = \frac{2}{0.7854 \times 0.5^2} = 10.2 \text{ ft per sec}$$

From Table 43, we find that for a 0.5-ft pipe when $v = 10.2, f = 0.02$. Therefore, a 6-in. pipe could be used. Actually a larger pipe would be more desirable to allow for fouling.

The mean velocity of a long pipe is

$$v = 8.02\sqrt{\frac{hd}{fl}} = 8.02\sqrt{\frac{500 \times 0.5}{0.02 \times 5000}} = 12.67 \text{ ft per sec}$$

$$H = \frac{v^2}{2g} + m\frac{v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g}$$

$$= \frac{12.67^2}{2g}\left(1 + 0.5 + \frac{0.02 \times 2000}{0.5}\right) = 202.9, \text{ or } 203 \text{ ft}$$

$$\begin{aligned} \text{Elevation of water in the tank} &= 3000 \text{ ft} \\ H &= \underline{-203} \text{ ft} \\ & \quad\; 2797 \end{aligned}$$

Therefore, the hydraulic gradient at A has an elevation of 2797 ft. As the pipe at A has an elevation of 2800 ft, it is 3 ft above the gradient.

Compound Pipe Lines. These are lines made of different-sized pipes. The change in size (reduction or increase) should be done gradually and should not be abrupt. It should, depending on the size, extend over a distance of several feet. This prevents a loss in head because of a sudden change in velocity.

Let d_1, d_2, d_3, etc. be the diameters; l_1, l_2, l_3, etc. be the lengths; and v_1, v_2, v_3, etc. the velocities in the various sections. Loss of head due to entrance and curvature may usually be neglected. The discharge q is the same at all sections for a given constant discharge at the end of the pipe. The head lost in friction is

$$h = f_1\frac{l_1v_1^2}{d_12g} + f_2\frac{l_2v_2^2}{d_22g} + f_3\frac{l_3v_3^2}{d_32g} + \cdots$$

If q equals the discharge and we have steady flow, then

$$v_1 = \frac{q}{\frac{\pi}{4}d_1^2} \qquad v_2 = \frac{q}{\frac{\pi}{4}d_2^2} \qquad v_3 = \frac{q}{\frac{\pi}{4}d_3^2}$$

Substituting these values in the formula for h and solving for q,

$$q = \frac{\pi}{4}\sqrt{\frac{2gh}{f_1\frac{l_1}{d_1^5} + f_2\frac{l_2}{d_2^5} + f_3\frac{l_3}{d_3^5}}}$$

Large-diameter pipe lines are usually made of wood, riveted steel plates or welded steel plates. The same formulas and principles apply for these materials as for the pipes previously discussed. Values for f are about the same. A limited number of values for f may be obtained from books on hydraulics.[1]

Pipe with a Nozzle.[2] Applications of pipes with nozzles are of interest to the mining engineer because of their use in hydraulicking.

It is desirable that the pressure at entrance to the nozzle be as great as possible; therefore, the loss of head in the pipe should be a minimum. Let

h = total head on end of nozzle
D = exit diameter of nozzle
V = velocity of issuing stream
d = diameter of pipe
v = velocity of water through pipe
l = length of pipe

Then the effective velocity head of the issuing stream equals $V^2/2g$, and the lost head is $h - (V^2/2g)$.

The lost head is caused by

1. Entrance
2. Friction in the pipe
3. The nozzle

From this we may set up the expression

$$h - \frac{V^2}{2g} = m\frac{v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g} + m'\frac{V^2}{2g}$$

where $m = \left(\frac{1}{c_1^2}\right) - 1$ (c_1 is the coefficient of velocity at entrance)

$m' = \left(\frac{1}{c_1^2}\right) - 1$ (c_1 is coefficient of velocity for the nozzle)

For m, we may use 0.5, as previously explained. For m', the coefficient for velocity is used. This value is 0.98. It takes into consideration all the losses in the nozzle; so the length need not be known. The velocities v and V vary inversely as the areas of the corresponding cross section, since the flow is steady.

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[1] MERRIMAN, *op. cit.*

[2] See also NEWMAN, W. A., Some Practical Points on Hydraulic Mining, *Eng. Mining J.*, **137**, No. 1, 10 (1936).

$$q = av = \frac{\pi}{4} d^2 v$$

$$q = AV = \frac{\pi}{4} D^2 V$$

Solving for V,

$$V = v \frac{d^2}{D^2}$$

Inserting this value for V in

$$h - \frac{V^2}{2g} = m \frac{v^2}{2g} + f \frac{l}{d} \frac{v^2}{2g} + m' \frac{V^2}{2g}$$

and solving for v, we get

$$v = \sqrt{\frac{2gh}{m + f\frac{l}{d} + \left(\frac{1}{c_1^2}\right)\left(\frac{d}{D}\right)^4}}$$

where $c_1 = 0.98$

The above expression for v gives the velocity in the pipe when the nozzle is attached. The velocity and discharge for the nozzle are found from $V = (d/D)^2 v$ and $q = (\pi/4)D^2 V$, respectively.

From $V = (d/D)^2 v$, we see that the greatest nozzle velocity is obtained when D is much smaller than d.

EXAMPLE

The effect of attaching a nozzle to a pipe under the following conditions will be examined.

$l = 1000$ ft $\quad d = 0.5$ ft $\quad D = 2$ in. $= 0.167$ ft
$h = 100$ ft $\quad m = 0.5$ $\quad c_1 = 0.98$ for the nozzle

The velocity in the pipe without the nozzle is

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}} = \sqrt{\frac{2 \times g \times 100}{1.5 + 0.02 \times \frac{1000}{0.5}}} = 12.5 \text{ ft per sec}$$

From Table 43, $f = 0.02$, so that no change in the factor used need be made. The velocity with the nozzle attached is

$$v = \sqrt{\frac{2 \times g \times 100}{0.5 + 0.02 \times \frac{1000}{0.5} + \left(\frac{1}{0.98^2}\right)\left(\frac{0.5}{0.167}\right)^4}} = 7.2 \text{ ft per sec}$$

The nozzle reduces the velocity in the pipe about 42.5 per cent.

$$V = \left(\frac{d}{D}\right)^2 v = 7.2 \times \left(\frac{0.5}{0.167}\right)^2 = 64.8 \text{ ft per sec}$$

The exit velocity is increased about 420 per cent by attaching a nozzle. The discharge of the pipe is

$$q = av = \frac{\pi}{4} \times 0.5^2 \times 12.5 = 2.46 \text{ cu ft per sec}$$

The discharge with the nozzle is

$$q = \frac{\pi}{4} D^2V = \frac{\pi}{4} \times 0.167^2 \times 64.8 = 1.42 \text{ cu ft per sec}$$

which is a decrease of about 42 per cent.

The velocity head corresponding to $v = 12.5$ ft per sec is

$$h = \frac{v^2}{2g} = \frac{12.5^2}{2g} = 2.43 \text{ ft}$$

For the V at the nozzle

$$h = \frac{64.8^2}{2g} = 65.5 \text{ ft}$$

The velocity head with the nozzle is about 27 times as great as in the pipe.

The diameter of the supply pipe is from 2.25 to 2.33 times the diameter of the nozzle.[1] Actually the ratio of reduction will depend on the purpose of the giant. A high velocity with corresponding decrease in quantity of water may be necessary to disintegrate tightly cemented material. On the other hand, a lower velocity with an increase in the quantity of water delivered would be desirable for loose gravel. Newman has found that the forgoing solution checks practical observations very closely.

FLOW OF WATER IN CONDUITS AND RIVERS

The necessary utilization of water in creeks and rivers, as well as of water conducted through ditches and other types of conduits, is in many instances of great importance to the successful operation of mineral properties. For this reason a brief discussion of this part of hydraulics will not be out of place.

TERMS USED

Conduit. By conduit is meant troughs, sewers, channels, large pipes, etc. They may be open or closed.

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[1] Young, G. J., "Elements of Mining," 4d ed., p. 463, McGraw-Hill Book Company, Inc., New York, 1946.

River. Practically the same principles apply to rivers as to open conduits. The friction coefficients usually differ.

Wetted Perimeter. By this is meant that part of the conduit or river-channel cross section which is actually wetted by the water; *e.g.*, if a circular sewer of diameter d is half full of water, the wetted perimeter is

$$p = \pi \frac{d}{2}$$

Hydraulic Radius. The hydraulic radius r of a conduit is the area of the water cross section divided by the wetted perimeter. If a is the area of the cross section, then $r = a/p$. The hydraulic radius of a circular cross section full of water is

$$r = \frac{a}{p} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$$

This is also true of a section half full of water.

Slope of the Water Surface. The slope of the water surface s is the ratio of the fall h to the slope distance l in which the fall occurs.

$$s = \frac{h}{l}$$

It must be remembered that l is the inclined distance and not the horizontal.

Mean Velocity. The flow of water in a channel is said to be steady when the same quantity of water per second passes any number of given sections.

$$q = a_1v_1 = a_2v_2 = a_3v_3 = \cdots$$

When q is known, the mean velocities are

$$v_1 = \frac{q}{a_1} \qquad v_2 = \frac{q}{a_2} \qquad v_3 = \frac{q}{a_3}$$

The mean velocity is therefore that velocity which, multiplied by the area of the cross section, gives the discharge, or

$$v = \frac{q}{a}$$

Formula for Mean Velocity. In order to determine the discharge of a conduit, the mean velocity must be known. There have been many experiments performed to study this subject. It is not yet thoroughly understood. For most purposes, the present knowledge of the subject is sufficiently accurate for the mining engineer.

Without going through the derivation of the formula,[1] we find that

$$v = \mathbf{c}\sqrt{rs}$$

where r = hydraulic radius
s = slope of water surface
$\mathbf{c}$ = coefficient that varies with roughness of conduit and other conditions

The expression $\mathbf{c}\sqrt{rs}$ is known as "Chézy's formula." The coefficient $\mathbf{c}$ is the Chézy coefficient. Chézy's formula may also be obtained as follows:

From the formula for pipes, $h = f\frac{l}{d}\frac{v^2}{2g}$, we substitute for d its value expressed as the hydraulic radius, which is $4r = d$.

$$h = \frac{f}{4}\frac{l}{r}\frac{v^2}{2g}$$

$$v^2 = 8g\frac{h}{l}\frac{r}{f}$$

and

$$v = \sqrt{\frac{8g}{f}} \cdot \sqrt{rs}$$

The Chézy coefficient is $\sqrt{8g/f}$, and f is the friction factor corresponding to an iron pipe of diameter equal to the conduit, and carrying water at the same velocity. In English units $\mathbf{c}$ lies between 30 and 160. It varies with the hydraulic radius, the slope, and the roughness of the conduit's surface.

Circular Conduits. Circular conduits are usually large wood, steel, clay, or concrete pipes. When a circular conduit runs either full or half full, the hydraulic radius is $d/4$, and the Chézy formulas for mean velocity and discharge are

$$v = \mathbf{c}\sqrt{rs} = \frac{\mathbf{c}}{2}\sqrt{ds} \qquad \text{and} \qquad q = av = \frac{\mathbf{c}}{2}a\sqrt{ds}$$

where $r = d/4$ and a is either the area of the circular section or one-half that area.

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[1] Merriman, *op. cit.*, p. 275.

The values of **c** in Table 45 are for conduits with fairly smooth surfaces and no sharp bends.[1]

In using the formula and the table, the same procedure is followed as in solving pipe problems. For this purpose a mean value of **c** = 125 is generally used. Various substitutions are made until v and q are determined. Usually two or three trials are necessary before the probable mean velocity is found.

TABLE 45. VALUES FOR **c** IN CHÉZY'S FORMULA

Dia., ft.	Velocity, ft. per second						
	1	2	3	4	6	10	15
1	96	104	109	112	116	121	124
1 5	103	111	116	119	123	129	132
2	109	116	121	124	129	134	138
2 5	113	120	125	128	133	139	143
3	117	124	128	132	136	143	147
3.5	120	127	131	135	139	146	151
4	123	130	134	137	142	150	155
5	128	134	139	142	145	155	
6	132	138	142	145	150		
7	135	141	145	149	153		
8	137	143	148	151			

EXAMPLE

The velocity and discharge of water from a semicircular conduit 5 ft in diameter, laid on a grade of 0.2 ft in 100 ft, are required.

$$r = \frac{d}{4} = \frac{5}{4} \qquad \text{and} \qquad s = \frac{0.2}{100} = 0.002$$

Using $v = 125\sqrt{rs} = 125\sqrt{5/4 \times 0.002} = 6.25$ ft per sec

From Table 45 a 5-ft conduit with a velocity of 6.25 ft per sec gives **c** = 145.

$$v = 145\sqrt{5/4 \times 0.002} = 7.25 \text{ ft per sec}$$

Again consulting the table, **c** = 146.6

$$v = 146.6\sqrt{5/4 \times 0.002} = 7.33 \text{ ft per sec}$$

which is probably close to being correct.

$$q = av = \frac{\pi \times 5^2}{2 \times 4} \times 7.33 = 71.9 \text{ cu ft per sec}$$

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[1] MERRIMAN, *op. cit.*, p. 277.

Diameter of a Circular Conduit. To find the diameter of a circular conduit to discharge a given quantity of water under a specified slope, the area a is expressed in terms of d in $q = a\mathrm{c}\sqrt{rs}$.

$$q = \frac{\pi}{4} d^2 \mathrm{c} \sqrt{\frac{d}{4} s} = \frac{\pi}{8} \mathrm{c} \sqrt{d^5} \cdot \sqrt{s}$$

$$d^5 = \left(\frac{8q}{\pi c \sqrt{s}}\right)^2$$

$$d = \left(\frac{8q}{\pi \mathrm{c} \sqrt{s}}\right)^{\frac{2}{5}}$$

For a conduit half full, $d = (16q/\pi\mathrm{c}\sqrt{s})^{\frac{2}{5}}$.

If a circular conduit is filled to any depth d', the following formulas are used to obtain the hydraulic radius.

$$\text{Wetted perimeter} = p = \frac{\pi}{2} d + d \arcsin \frac{2d' - d}{d}$$

$$\text{Area of water surface} = a = \frac{d}{4} p + \left(d' - \frac{d}{2}\right) \sqrt{d'(d - d')}$$

By higher mathematics it can be shown that the velocity is greatest when $d' = 0.81d$ and the discharge is a maximum when $d' = 0.95d$, where d is the diameter. Therefore, for maximum discharge the conduit should not run completely full of water.

Rectangular Conduits. In designing an open rectangular conduit, there is a certain ratio of width to depth that gives maximum efficiency; *i.e.*, the discharge is greatest for the minimum amount of material required for construction.

Let b = breadth of water section
d = depth of water section
area $= a = bd$
$p = 2d + b$

By calculus the most economical ratio is found to be $b = 2d$. The velocity and discharge are

$$v = \mathrm{c}\sqrt{rs} \qquad \text{and} \qquad q = av = a\mathrm{c}\sqrt{rs}$$

The determination of values for **c** is difficult. They vary with the slope and roughness in addition to the hydraulic radius. For conduits where $b = 2d$, the values of **c** for circular conduits may be used with sufficient accuracy for most practical purposes. If b is not equal to $2d$, other values must be used for **c**. Table 46 gives such values.[1] These conduits had a slope of $s = 0.0049$.

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[1] MERRIMAN, *op. cit.*, p. 284.

TABLE 46. COEFFICIENTS OF c FOR RECTANGULAR CONDUITS

Unplaned plank b = 3.93 ft.		Unplaned plank b = 6.53 ft.		Neat cement b = 5.94 ft.		Brick b = 6.27 ft.	
d	c	d	c	d	c	d	c
0.27	99	0.20	89	0.18	116	0.20	89
0.41	108	0.30	101	0.28	125	0.31	98
0.67	112	0.46	109	0.43	132	0.49	104
0.89	114	0.60	113	0.56	135	0.57	105
1.00	114	0.72	116	0.63	136	0.65	105
1.19	116	0.78	116	0.69	136	0.71	106
1 29	117	0.89	118	0.80	137	0.85	107
1 46	118	0.94	120	0.91	138	0 97	110

Kutter's Formula. Ganguillet and Kutter discussed and correlated all the known gagings of channels in 1869. From these data they deduced a formula for calculating c in Chézy's formula. They expressed c in terms of the hydraulic radius, slope, and degree of roughness of the surface. The coefficient c can be computed when these factors are known.

When r is in feet and v is in feet per second, Kutter's formula for c is expressed as follows:

$$\mathbf{c} = \frac{\dfrac{1.811}{n} + 41.65 + \dfrac{0.00281}{s}}{1 + \dfrac{n}{\sqrt{r}}\left(41.65 + \dfrac{0.00281}{s}\right)}$$

Values for n depend on the roughness of the surface. Values of n for materials of interest in mining work are[1]

n = 0.009 for well-planed timber
n = 0.010 for neat cement
n = 0.011 for cement with ⅓ sand
n = 0.012 for unplaned timber
n = 0.020 for canals in very firm ground
n = 0.025 for canals and rivers free of stones and weeds
n = 0.030 for canals and rivers with some stones and weeds
n = 0.035 for canals and rivers in bad condition

Kutter's formula is very widely used. For practical purposes it gives good results. It does not give extremely accurate values for streams moving at very low velocities or very high velocities. It should be used as a check against all values of c obtained from other sources. The formula may be in error from 5 to 10 per cent.

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[1] MERRIMAN, *op. cit.*, p. 288*a*. (Abstracted from the table given.)

Ditches and Canals. These types of conduits usually have trapezoidal cross sections. The slope depends on the difference in elevation between the inlet and outlet. For work of this kind, **c** in Chézy's formula is best computed by means of Kutter's formula.

EXAMPLE

It is required to find the discharge for a trapezoidal ditch in gravel and sand, where

$$s = 0.001 \qquad b = 5.0 \text{ ft (bottom)} \qquad d = 4.0 \text{ ft} \qquad n = 0.025$$

The sides make an angle of 40 deg with the horizontal.

The width of the water surface at the top is

$$2 \times \left(\frac{4}{\cot 50°}\right) + 5.0 = 14.5 \text{ ft}$$

$$\text{Area} = a = \frac{5 + 14.5}{2} \times 4 = 39.0 \text{ sq ft}$$

$$\text{Wetted perimeter} = p = 5 + 2 \times \left(\frac{4}{\sin 40°}\right) = 17.5 \text{ ft}$$

$$\text{Hydraulic radius} = r = \frac{a}{p} = \frac{39.0}{17.5} = 2.23$$

Substituting these values in Kutter's formula,

$$\mathbf{c} = \frac{\frac{1.811}{0.025} + 41.65 + \frac{0.00281}{0.001}}{1 + \frac{0.025}{\sqrt{2.23}}\left(41.65 + \frac{0.00281}{0.001}\right)} = 66.9 \text{ or } 67$$

$$v = \mathbf{c}\sqrt{rs} = 67\sqrt{2.23 \times 0.001} = 3.2 \text{ ft per sec}$$

$$q = av = 39 \times 3.2 = 124.9 \text{ cu ft per sec}$$

From Table 47 it is found that a velocity of 3.2 ft per sec is too great for sand and gravel.[1] It would be advisable to widen the ditch and thus reduce the velocity.

Transporting Capacity.[2] Material is carried either in suspension or by rolling; or by both. It has been found that the diameter of an object, which is just moved by the pressure of a current of water, varies as the square of the velocity. The weight of the body that is moved varies as the sixth power of the velocity.

When water causes sand or pebbles, or other material, to roll along the bed of a channel, it must exert a force approximately proportional to the square of the velocity and to the area exposed. If d is the diameter of the body and **C** is a constant, then the force **F** required to

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[1] YOUNG, *op. cit.*, p. 440.

[2] MERRIMAN, *op. cit.*, pp. 294, 339.

TABLE 47. MATERIAL AND VELOCITY TO MOVE IT

Velocity, Ft per Sec	Material
0.25	Will just erode fine clay
0.5	Will lift fine sand or loam and earth
0.75	Will lift sand as coarse as linseed
1.00	Will move fine gravel
2.00	Will roll rounded pebbles 1 in. in diameter
3.00	Will sweep along slippery, angular stones the size of an egg
5.3	Will move pieces 3 to 4 in. in diameter
6.3	Will move pieces 6 to 8 in. in diameter
10.0	Will move pieces 12 to 18 in. in diameter

move the object in a horizontal direction is

$$F = Cd^2v^2$$

When motion just occurs, the force is proportional to the weight. (Frictional resistance is proportional to weight.) The weight of a sphere is proportional to the cube of the diameter. Therefore

$$d^3 = Cd^2v^2$$

or

$$d = Cv^2$$

Since d varies as v^2, the weight, which is proportional to d^3, must vary as v^6. From this it is seen that a small increase in velocity causes a much greater increase in transporting capacity.

EXAMPLE

A rock 3 in. in diameter is rolled by a current having a velocity of 5 ft per sec. What size stone will a current of three times this velocity roll?

$$\begin{aligned} d_1:d &= Cv_1^2:Cv^2 \\ d_1:3 &= C(3 \times 5)^2:C \times 5^2 \\ d_1 &= 3\left(\frac{C \times (3 \times 5)^2}{C \times 5^2}\right) = 27 \text{ in.} \end{aligned}$$

The subject of transporting capacity is of more than passing importance in many alluvial mining methods. It is necessary that the water have sufficient velocity to move the largest stones that are to pass through the sluice box. Or, if a hydraulic method is used, the stream of water must attain sufficient velocity to move the largest material to be handled.

The foregoing discussion, while more or less approximate, will if applied save considerably on cut-and-try methods.

MEASUREMENT OF WATER

The quantity of water passing a specified point is measured by either direct or indirect methods.

Direct Methods

1. The water is measured by catching the water in a vessel and weighing. The time to fill is recorded.

2. The water is measured by catching in a vessel of known capacity. The time is recorded.

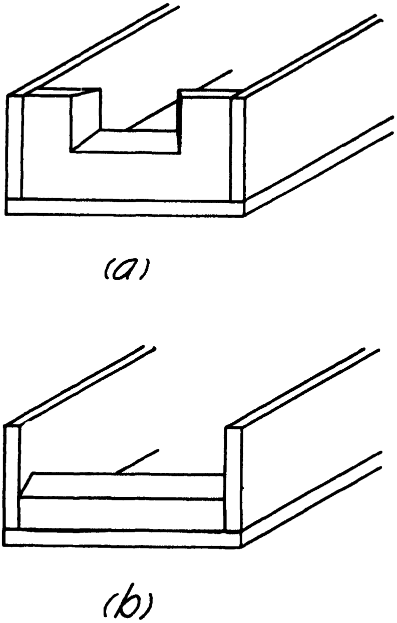

FIG. 153. Standard weirs: (a) contracted; (b) end contractions suppressed.

3. The water is measured by measuring the quantity in a reservoir. The time to fill is noted.

4. The velocity is measured. From $q = av$, the quantity is determined.

Indirect Methods. A device known as the "weir" is the most satisfactory; it is the only one that will be discussed. The weir is indirect because certain coefficients are necessary, depending on the head and the width of the weir crest.

Weirs. The notch in the weir may be almost any shape. The one most commonly used is the standard weir. It has a rectangular opening and may be one of two types: (1) with end contractions; (2) with end contractions suppressed. Figure 153 illustrates the two types. The lower edge, over which the water passes, is called the "crest." The head H on the crest is measured by means of a hook gage or other device, depending on the accuracy desired. The measurement must take place several feet upstream from the crest in order to approach the true head. This distance is 2 to 3 ft for small weirs, and 6 to 8 ft for very large ones. The hook gage should not be near the sides of the channel or weir. Figure 154 shows the position of the gage and the head H which is measured.

The value of H given by the hook gage is not the true one. It is a very slight amount too small. This is because the water surface slopes back from the weir for a considerable distance, giving a velocity of approach.[1] However, this correction is so small that, for most practical purposes, it is disregarded.[1] For refined measurements standard references should be consulted.

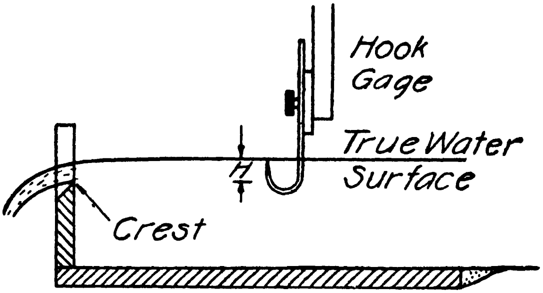

FIG. 154. Location of hook gage for weir.

Discharge over a Weir. For a weir with end contractions, length of crest b, and head H on the crest, the theoretical discharge, with certain modifications, is found from the formula for rectangular vertical orifices.

The theoretical discharge is

$$Q = \tfrac{2}{3} \sqrt{2g}\, bH^{\frac{3}{2}}$$

The actual discharge is

$$q = \mathbf{c}\tfrac{2}{3} \sqrt{2g}\, bH^{\frac{3}{2}}$$

where **c** is the coefficient of discharge for a contracted weir. The same formula is used for suppressed weirs, except that different values for **c** must be used. Table 48 gives values of **c** for contracted weirs.[2] Table 49 gives values of **c** for suppressed weirs.[2] Interpolation can be made in the tables, for either H or b, by assuming that the values vary uniformly.

EXAMPLE

Find the discharge over a contracted weir 2.5 ft long, when H is 0.35 ft.

From Table 48, **c** is 0.6175 for $H = 0.3$ ft; and 0.611 for $H = 0.4$ ft, which gives $(0.6175 + 0.611)/2 = 0.6143$ for $H = 0.35$ ft.

$$q = \mathbf{c}\tfrac{2}{3}\sqrt{2g}\, bH^{\frac{3}{2}}$$
$$q = 0.6143 \times \tfrac{2}{3} \times \sqrt{2g} \times 2.5 \times 0.35^{\frac{3}{2}} = 1.68 \text{ cu ft per sec}$$

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[1] MERRIMAN, *op. cit.*, p. 147.

KING and WISLER, "Hydraulics," 2d ed., p. 112, John Wiley & Sons, Inc., New York.

[2] MERRIMAN, *op. cit.*, pp. 150, 152.

. TABLE 48. COEFFICIENTS FOR CONTRACTED WEIRS

Effective head, ft. = H	Length of weir, ft. = b						
	0.66	1	2	3	5	10	19
0 1	0.632	0.639	0.646	0.652	0.653	0.655	0.656
0 15	0.619	0.625	0.634	0.638	0.640	0.641	0.642
0 2	0.611	0.618	0.626	0.630	0.631	0.633	0.634
0 25	0.605	0.612	0.621	0.624	0.626	0.628	0.629
0.3	0.601	0.608	0.616	0.619	0.621	0.624	0.625
0 4	0.595	0.601	0.609	0.613	0.615	0.618	0.620
0 5	0.590	0.596	0.605	0.608	0.611	0.615	0.617
0 6	0 587	0.593	0.601	0.605	0.608	0.613	0.615
0.7	.....	0.590	0.598	0.603	0.606	0.612	0.614
0 8	.....	.....	0.595	0.600	0.604	0.611	0.613
0 9	. . .	.....	0.592	0.598	0.603	0.609	0.612
1 0	.	.....	0.590	0.595	0.601	0.608	0.611
1 2	.	...	0.585	0.591	0.597	0.605	0.610
1 4		.	0.580	0.587	0.594	0.602	0.609
1 6	..	. .	.....	0.582	0.591	0.600	0.607

TABLE 49. COEFFICIENTS FOR SUPPRESSED WEIRS

Effective head ft. = H	Length of weir, ft. = b						
	19	10	7	5	4	3	2
0 1	0.657	0.658	0.658	0.659			
0 15	0.643	0.644	0.645	0.645	0.647	0.649	0.652
0 2	0.635	0.637	0.637	0.638	0.641	0.642	0.645
0 25	0.630	0.632	0.633	0.634	0.636	0.638	0.641
0 3	0.626	0.628	0.629	0.631	0.633	0.636	0.639
0 4	0.621	0.623	0.625	0.628	0.630	0.633	0.636
0.5	0.619	0.621	0.624	0.627	0.630	0.633	0.637
0.6	0.618	0.620	0.623	0.627	0.630	0.634	0.638
0.7	0.618	0.620	0.624	0.628	0.631	0.635	0.640
0 8	0.618	0.621	0.625	0.629	0.633	0.637	0.643
0 9	0.619	0.622	0.627	0.631	0.635	0.639	0.645
1 0	0.619	0.624	0.628	0.633	0.637	0.641	0.648
1.2	0.620	0.626	0.632	0.636	0.641	0.646	
1.4	0.622	0.629	0.634	0.640	0.644		
1 6	0.623	0.631	0.637	0.642	0.647		

Francis' Formula. For weirs where the length of the crest is less than 4 ft and the head less than 0.4 ft Francis' formula may be used.

For suppressed weirs,

$$q = 3.33bH^{3/2}$$

For contracted weirs,

$$q = 3.33(b - 0.2H)H^{3/2}$$

The number 3.33 is equal to $\mathbf{c} \cdot \frac{2}{3}\sqrt{2g}$, where **c** is the true coefficient of discharge. If an error of about 1 per cent is of no consequence, Francis' formula may be used under the restrictions previously given.

For ordinary mining purposes: underground drainage ditches, surface conduits, etc., the contracted weir is usually used.

The following example illustrates the application of a contracted weir for measuring the flow of water in an underground drainage ditch. Such a determination is necessary in order to measure water handled by pumps; water being made on various levels; or water from adjoining properties which must be handled. A permanent measuring gage is attached to the weir box. By computing for various heads on the weir crest, the gallons per minute can be read directly from the gage.

EXAMPLE

Figure 155 shows the weir. Taking $b = 1$ ft and values of H equal to 0.1; 0.15; 0.2; 0.25; 0.3; and 0.4, compute q in cubic feet per second for the six values of H and convert to gallons per minute. Values of H are then plotted against the values of q in gallons per minute. Values of H to the nearest 0.01-ft can then be interpolated from the curve obtained.

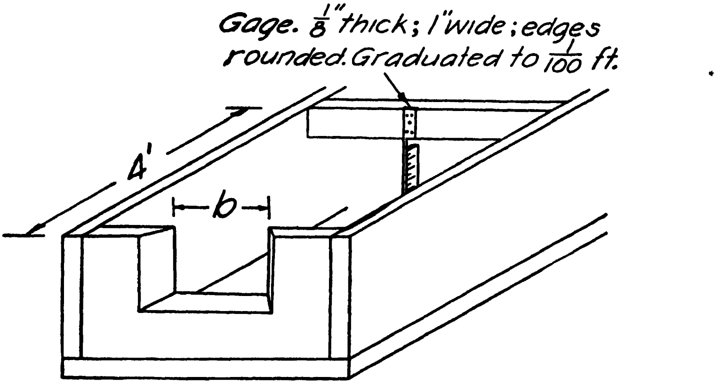
Gage. $\frac{1}{8}$" thick; 1" wide; edges rounded. Graduated to $\frac{1}{100}$ ft.
4'
b

FIG. 155. Weir and gage for measuring water in an underground ditch.

$$q = \tfrac{2}{3}\mathbf{c}\sqrt{2g}bH^{3/2}$$

From Table 48 and the above formula, we get the following:

H	c when $b = 1$ ft.	q, cu. ft. per sec.	G.p.m.
0 1	0.639	0.108	48.5
0 15	0.625	0.184	87.1
0 2	0.618	0.295	132.5
0 25	0.612	0.409	184.0
0 3	0.608	0.534	239.5
0 4	0.601	0.812	364.5

$$\text{Gpm} = q \times 7.48 \times 60$$

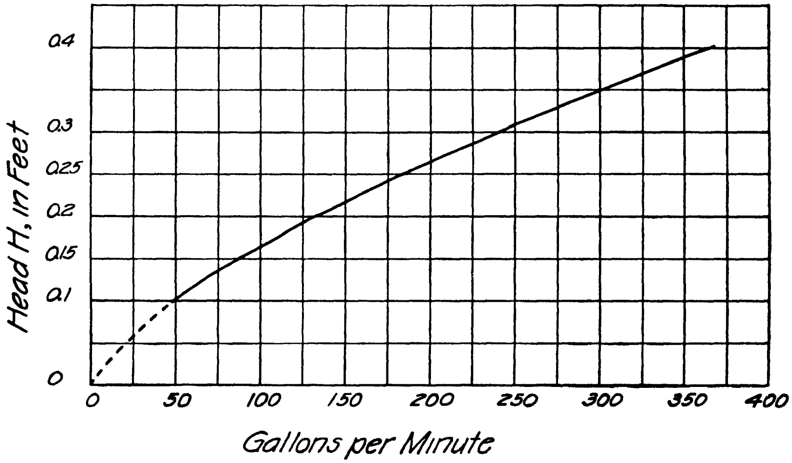

FIG. 156. Curve for converting head to gallons per minute. Crest equals 1 ft.

Figure 156 shows the plotting of the above data. From this curve we make up the following table:

H	Gpm
0.15	87.0
0.16	97.0
0.17	105.0
0.18	114.0
0.19	122 5
0.20	132.5
0.21	142.5
0.22	152.5
0.23	162.5
0.24	172.5
0.25	184.0

To use this table it is necessary only to read the gage and then refer to the table or curve in Fig. 156 to determine the gallons per minute equivalent to the head shown on the gage. In reading the gage, care should be taken not to confuse the true reading with that indicated by the surface tension (meniscus) of the water.

PUMPING THROUGH PIPES

In order to reduce as much as possible the horsepower necessary to operate the pump, the diameter of the pipes carrying the water should be carefully selected.[1] This reduces the velocity and in turn reduces the friction loss.

There are in general three types of pumps used in mining operations:

1. Piston or reciprocating
2. Plunger (also a type of reciprocating pump)
3. Centrifugal

Piston. The piston, or, as it is commonly known, the reciprocating pump, may be single- or double-acting. It may consist of one cylinder, two (duplex), three (triplex), etc. This type offers more trouble because of leaks, friction, etc., than the plunger pump. This is because the cylinder must be accurately bored and the piston well packed. Many mines use the reciprocating pump.

Plunger. The essential difference between the plunger and piston pump is that the plunger works through a small stuffing box of short length. Only the stuffing box is packed.

Both these pumps are of the reciprocating type. At one time practically all mine pumps were of the reciprocating type. In recent years the centrifugal pump has become highly developed and is rapidly replacing the reciprocating pump for most mine requirements. Under extremely high heads, when it is desirable to raise the water in one lift, the reciprocating pump compares favorably with the centrifugal. For most applications the reciprocating pump is very much heavier, requires a great deal more space, and takes, as a rule, a more expensive power unit. These disadvantages are offset by a much higher efficiency.

Centrifugal Pumps. Of the three types mentioned, the centrifugal pump has the lowest efficiency. It operates at a very much higher speed. It may be had in single-stage, double-stage, three-stage, etc., installations.[2] The size of a centrifugal pump is indicated by the diameter in inches of the discharge opening, and the number of stages. Centrifugal pumps are used extensively for unwatering mine workings. They are lighter, occupy less space, and cost less than the other types.

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[1] On p. 393 will be presented a comparison of pipes enlarging this statement.

[2] Usually about 250 ft of head per stage is allowed for in practice. This is not the limit by any means.

They are not subjected to damage if started with the discharge valve closed. The main drawback is probably the necessity of priming before starting. Many ingenious devices for ensuring automatic priming are described in the books and articles listed at the end of this chapter.

Centrifugal pumps are designed to perform at maximum efficiency under certain rather definite conditions. The characteristic curves (capacity head, efficiency, and brake horsepower) for each type of pump give a clear picture of the pumps performance. For maximum results, a pump must be selected whose characteristic curves satisfy the pipe-line characteristic. It is easy to select a pump with a rather wide range of capacity and head without sacrificing too great a change in efficiency or having too great a variation in the horsepower. Ordinarily a mine pump will operate under a definite head with practically a constant capacity. Pumps supplying reservoirs and tanks, in many cases, pump against a minimum and a maximum head. The usual mine pump will have to operate against a minimum and maximum suction head—although this variation will be small when compared with the total head. Figure 157 shows the characteristic curves of a typical centrifugal pump.[1] It will be noted in the figure that the head may vary from about 136 ft to about 105 ft with only an extreme change of from 68 to 70 per cent in efficiency and from a little over 12 hp to a maximum of about 13 hp. The change in capacity ranges

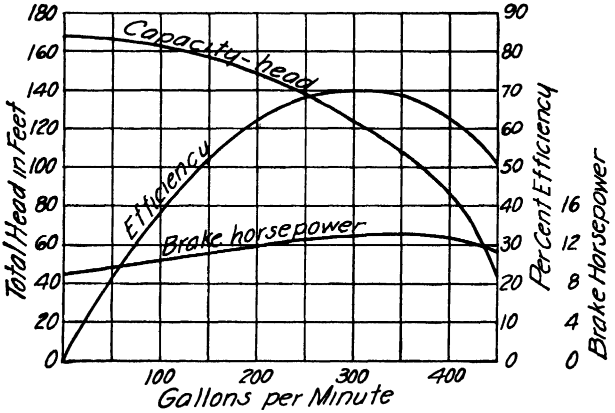

FIG. 157. Characteristic curve for centrifugal pump. (*Kristal and Annett, "Pumps."*)

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[1] KRISTAL, F. A., and F. A. ANNETT, "Pumps," Fig. 339, McGraw-Hill Book Company, Inc., New York.

from 250 to 350 gpm. It is worth noting that the brake-horsepower curve is of the nonoverloading type. This pump would properly be selected for 325 gpm against a *total* head of about 115 ft. The efficiency would be 70 per cent. By total head is meant the pipe-line characteristic. This is made up of the suction head, static head, and pipe-line friction losses in the suction and discharge lines, and in some cases the velocity head in the suction pipe, if this pipe is small and the lift is high.

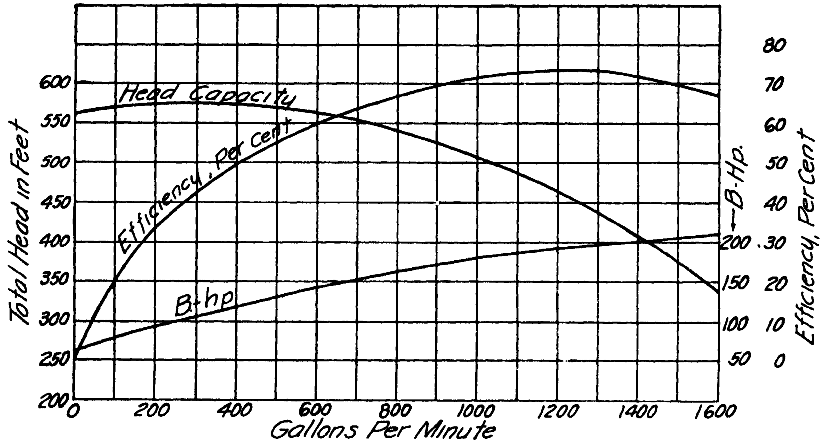

FIG. 158. Characteristic curves of a centrifugal mine pump. (*Courtesy of Eng. Mining J.*)

Figure 158 shows the characteristic curves[1] for a pump to meet the following requirements. Mine workings that contained 53,000,000 gal with a daily increase of 40,000 gal were to be unwatered. The maximum lift was 500 ft. The mine was to be unwatered in 2 months or less. Calculations indicated that 1000 gpm should be handled. The equipment chosen was a 5-in. two-stage vertical sinker pump complete with pipe and connections and a 200-hp 1750 rpm 2300-volt 60-cycle three-phase vertical, solid-shaft, squirrel-cage induction motor with a hand-compensator type of starter. The pump was equipped with bronze runners, wear rings, shaft sleeves, and cases.

The horsepower necessary to operate a pump is obtained from the following formula:

$$\text{Hp} = \frac{G \times \text{head} \times 8.33}{33{,}000 \times E}$$

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[1] WEAVER, LAMAR, Mine Pumps and Their Selection, Part I, *Eng. Mining J.*, **137,** No. 12, 621 (1936).

where G = gpm
head = discharge head + suction head + friction head in pipes (this should include the fittings)
8.33 = weight of 1 gal of water, lb
E = efficiency of pump, drive, and motor

In general, the duty of a centrifugal pump is affected in the following way (theoretically),

1. Change in diameter of impeller
    - *a*. Capacity varies as the diameter.
    - *b*. Head varies as the square of the diameter.
    - *c*. Power varies as the cube of the diameter.
2. Change in speed
    - *a*. Capacity varies as the speed.
    - *b*. Head varies as the square of the speed.
    - *c*. Power varies as the cube of the speed.
    - *d*. Efficiency remains the same.

There are many instances where multistage centrifugal-pump installations are operating against heads in excess of 2000 ft.

Raising Water by Suction. Mean atmospheric pressure at sea level is 14.75 lb per sq in. The weight of 1 cu ft of water may be taken as 62.4 lb. The pressure per square inch exerted by a column of water 1 ft high is

$$\frac{62.4}{144} = 0.434 \text{ lb per sq in.}$$

From this we can find the height of the water column necessary to balance the atmospheric pressure at any given elevation, for example, at sea level

$$\frac{14.75}{0.434} = 34 \text{ ft (approximately)}$$

At an elevation of 5000 ft above sea level, the atmospheric pressure is 12.20 lb per sq in., and the height of the water column is

$$\frac{12.20}{0.434} = 28.2 \text{ ft}$$

Table 50 gives the barometric pressure for different altitudes.

In practice it has been found impossible to raise water the theoretical height of 34 ft because a perfect vacuum cannot be obtained. Under ideal conditions a lift of about 28 ft is the maximum that can be made when at sea level (about 80 per cent of the theoretical).[1] This lift

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[1] For practical work, 63 per cent of the theoretical.

TABLE 50. ATMOSPHERIC PRESSURES AT VARIOUS ALTITUDES

Elevation, ft.	Barometric pressure, in.	Atmospheric pressure, lb. per sq. in.	Elevation, ft.	Barometric pressure, in.	Atmospheric pressure, lb. per sq. in.
0	29.92	14.7	5,000	24.78	12.18
500	29.40	14.44	6,000	23 86	11.72
1,000	28.87	14.18	7,000	22.96	11.28
1,500	28.34	13.91	8,000	22.10	10.86
2,000	27.80	13.67	9,000	21.29	10.45
2,500	27.26	13.38	10,000	20.47	10.06
3,000	26.76	13.16	11,000	19 72	9 70
3,500	26.26	12.90	12,000	18.98	9.34
4,000	25.76	12 67	13,000	18.27	8 98
4,500	25.26	12.41	14,000	17.59	8.32

can be made when the temperature of the water is about 32°F. As the temperature increases, the height of lift decreases. The following table[1] gives a general idea as to this decrease.

Temperature, °F	Height of Water Column*
32	34 0 ft
60	33 4
90	32 4
120	30 0
160	23.0
200	6.0
212	0 0

* Barometer at 30 in. of mercury in all cases.

From the foregoing discussion it is seen that the theoretical height must be considerably corrected. The pump should be placed as close to the water surface as possible (in case of suction lift), and in the case of very hot water the suction head should be negative; *i.e.*, the water should flow into the pump by aid of gravity.

EXAMPLE

Select a pump and motor for the following duty: The static head (suction and discharge) is 1000 ft; the maximum quantity of water is 300 gpm; the pump is 20 ft from the shaft; the suction head does not exceed 10 ft; 20 ft of pipe from sump to pump; in the line there is one gate valve, six standard elbows, one check valve, and one foot valve. Assume pump efficiency at 61 per cent for calculations.

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[1] MERRIMAN, *op. cit.*, p. 507.

The economical pipe size must first be decided upon. The friction loss in the pipe is found from

$$v = \frac{0.002225G}{\pi d^2/4} \quad \text{fps}$$

and

$$h'' = 0.000000125 \frac{fl}{d^5 G^2} \quad \text{ft}$$

or tables such as those in "Cameron Hydraulic Data" may be used.

The loss in 4-in. and 5-in. pipes will be compared.

For the 4-in. pipe,

$$v = 7.7 \text{ fps}$$

From Table 43, $f = 0.023$. Increasing this value of f to allow for future fouling of the pipe,

$$f = 0.023 \times 1\tfrac{1}{2} = 0.0345$$

and

$$h'' = 91 \text{ ft} \qquad \text{per 1000 ft}$$

For the 5-in. pipe,

$$v = 4.9 \text{ fps}$$

From Table 43, $f = 0.023$. Increasing this value of f to allow for future fouling,

$$f = 0.023 \times 1\tfrac{1}{2} = 0.0345$$

and

$$h'' = 31 \text{ ft} \qquad \text{per 1000 ft}$$

By using the 5-in. pipe, 60 ft of friction head is saved. This amounts to

$$\begin{aligned} \text{Hp} &= \frac{G \times H \times 8.33}{33{,}000 \times E} \\ &= \frac{300 \times (91 - 31) \times 8.33}{33{,}000 \times 0.61} = 7.5 \text{ hp} \end{aligned}$$

With a motor efficiency of 90 per cent,

$$\text{Kw input} = \frac{7.5 \times 0.746}{0.90} = 6.2 \text{ kw}$$

If the power costs 1 cent per kilowatt-hour, the 5-in. pipe will save

$$6.2 \times 24 \text{ hr} \times 1 \text{ cent} = \$1.49 \text{ per day.}$$

A 5-in. pipe will be used.

Total pipe and fittings resistance

Total pipe (1000 + 20 + 20)	1040 ft
From Table 44:	
5-in. gate valve	3 ft
Six 5-in. elbows at 13 ft	78 ft
5-in. check valve	34 ft
5-in. foot valve (same as check valve)	34 ft
Total	1189 ft

$$\text{Total friction loss} = 31 \times {}^{1189}\!/_{1000} = 37 \text{ ft}$$

$$\text{Pumping head} = 1000 + 37 = 1037 \text{ ft}$$

To this is added 40 ft for contingencies.

$$\text{Total head} = 1037 + 40 = 1077 \text{ ft}$$

Consulting a manufactuer's catalogue, a pump whose characteristic curves satisfy the requirements of 300 gpm under a 1077-ft head is selected. From such curves the efficiency is found to be 65 per cent at 3450 rpm for a No. 2, six-stage pump.

The No. 2 pump has a 3-in. intake and a 2-in. discharge. By the use of eccentric reducers, the pump is connected to the suction pipe and the discharge pipe.

$$\text{Hp} = \frac{300 \times 1077 \times 8.33}{33{,}000 \times 0.65} = 125.4 \text{ hp}$$

A 125-hp motor would prove satisfactory.

Two two-stage pumps with 65 per cent efficiency might also be used.

Before final selection of the pump is made, the motor speed must be known by the manufacturer so that he can correct the impeller diameter for developing the necessary head.

The preceding example has been worked on the basis of 24-hr operation of the pump. Mine pumps usually do not run continuously. A sump is provided of such capacity that the pump has occasional periods for "resting." Intermittent or continuous operation depends on the quantity of water to be handled. The capacity of the pump will depend on the amount of water the mine is making, the capacity of the sump or storage reservoir, and on the time it takes the sump to fill. These various costs should be investigated and the most economical setup chosen.

As a rule, stand-by pumps are provided. In recent years the practice has developed of installing identical units. This simplifies maintenance and reduces the warehouse inventory. Occasionally the pumps are so selected that one will handle the minimum flow and the other the maximum flow. During an emergency both pumps would operate. At some mines the flow of water is seasonal.

Pipe-line Characteristic. The pipe-line characteristic arises from the static head, suction and discharge heads; pipe friction and friction of fittings; entrance loss; and in exceptional cases velocity head. Loss at entrance may be disregarded if the foot valves or strainer head is properly selected. The area of the entrance to the suction pipe should be at least three times the suction pipe.[1] The size of the suction pipe should be such that the velocity of the water does not exceed 8 ft per sec. This usually means a suction pipe one or more sizes larger than the pump intake. The suction pipe should slope gently down from the pump intake before turning into the sump. An eccentric reducer should be used for connecting the pipe to the intake and a long-radius elbow is used for changing the direction of the pipe. Gate valves should not be put in the suction pipe. When the eccentric reducer is used, the horizontal side must be on top. The slope of the pipe from the pump is not less than a 1-in. drop per 12 in. Every precaution should be taken to prevent air pockets from forming in the suction line.

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[1] "Cameron Pump Operators Data," 1st ed., p. 28, Ingersoll-Rand Company.

The friction losses may be obtained from tables usually found in manufacturers' catalogues or, in the absence of such tables, from

$$h'' = \frac{fl}{d}\frac{v^2}{2g}$$

already explained on page 361. Careful consideration must be given to the value assigned to f (page 363).

When the sump is lower than the pump intake (which is usually the case at mining properties), the pump should be located as near the sump as possible to reduce friction losses in the suction line. The ideal arrangement would place the pump lower than the sump.

To find data for the pipe-line characteristic, a pipe size is selected. Then from $q = av$ the velocity is determined for a range of q through which the pump may be called upon to operate. Consulting the friction-loss tables, the head equivalent to the friction is found; or the formula for h'' may be used. Most of the tables have gallons per minute plotted against pipe diameter so that the determination of v is not necessary.

When tables are not available, h'' may be found from

$$q = av$$

$$v = \frac{q}{a} = \frac{q}{\pi \frac{d^2}{4}} = \frac{0.002225G}{\pi \frac{d^2}{4}} \quad \text{ft per sec}$$

Converting over to units involving gallons per minute,

$$h'' = 0.000000125f\frac{l}{d^5}G^2$$

where G = gpm and the other members are in the same units as $h'' = (fl/d)\ (v^2/2g)$, the formula from which it was derived.

Pumps in Series or Parallel. Figure 159 shows the results obtained by operating two pumps of equal rating in series or parallel.[1] Curve A is for a single pump; curve *B* for parallel operation; and curve *C* for series operation. It will be observed that in parallel the head remains the same while the capacity doubles. But with the pumps in series, the head doubles and the capacity remains unchanged. The efficiency of either of these arrangements is practically the same as for a single pump. If separate motors for each pump are used, they should have practically the same speed characteristics.

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[1] KRISTAL and ANNETT, *op. cit.*, Fig. 353, p. 283.

The first consideration in connecting pumps in series is to be sure that safe suction pressures are not exceeded on the second pump. The limit on mine pumps, ordinarily, is about 50 lb per sq in. If the suction flange is reinforced, the two pumps may be placed side by side. With ordinary pipe, the second pump will have to be sufficiently above the other pump to reduce the intake pressure. The lower pump must be started first, thus ensuring that the second pump becomes primed.

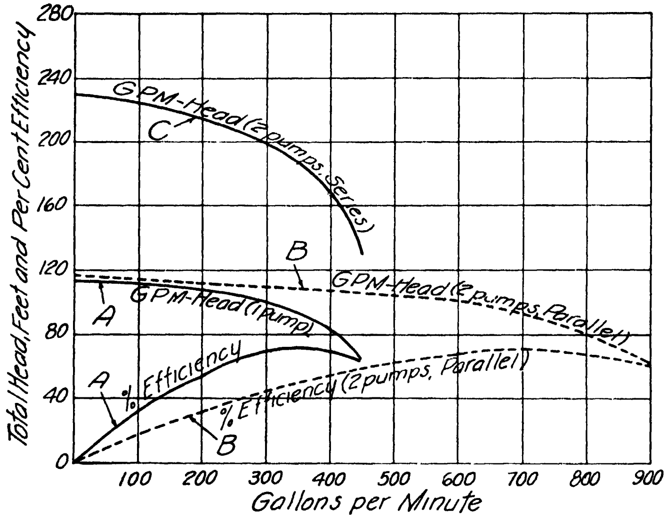

FIG. 159. Centrifugal pumps in parallel and series. (*Kristal and Annett, "Pumps."*)

Pumps can be connected in series when both are of the same head and capacity or when they have different heads and the same capacity. If they do not have equal capacities, then the series operation is limited to the output of the smaller pump.

In parallel operation the pumps should have separate suction lines, as trouble develops with a common suction header. The size of the discharge line should be increased to handle the double volume flowing from the two pumps.

CHOICE OF PIPE DIAMETER WHEN PUMPING WATER[1]

The following table[1] illustrates a method for determining the most economical pipe size. It is based on cost of the pipe, depreciation of the pipe, and cost of the power. Small pipe costs less than large pipe, but on the other hand the friction head is greater, thus increasing the power cost.

Dia. of pipe, in.	Cost of pipe installed	12 Per cent of pipe cost (yearly cost)	Pipe fric. head, ft.	Kw.-hr. per year lost in pipe friction	Cost of power at 4¢ a kw.-hr.	Total cost per year
2	$ 800	$ 96	358	28,900	$1156	$1252
2½	1150	138	120	9700	388	526
3	1500	180	49 6	4000	160	340
4	2300	276	12.2	987	39	315
5	3000	360	4 1	331	13	373

The above table is based on a pump supplying 100 gpm through a pipe 1000 ft long. The efficiency of the pump and motor is taken at 70 per cent, and the cost of electrical energy at 4 cents per kilowatt-hour.

Explanation of Items. The cost of the pipe includes labor and materials to install the pipe ready for use. The interest rate on this work is 7 per cent. The life of the pipe is taken at 25 years, so that 4 per cent each year must be put aside to replace the pipe at the end of this period. One per cent is allowed for minor repairs, such as painting and usual maintenance. Thus the total (7 + 4 + 1) of 12 per cent of the pipe cost is obtained. The pipe friction is obtained either from tables or by means of the formula

$$h'' = f \frac{l}{d} \frac{v^2}{2g}$$

The values in column (5) are found from

$$\text{Kw-hr} = \frac{\text{gpm} \times \text{head} \times 8.33 \times 0.746 \times 3000}{33{,}000 \times 0.70}$$

The head used is the friction head. It alone should be considered for the purpose of this comparison. The gravity head would be the same regardless of the pipe size selected.

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[1] Pump-Fax, Gould Pump Company, Inc., *Bull.* 400, p. 30. (By permission.)

The 3000 is the number of hours that the pump operates each year. From the power cost of 4 cents per kilowatt-hour the sixth column is obtained. The total cost per year is column (3) plus column (6).

At face value it would appear that the 4-in. pipe would be the cheapest size. The cost of a pump for the 5-in. pipe might be cheaper. However, with the comparison of pipes used, there would be required a reduction of $400 in pump cost to make the 5-in. pipe compare with the 4-in. pipe. For the pipes considered this is impossible.

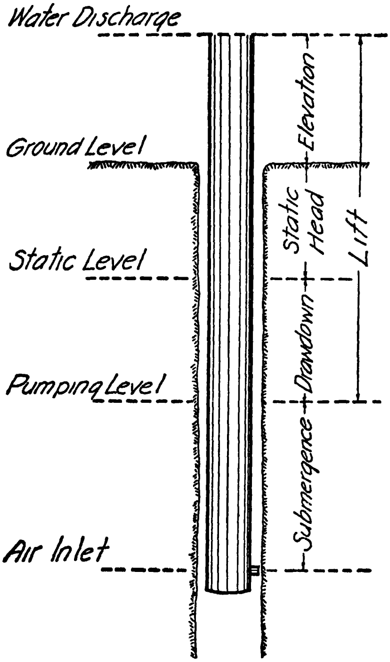

Fig. 160. The air-lift pump. (*Courtesy of Compressed Air Mag.*)

A complete comparison would require that the pump and motor necessary for each installation be determined, and about 15 per cent of their cost added to the total yearly cost. It is difficult to do this because of lack of information regarding the cost of pumps. The information used gives a sufficiently close comparison.

If the operating time had been 8760 hr per year instead of 3000, the 5-in. pipe would be selected. The lower power cost for the 5-in. pipe would pull the total down.

Air-lift Pumping. The air-lift pump has a wide use in pumping water, oil, or mill tailings or pulps, and various chemical solutions. It may be used in wells, rivers, ponds, or lakes, and mine workings. Figure 160 illustrates the terms used in air-lift pumping.[1] The effectiveness of the air lift depends on the submergence of the foot piece below the pumping level. Submergence is expressed as a percentage of the total depth (discharge to bottom of air inlet).

It is found from

$$\frac{\text{Depth} - \text{lift}}{\text{Depth}} \times 100 = \text{per cent of submergence}$$

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[1] O'Neil, F. W. (ed.), "Compressed Air Data," 4th ed., p. 167, Compressed Air Magazine, New York, 1934.

Table 51 gives submergence data.[1]

The greatest efficiency is obtained when the pumping level remains nearly constant. This may be closely approached in pumping many wells or handling mill pulps or tailings. It will be more difficult when unwatering flooded mine workings. The length of the discharge pipe or total depth may be found from Table 51.

TABLE 51. CUSTOMARY ALLOWABLE AND BEST SUBMERGENCES

Lift, ft	Customary allowable percentage of submergence	Best percentage of submergence	Single-stage or compound air compressors
20	55–70	65–70	Single
30	55–70	65–70	Single
40	50–70	65–70	Single
50	50–70	65–70	Single
60	50–70	65–70	Single
80	50–70	65–70	Single
100	45–70	65–70	Single
125	45–65	65	Single
150	40–65	60–65	Single
175	40–60	55–60	Single
200	40–60	55–60	Compound
250	40–60	55–60	Compound
300	37–55	50–55	Compound
350	37–55	50–55	Compound
400	37–50	45–50	Compound
450	35–45	40–45	Compound
500	35–45	40–45	Compound
550	35–45	40–45	Compound
600	35–45	40–45	Compound
650	35–45	40–45	Compound
700	35–40	40	Compound

$$\text{Depth} = \text{submergence} + \text{lift} = \frac{\text{submergence}}{\text{per cent of submergence}}$$

Let d = depth
s = submergence
l = lift
p = per cent of submergence

---

[1] *Ibid.*, p. 170.

then

$$s + l = \frac{s}{p}$$

and

$$s = \frac{pl}{(1 - p)}$$

$$d = s + l = \frac{pl}{(1 - p)} + l$$

$$d = \frac{l}{(1 - p)}$$

Slippage of air through the water and friction of water in the discharge line are largely responsible for losses. As the discharge velocity increases, the slippage decreases and the friction increases. Friction causes an entrance loss at the foot piece. A constant velocity of fluid through the discharge pipe is most desirable. This may require a tapered discharge pipe.

With a straight pipe, the best discharge velocity of the air-water mixture is as follows:

The *lift* is 40 to 200 ft; the *velocity* is 2000 ft per min at 35 per cent submergence to 700 ft per min at 70 per cent submergence.

With a tapered discharge pipe:

The *lift* is 40 to 200 ft; the *velocity* is 1400 ft per min at 35 per cent submergence and 550 ft per min at 70 per cent submergence.

The best velocity at entrance to the discharge pipe for the air-water mixture is as follows:

The *lift* is 40 to 200 ft; the *velocity* is 800 ft per min at 35 per cent submergence and 450 ft per min at 70 per cent submergence.

The diameter of the discharge pipe is found from

$$Q = av = \frac{\pi d^2}{4} v$$

$$d = 13.54 \sqrt{\frac{Q}{v}} \qquad \text{inside diameter of pipe, in.}$$

Q = volume of the air-water mixture cu ft per min
v = velocity of mixture in pipe ft per min

The quantity Q is the volume of the water being pumped plus the volume of compressed air at any particular point.

The diameter of the air pipe should be such that the velocity of the transmitted air is 30 to 40 ft per sec. In general the friction drop should not exceed 3 lb.

Experiments indicate that the quantity of air required may be found from

$$V_a = \frac{H_1}{C \log_{10}\left(\frac{H_s + 34}{34}\right)}$$

where V_a = cu ft free air required per gal of water
H_1 = total lift, ft
H_s = working submergence, ft
C = a constant (see Table 52)

TABLE 52. SUBMERGENCE CONSTANTS

Submergence, per cent	75	70	65	60	55	50	45	40	35
C, outside air line. .	366	358	348	335	318	296	272	246	216
C, inside air line	330	322	306	285	262	238	214	185	162

The air pressure is found from

$P_s = 0.434\, H_m$ lb per sq in. starting pressure
$P_m = 0.434\, H_s + P_f$ lb per sq in. working pressure

where H_m = starting submergence, ft
H_s = working submergence, ft
P_f = friction drop in air line, compressor to foot piece, lb per sq in.

If a fluid other than water, or a mixture of solids and water is being pumped, then the above formulas must be multiplied by the specific gravity of the mixture.

In some operations the starting pressure may be so great that the compressor becomes dangerously overloaded. This would result when there is a considerable difference between the static level and the pumping level (see Fig. 160). This difficulty may be overcome by placing an auxiliary air lift above the main pump. The auxiliary would have a submergence with respect to the static head that would take the maximum pressure of the compressor. When the static level is reduced, the auxiliary pump is gradually turned off and the main pump on.

Foot Piece. The device used to connect the air line to the discharge pipe is known as the "foot piece," or "pump." When selecting a foot piece, the following points should be kept in mind:[1]

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[1] *Ibid.*, p. 174.

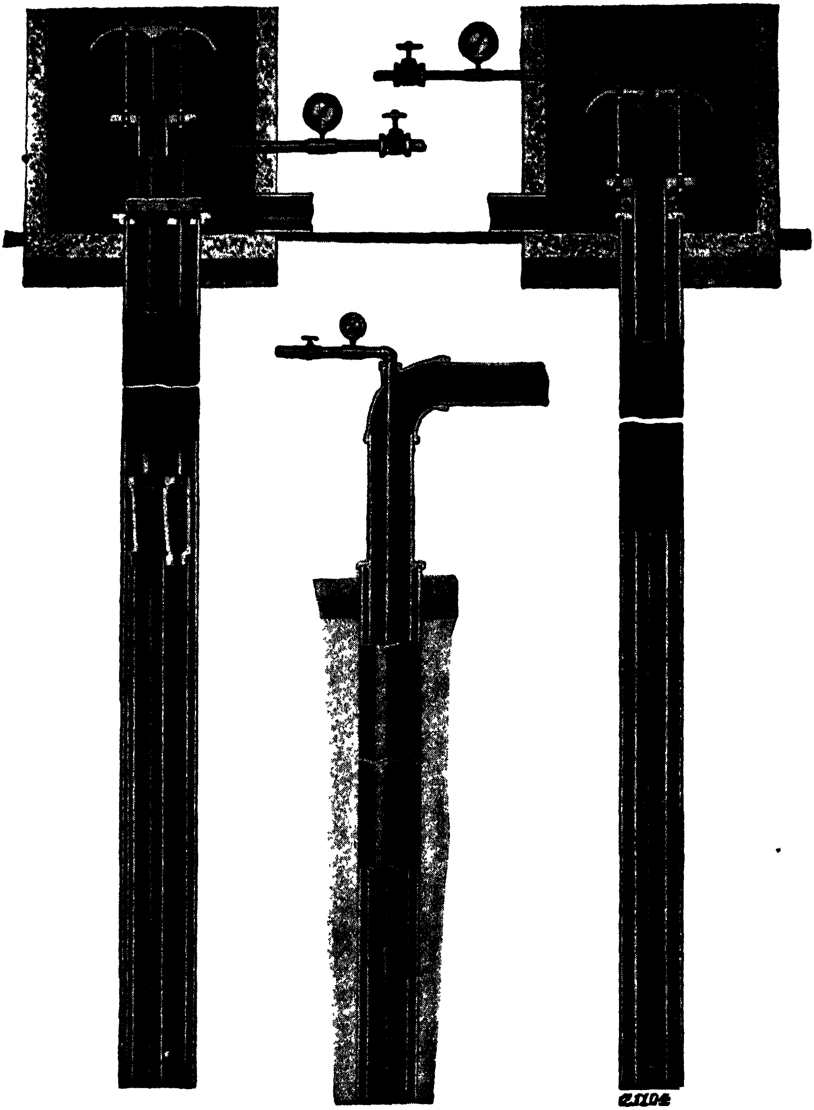

FIG. 161. Types of air-lift foot pieces. (*Ingersoll-Rand Company.*)

1. It should divide the air into small streams without loss in pressure.
2. The air outlet should be spaced properly to prevent any sudden rush of air and lightening of the discharge column.
3. There should be no moving parts present.
4. It should be designed to give an unobstructed water passage.

The cross-sectional area should equal that of the discharge pipe at the bottom.

5. It should be designed so that scale and dirt passing down the air or discharge pipe will not clog the holes but will pass through the pump and fall to the bottom of the well.

Figure 161[1] shows several types of foot pieces. The outside air line is the most desirable. When the well or opening is small, the inside air line is used. In some wells an air line with a perforated end is simply run down inside the casing and the casing used for a discharge pipe. It is best to have the discharge pipe connect to a sump with a deflector headpiece over the discharge end.

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[1] Ingersoll-Rand Catalogue.

CHAPTER X

COMPRESSED AIR

Compressed air is one of the most important adjuncts of up-to-date mining operations. It might even be stated, without fear of exaggeration, to be indispensable. Practically all drilling for underground blasting purposes, with the exception of electrical coal cutters, is done through the agency of compressed air. Underground hoists, blowers for ventilation, drill sharpeners, operation of pumps, and operation of diamond drills may be mentioned among its important uses.

The purpose of this chapter is not to discuss the design or operation of compressors and auxiliary equipment, but to present general data which will aid the selection of a compressor from a capacity standpoint. For a thorough treatise on compressed air, it is suggested that the reader consult the books and articles used as a reference for this chapter.

Before continuing further it is well that a few terms be defined. This will prevent possible confusion.

DEFINITIONS

Gage Pressure and Absolute Pressure. The ordinary gage registers the pressure above atmospheric pressure. The absolute pressure is the gage pressure plus the atmospheric pressure; for example, if the gage on a compressor situated at Moscow (Idaho) reads 70 lb per sq in., the absolute pressure will be

$$70 + 13.2 = 83.2 \text{ lb per sq in.}$$

(atmospheric pressure at the elevation of Moscow is about 13.2 per sq in.). For finding the equivalent atmospheric pressure at various elevations, Table 50 may be consulted.

For elevations not given in the table, the following formula can be used for determining the atmospheric pressure.[1]

$$\log P_2 = \log P_1 - \frac{h}{122.4(°\text{F} + 461)}$$

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[1] SIMONS, T., "Compressed Air," 2d ed., p. 2, McGraw-Hill Book Company, Inc., New York.

For an average temperature of 60°F,

$$\log P_2 = \log P_1 - 0.0000157h$$

where P_2 = atmospheric pressure at elevation h

h = elevation above sea level, or difference in elevation between two points, ft

P_1 = atmospheric pressure at sea level (usually taken at 14.7 lb per sq in.) or other known point

°F is usually taken with an average value of 60°F

As the laws governing expansion and compression (Boyle's and Charles's) are based on absolute temperature and pressure, it is necessary to correct ordinary temperature values to absolute and to correct gage pressure to absolute before a true solution of compressor problems can be made.

Free Air. This is the air at normal atmospheric pressure which is taken into the compressor. It is compressed to the desired gage pressure. Manufacturers' catalogues usually give the rating (capacity) of compressors in cubic feet of free air per minute at sea level. As will be shown later, this rating will be seriously in error if used for elevations other than sea level.

Isothermal Compression or Expansion. When air is compressed, heat is generated. If this heat could be removed as fast as it was generated and if the air could be kept at a constant temperature, we would have isothermal compression. When air expands, it gives up heat and the air cools below the starting temperature. If this heat could be supplied as rapidly as required, so that the air remained at a constant temperature, we would have isothermal expansion. From the above we find that an isothermal change is one in which the temperature of the gas remains constant.

Adiabatic Compression or Expansion. If the cylinder could be insulated so that none of the heat generated by compression could escape, but was retained by the air, thus raising its temperature, we would have adiabatic compression. Conversely, if we prevent the addition of heat from any external source during expansion, we would have adiabatic expansion.

Theoretically, a gas may be compressed by either of the above methods. Actually the compressor operates in such a way that the compression curve follows very closely that of perfect adiabatic compression. For most practical problems, the compressor may be considered as giving adiabatic compression, and the adiabatic constant for air is used.

For isothermal changes, PV = a constant; but for adiabatic changes PV^n = a constant, where the constant n is the ratio of the specific heat of air at constant pressure to the specific heat of air at constant volume.

$$n = \frac{C_p}{C_v} = \frac{0.2375}{0.1689} = 1.406$$

Therefore, for air

$$PV^{1.406} = \text{a constant}$$

Volumetric Efficiency. The volumetric efficiency is the ratio of the volume of free air actually admitted to the theoretical volume resulting from the displacement of the piston. Because of clearance, valve action, expansion of clearance air, etc., the volume of free air admitted is less than the displacement.

Simons[1] states that large high-class compressors should have a volumetric efficiency of 90 per cent or better; whereas small single-stage compressors will usually have a much lower efficiency.

Clearance and Capacity. The clearance is the space between the piston and the cylinder head at the end of the stroke. It is necessary to provide a space for the water that usually accumulates in the cylinder and to provide a passage for the admission and delivery of the air. Clearance may be expressed as the ratio between clearance volume and cylinder volume. It varies from 1 to 2 per cent according to Simons;[2] and from 1.5 to 2.5 per cent according to Peele.[3] The lower values are for large long-stroke compressors; and the higher values are for small short-stroke machines. The free-air capacity of a compressor is often based on the volume swept through by the piston (piston displacement), with no allowance for clearance. In selecting a compressor and driving motor, it must be known whether or not the rating is based on such an assumption. The following example taken from Simons[4] will illustrate the importance of this statement.

If the discharge pressure is 75 lb gage or 89.7 lb absolute and the initial pressure is atmospheric at sea level, *i.e.*, 14.7 lb absolute, the air remaining in the 20 cu in. clearance space will expand on the return stroke of the piston to about six times the clearance volume,* or to 120 cu in., and will, therefore, take up an additional 100 cu in. from the intake cylinder. That is, in a cylinder of 1000-cu in. piston displacement the piston must travel back 10 per cent of the return stroke before the clearance air has expanded to atmospheric pressure and before the atmospheric air is allowed to flow into the cylinder.

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[1] SIMONS, *op. cit.*, p. 40.

[2] SIMONS, *op. cit.*, p. 38.

[3] PEELE, R., "Compressed Air Plant for Mines," 5th ed., John Wiley & Sons, Inc., New York.

[4] SIMONS, *op. cit.*, pp. 38, 40. (Abstracted.)

* $\frac{89.7}{14.7} \times 20 = 122$ cu in.

The actual room for the admission of new free air is, therefore, only

$$1000 - 100 = 900 \text{ cu in.}$$

Or, as commonly stated, the volumetric efficiency of the compressor is 90 per cent ($^{900}/_{1000} \times 100 = 90$ per cent).

If we now take a compressor with a free-air rating based on piston displacement, having a piston with an area of 2 sq ft, traveling 500 ft per min the free-air capacity is $2 \times 500 = 1000$ cu ft, per min. We previously found that for a machine compressing to 75 lb gage, with a clearance of 2 per cent, the actual capacity was only 900 cu ft per min. To deliver 1000 cu ft the compressor must be speeded up to

$$^{1000}/_{900} \times 500 = 555 \text{ fpm}$$

Diameter of Cylinder for Compressing a Given Amount of Air.[1] The size of the cylinders for multistage compressors are based on the amount of free air to be compressed. The low- or initial-pressure cylinder is first determined. The diameters of the remaining cylinders are then proportioned to the low-pressure cylinder so that the work done in each cylinder is the same. The length of the stroke is the same for all cylinders. Only data for a two-stage compressor will be given. Let

V_1 = volume of free air taken into low-pressure cylinder per stroke, cu ft
V_2 = piston displacement of high-pressure cylinder
d_1 = diameter of low-pressure cylinder, in.
d_2 = diameter of high-pressure cylinder, in.
A = area of piston in low-pressure cylinder, sq in.
L = length of stroke, in.
P_a = absolute atmospheric pressure
P_1 = discharge pressure of low-pressure cylinder
P_2 = discharge pressure of high-pressure cylinder
r = ratio of compression

Then

$$V_1 = \frac{A}{144} \times \frac{L}{12} = \frac{0.7854 d_1^2 L}{1728}$$

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[1] Simons, *op. cit.*, p. 50.

or

$$d_1 = 47\sqrt{\frac{V_1}{L}} \text{ in.}$$

As the stroke is the same, the ratio of the volumes of the cylinders is proportional to the ratio of the squares of the diameters, or the inverse ratio of the pressures.

$$\frac{d_2^2}{d_1^2} = \frac{V_2}{V_1} = \frac{P_a}{P_1} = \frac{1}{\frac{P_1}{P_a}} = \frac{1}{r}$$

For a two-stage compressor,

$$\frac{1}{r} = \frac{1}{\left(\frac{P_2}{P_a}\right)^{\frac{1}{2}}} = \left(\frac{P_a}{P_2}\right)^{\frac{1}{2}}$$

or

$$\frac{d_2^2}{d_1^2} = \left(\frac{P_a}{P_2}\right)^{\frac{1}{2}}$$

and

$$d_2 = d_1\left(\frac{P_a}{P_2}\right)^{\frac{1}{4}} \text{ in.}$$

As stated before, this provides a high-pressure cylinder of such diameter that the theoretical work in both the low- and high-pressure cylinders is the same.

The correction for volumetric efficiency is made in selecting the diameter of the low-pressure cylinder. Volumetric efficiency is not considered in the high-pressure cylinder. The diameter of the low-pressure cylinder is increased an amount sufficient to correct for clearance, etc.

The actual diameter is found as follows: Let

d_1 = diameter of low-pressure cylinder, in.

x = diameter of low-pressure cylinder after correcting for volumetric efficiency, in.

E = volumetric efficiency, per cent

Then

$$\frac{x^2}{d_1^2} = \frac{100\%}{E}$$

or

$$x = d_1\sqrt{\frac{100}{E}}$$

Horsepower for Single-stage Compressor. The theoretical horsepower required for single-stage compression is found from[1]

$$\text{Hp} = \frac{144P_1V_1n}{33{,}000(n-1)}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]\frac{1}{E}$$

where V_1 = volume of free air to be compressed, cu ft per min
P_1 = initial absolute pressure, lb per sq in.
P_2 = terminal (desired) pressure, lb per sq in.
n = exponent of adiabatic compression = 1.406
E = 85 to 93 per cent, depending on the compressor; for practical problems E is usually taken at 85 per cent.

Horsepower for Two-stage Compression. The theoretical horsepower is given by[2]

$$\text{Hp} = 2\frac{144nV_aP_a}{33{,}000(n-1)}\left[\left(\frac{P_2}{P_a}\right)^{\frac{n-1}{2n}} - 1\right]\frac{1}{E}$$

where V_a = free air per minute taken into low-pressure cylinder, cu ft
P_a = atmospheric pressure in low-pressure cylinder, lb per sq in. absolute
P_2 = final absolute pressure desired from high-pressure cylinder
n = adiabatic compression exponent = 1.406
E = efficiency = 85 per cent for practical work

Choice between Single-stage and Two-stage Compression. When delivery pressures of 60 to 70 lb gage or less are required, and the compressor is situated at low altitudes, single-stage compressors are suitable. Above 70 lb at high altitudes, and up to 500 lb, two-stage compressors are usually the most economical. Above 500 lb multistage compressors should be used. Altitudes of 3000 ft or less above sea level would be considered low. Economical operation, where large volumes of air are demanded, also requires use of two- or more-stage compressors.

Effect of Altitude on Compressor. The volumetric efficiency, expressed in terms of free air, is the same at all altitudes because the piston displacement for a given size does not change. When expressed in terms of compressed air, it decreases with an increase in altitude. An example will illustrate this. One hundred cubic feet of air at atmospheric pressure at sea level (14.7 lb) is compressed isothermally to 100 lb gage. The same volume is compressed at 10,000 ft (10.07 lb).

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[1] SIMONS, *op. cit.*, p. 34.

[2] SIMONS, *op. cit.*, p. 55.

The volumes occupied are

$$\text{At sea level} = 100 \times \frac{14.7}{100 + 14.7} = 12.8 \text{ cu ft}$$

$$\text{At 10,000 ft} = 100 \times \frac{10.07}{100 + 10.07} = 9.1 \text{ cu ft}$$

The volumetric efficiency in terms of compressed air is

$$\frac{9.1}{12.8} \times 100 = 71.7\%$$

In order that the compressor at 10,000 ft altitude deliver the same volume of compressed air as at sea level, the intake cylinder must be made proportionally larger. This requires factors for converting free air at sea level to an equivalent amount of free air at various altitudes.

Factors for Altitude Compensation. The necessary factor is easily determined as follows:[1] Let

V = volume of free air to be compressed, both at sea level and any elevation above sea level, cu ft
P_a = absolute atmospheric pressure at sea level = 14.75 lb per sq in.
P_1 = absolute atmospheric pressure at desired elevation
p = gage pressure of delivered air
V_1 = volume of air after being compressed to p lb at sea level
V_2 = volume of air after being compressed to p lb at desired elevation

Then

$$V_1 = V \frac{P_a}{(p + P_a)}$$

and

$$V_2 = V \frac{P_1}{(p + P_1)}$$

In order that V_2 may equal V_1, it must be multiplied by some factor, F.

$$FV_2 = V_1$$

Substituting values in the above,

$$FV \frac{P_1}{(p + P_1)} = V \frac{P_a}{(p + P_a)}$$

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[1] Simons, *op. cit.*, p. 60.

from which

$$F = \frac{P_a(p + P_1)}{P_1(p + P_a)}$$

EXAMPLE

What is the factor at 8000 ft elevation and 100 lb gage?

$$F = \frac{14.75(100 + 10.87)}{10.87(100 + 14.75)} = 1.311$$

One hundred cubic feet of free air at sea level requires

$$100 \times 1.311 \doteq 131.1 \text{ cu ft}$$

at 8000 ft elevation to give the same effect.

AIR REQUIRED BY ROCK DRILLS

The air consumed by the various drills used for blasting purposes is at best but an approximation. It depends on the bore of the drill's air chamber; the length of the stroke; the speed of the hammer or other operating mechanism; the hardness of the rock being drilled; the experience of the operator; the condition of the drill; etc. For estimating purposes, as a guide toward selecting a compressor, the data issued by various drill manufacturing companies are usually considered sufficient. The data from various sources agree by no means. Table 53 is probably as satisfactory as any.[1]

TABLE 53. CUBIC FEET OF FREE AIR REQUIRED TO RUN ONE RECIPROCATING PISTON DRILL

Gage press., lb.	Cylinder diameter of drill, in.												
	2	2¼	2½	2¾	3	3⅛	$3\frac{3}{16}$	3¼	3½	3⅝	4¼	5	5½
60	50	60	68	82	90	95	97	100	108	113	130	150	164
70	56	68	77	93	102	108	110	113	124	129	147	170	181
80	63	76	86	104	114	120	123	127	131	143	164	190	207
90	70	84	95	115	126	133	136	141	152	159	182	210	230
100	77	92	104	126	138	146	149	154	166	174	199	240	252

The air consumption varies almost directly as the absolute pressure; for example, the consumption for a 2-in. drill at 100 lb gage, when the consumption at 60 lb gage is known, is

$$\frac{100 + 14.7}{60 + 14.7} \times 50 = 76.7, \text{ or } 77 \text{ cu ft}$$

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[1] Ingersoll-Rand Company, Products Catalog 9207, Engineering Data, p. 18.

Air Consumption of Drills at Altitudes. The free-air consumption at sea level (Table 53) must be multiplied by the proper factor to correct the consumption over to equivalent consumption at the desired elevation; for instance, the 2-in. drill at 90 lb gage requires 70 cu ft of free air per minute at sea level. At 8000 ft elevation, it would require

$$70 \times 1.26 = 88.2, \text{ or } 88 \text{ cu ft}$$

Table 54 gives the multipliers for various elevations.[1]

Capacity of Compressor to Operate More Than One Drill. It has been found from experience that, as a rule, the load factor for operating more than one drill depends on the number of drills in use at one time. For only one drill, a 100 per cent load factor is necessary. For two drills, it would undoubtedly be the same. When more than two drills are in use, all of them will probably not be in operation at the same time.

The percentage of the total number of drills actually in operation at the same time is

No. of Drills	Per Cent
1	100
2	90–100
3	90
4	85
5	82
6	80
7	77
8	75
9	72
10	71
15	63
25	55
50	51
70	47

The loss in capacity for an increase in elevation is about 3 per cent for each 1000 ft of elevation. The "number-of-drills" factor must be increased by this amount.

EXAMPLE

If 15 drills are to be operated, the factor will be 63 per cent × 15 = 9.45. If operation is at 6000 ft elevation, the factor becomes

$$100 + 3 \times {}^{6000}\!/_{1000} = 118\%$$
$$1.18 \times 9.45 = 11.15$$

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[1] Ingersoll-Rand Company, Products Catalog 9207, Engineering Data, p. 18.

TABLE 54. MULTIPLIERS TO DETERMINE CAPACITY OF COMPRESSORS REQUIRED TO OPERATE ROCK DRILLS AT ALTITUDES COMPARED WITH SEA LEVEL

Altitude above sea level, ft.	Number of drills																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	50	60	70
	Multipliers																		
0	1.00	1.80	2.70	3.40	4.10	4.80	5.40	6.00	6.50	7.10	8.10	9.50	11.70	13.70	15.80	21.40	25.50	29.40	33.20
1,000	1.03	1.85	2.78	3.50	4.22	4.94	5.56	6.18	6.69	7.30	8.34	9.78	12.05	14.10	16.30	22.00	26.26	30.30	34.20
2,000	1.07	1.92	2.89	3.64	4.39	5.14	5.78	6.42	6.95	7.60	8.67	10.17	12.52	14.66	16.90	22.90	27.28	31.46	35.52
3,000	1.10	1.98	2.97	3.74	4.51	5.28	5.94	6.60	7.15	7.81	8.91	10.45	12.87	15.07	17.38	23.54	28.05	32.34	36.52
4,000	1.14	2.05	3.08	3.88	4.67	5.47	6.15	6.84	7.41	8.09	9.23	10.83	13.34	15.62	18.01	24.40	29.07	33.52	37.80
5,000	1.17	2.10	3.16	3.98	4.80	5.62	6.32	7.02	7.61	8.31	9.48	11.12	13.69	16.03	18.49	25.04	29.84	34.40	38.84
6,000	1.20	2.16	3.24	4.08	4.90	5.76	6.48	7.20	7.80	8.52	9.72	11.40	14.04	16.44	18.96	25.68	30.60	35.40	39.84
7,000	1.23	2.21	3.32	4.18	5.04	5.90	6.64	7.38	7.99	8.73	9.96	11.68	14.39	16.85	19.43	26.32	31.36	36.16	40.84
8,000	1.26	2.27	3.40	4.28	5.17	6.05	6.80	7.56	8.19	8.95	10.21	11.97	14.74	17.26	19.90	26.96	32.13	37.04	41.83
9,000	1.29	2.32	3.48	4.39	5.29	6.19	6.96	7.74	8.38	9.16	10.45	12.26	15.09	17.67	20.38	27.60	32.90	37.92	42.83
10,000	1.32	2.38	3.56	4.49	5.41	6.34	7.13	7.92	8.58	9.37	10.69	12.54	15.44	18.08	20.86	28.25	33.66	38.80	43.82
12,000	1.37	2.47	3.70	4.66	5.62	6.57	7.40	8.22	8.90	9.73	11.10	13.02	16.03	18.77	21.64	29.32	34.94	40.28	45.48
15,000	1.43	2.57	3.86	4.86	5.86	6.86	7.72	8.58	9.30	10.15	11.58	13.58	16.73	19.59	22.59	30.60	36.46	42.04	47.47

Drill company catalogues usually contain tables (see Table 54) with the above factors, or multipliers as they term them, calculated for various altitudes.[1] Under ordinary circumstances they would be used. If such tables were not available, the foregoing discussion would suffice.

A comparison of such tables with the method outlined shows the following difference.

EXAMPLE

Ten 3-in. drills are to be operated at a gage pressure of 80 lb at 7000 ft elevation. What must be the capacity of the compressor (*a*) from Table 54; (*b*) by calculating from previous data?

a. From Table 53 a 3-in. drill when operating at 80 lb requires 114 cu ft of air per minute. Ten of these drills have a factor (Table 54) of 8.73 at 7000 ft.

114 × 8.73 = 995 cu ft of free air per minute to be delivered by the compressor.

b. As in (*a*), one drill requires 114 cu ft of free air.

The factor at 7000 ft is

$$F = \frac{14.75(80 + 11.3)}{11.3(80 + 14.75)} = 1.258$$

The load factor for operating 10 drills is 71 per cent. Therefore, air is required for but 71 per cent × 10 = 7.1 drills.

The factor for 7000 ft elevation = 1.258 × 7.1 = 8.93.

8.93 × 114 = 1018 cu ft of free air per minute to be delivered.

Another method would be as follows:

For each 1000 ft of elevation, the factor for sea-level operation must be increased by 3 per cent. The factor for sea level for 10 drills is

$$71\% \times 10 = 7.1$$

7000 ft elevation would give

$$7000/1000 \times 3 = 21\%$$

Factor at 7000 ft = 1.21 × 7.1 = 8.59.

Volume of air required = 8.59 × 114 = 979 cu ft.

We find then that, using (*a*) as an index, the first part of (*b*) gives 2.4 per cent more air than required; and the second part of (*b*) gives 1.6 per cent less air than required. Either of the three methods is sufficiently accurate when we consider that the capacity of the compressor would be increased by 25 per cent as a factor of safety.

Drilling Rate. The size of the compressor will depend, as a rule, on the number of drills. To determine the number of drills required, we must know the average drilling rate of each type of drill and the tonnage or cubic feet of rock broken by the drill holes. There is a very meager amount of information available on drilling rates. Seaman[1] gives a limited amount, but does not state the nature of the rock.

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[1] The multipliers in Table 54 correct for both "number of drills" and increase in altitude.

Table 55 is composed of data taken from a U.S. Bureau of Mines publication.[2]

TABLE 55. COMPARATIVE DRILLING SPEED OF PISTON AND HAMMER DRILLS IN VARIOUS TYPES OF ROCKS*

Rock	Drilling speed, in. per min.		Remarks
	Hammer	Piston	
Granite	8–20	8–10	
Shale	.....	8–13	
Shale and sandstone	30	....	Medium hard
Gneiss	8–16	8–10	
Trap	.....	2	Exceptionally hard
Breccia	12–15		
Rhyolite	10	8–10	
Andesite	15–20	5–10	
Diabase	.....	8–10	Hammer machine probably drills about the same
Quartzite	.....	6–8	

* Includes time used in setting up and tearing down column and bar; in shifting machine to new holes; and in changing steel; but does not include time used in mucking for set-up or in loading, blasting, and clearing smoke. Drifter type of machine used.

Mills[3] gives the information presented in Table 56 and Table 57.

TABLE 56. DRILLING SPEED WITH INGERSOLL-RAND 248 MACHINE (DRIFTER) Air Pressure 90 Lb

Ground	Average drilling speed, in. per minute	
	Gage of bit $2\frac{1}{4}$ in.	Gage of bit $1\frac{15}{16}$ in.
Massive sulfide	3.3	4.2
Fresh porphyry	5.5	7.9

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[1] SEAMAN, H. W., Relation of Air Pressure to Drilling Speed of Hammer Drills, *Trans. AIMME*, **66**, 743 (1921).

[2] BRUNTON and DAVIS, Safety and Efficiency in Mine Tunneling, *U.S. Bur. Mines Bull.* **57**, 106 (1914).

[3] MILLS, C. E., Drilling and Blasting Underground at United Verde Mine, *Eng. Mining J.*, **121**, 877 (1926).

TABLE 57. DRILLING SPEED IN VARIOUS CLASSES OF GROUND

Ground Classification	Approximate Drilling Speed, In. per Min
Very good	18
Good	12
Medium	8
Hard	4

From the same article the following information was obtained:

	Hard iron	Fresh porphyry	Schist and medium porphyry
Average drilling speed, in. per min	6	10	14

If the drilling rate can be approximated, the total footage of holes[1] and thus the tonnage may be found from Figs. 162, 163, and 164.

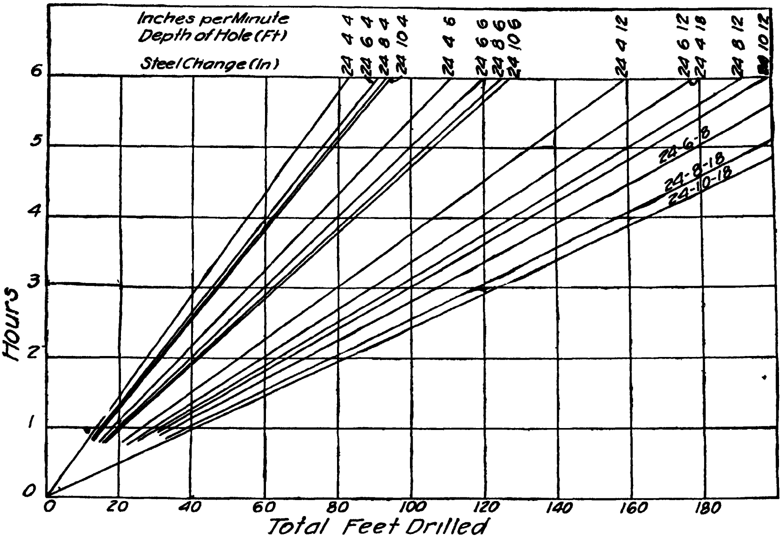

FIG. 162. Footage drilled per shift with stopers. (*Young*, "*Elements of Mining.*")

The cubic feet of material to be assigned to the drilled footage is difficult to determine. It will depend upon the nature of the rock; the type of round; number of free faces; strength and characteristics of explosives used; etc. Table 58, condensed from the original table in Peele,[2] gives data that may be useful for estimating purposes.

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[1] YOUNG, G. J., "Elements of Mining," p. 115.

[2] PEELE, R. (ed.), "Mining Engineers' Handbook," 2d ed., p. 235, John Wiley & Sons, Inc., New York.

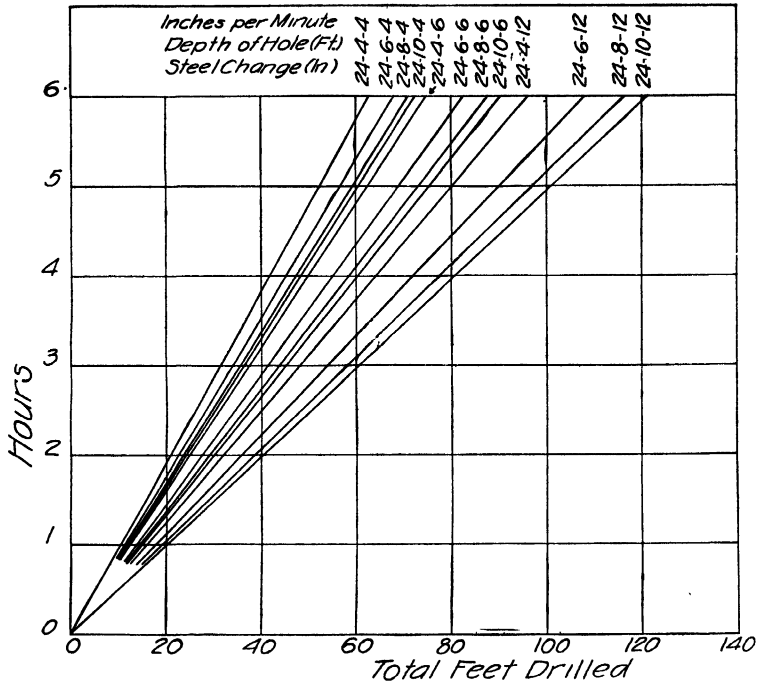

FIG. 163. Footage drilled per shift with drifters. (*Young, "Elements of Mining."*)

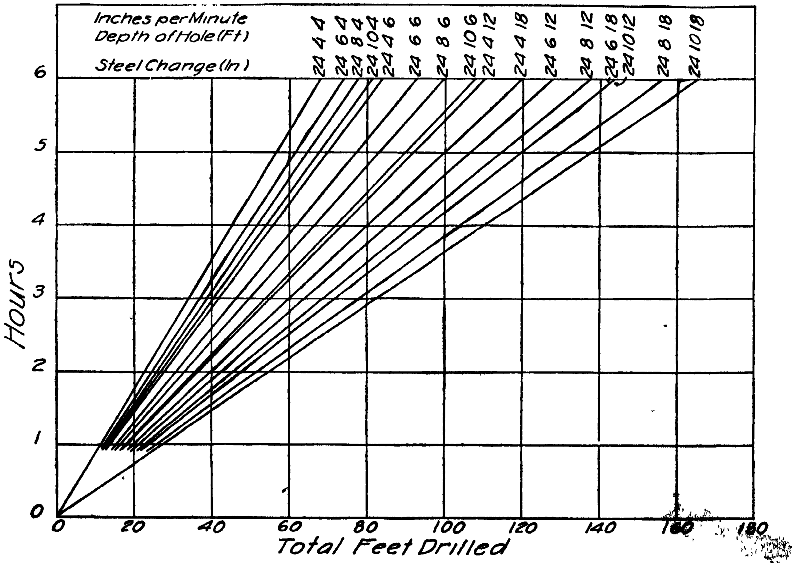

FIG. 164. Footage drilled per shift with jackhammers. (*Young, "Elements of Mining."*)

TABLE 58. NUMBER OF HOLES PER ROUND IN AMERICAN TUNNEL HEADINGS

Number of holes	Nature of rock	Area of heading, sq. ft.	Sq. ft. of heading per hole
16	Granite and gneiss	42	2.6
22	Limestone	120	5.5
10–11	Gneiss, granite, porphyry	41	3.7–4.1
22	Limestone, sandstone, shale	120	5.5
24	Shale	120	5.0
24	Sandstone and shale	120	5.0
21	Gneiss	120	5.7
18–24	Gneiss	35	1.5–1.9
14–20	Basalt	35	1.7–2.5
12	Gneiss and granite	48	4.0
18	Gneiss	40	2.2
24	Altered granite	60	2.5
15–21	Shale, conglomerate and coal	85	4.0–5.6
25	Granite	145	5.8
14–16	Medium granite	90	5.6–6.4
20–21	Hard granite	90	4.3–4.5
12–14	Shale and slate	37	2.6–3.1
18	Rhyolite	95	5.2
25–27	Andesite	55	2.0–2.2
16	Diabase	65	4.0
21	Limestone	175	8.4
16	Conglomerate and andesite	50	3.1
12–16	Quartzite	80	5.0–6.6

TRANSMISSION OF AIR[1]

The selection of pipe to transmit compressed air should receive the same consideration as pipe for carrying water. It is a subject that is too lengthy to be treated in detail here. Because of friction, there is a drop in pressure between the receiver and the end of the line (assuming no losses due to leakage). In choosing the pipe size, very much the same procedure is followed as in selecting pipes for carrying water. The smaller the pipe diameter, the lower the pipe cost will be. On the other hand, too small a pipe means heavy friction loss, which will call for a more powerful compressor to maintain the terminal pressure; too large a pipe requires a large initial outlay. Most compressors are built to deliver air between certain minimum and maximum pressures. As an aid toward selecting the pipe size it would be advisable to start with a choice of pipe diameter which, with the total length of pipe required, would not cause a drop in pressure, because of friction, greater than one-half the difference between the required terminal pressure at the drills and the maximum pressure delivered by the compressor. It is possible that such calculations would indicate that either a more powerful compressor or a larger pipe was necessary. In such cases, the extra cost of the pipe should be compared with the extra cost of the compressor with its additional operating cost in order to make the final selection.

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[1] Simons, *op. cit.*, p. 88.

Tables may be found that show the loss due to friction (expressed in pounds per square inch) per 1000 ft of pipe, depending on the diameter, pressure, and volume of air transmitted.[1]

If tables of transmission losses are not available, the data required may be calculated from the following formulas:[2] Let

P_1 = absolute pressure at intake (receiver)
P_2 = absolute pressure required at drill (terminal)
V = volume of free air passing through pipe, cu ft per min
L = length of pipe line, ft
D = diameter of pipe, in.

Then

$$P_1^2 - P_2^2 = \frac{V^2L}{2000D^5}$$

$$D = \left(\frac{V^2L}{2000(P_1^2 - P_2^2)}\right)^{\frac{1}{5}}$$

$$V = \left(\frac{2000D^5(P_1^2 - P_2^2)}{L}\right)^{\frac{1}{2}}$$

$$L = \frac{2000D^5(P_1^2 - P_2^2)}{V^2}$$

$$P_1 = \left(\frac{V^2L}{2000D^5} + P_2^2\right)^{\frac{1}{2}}$$

$$P_2 = \left(P_1^2 - \frac{V^2L}{2000D^5}\right)^{\frac{1}{2}}$$

Effect of Change in Altitude on the Transmission of Compressed Air. At most mines the compressed air is not used at the same elevation at which it is compressed. If there is very much variation between the terminal and compressor elevation, the difference in pressure because of altitude should be determined. This may be done by using the formula given on page 400. To illustrate this point, the following examples will be solved:

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[1] PEELE, (ed.), *op. cit.*, 3d ed., Sec. 15, p. 10.
Drill companies' catalogues.

[2] SIMONS, *op. cit.*, p. 89.

EXAMPLE 1

The elevation of the compressor house is 6000 ft. The lowest level in the mine has an elevation of 3000 ft, and the gage pressure is to be not less than 80 lb. What should be the gage pressure at the compressor receiver?

$$\log P_2 = \log P_1 - 0.0000157h$$

Atmospheric pressure at 6000 ft = 11.73 lb per sq in.
Atmospheric pressure at 3000 ft = 13.16 lb per sq in.

$$\log P_2 = \log (80 + 13.16) - 0.0000157(6000 - 3000)$$

$$P_2 = 83.59 \text{ lb per sq in. absolute}$$

$$\text{Gage pressure} = 83.59 - 11.73 = 71.86 \text{ lb}$$

EXAMPLE 2

The elevation of the compressor is 3000 ft. The air is to be used on a level whose elevation is 6000 ft. The minimum gage pressure at 6000 ft is to be not less than 80 lb. What must be the gage pressure at the compressor receiver?

$$\log P_1 = \log (80 + 11.73) + 0.0000157(6000 - 3000)$$

$$P_1 = 102.24 \text{ lb per sq in. absolute}$$

$$\text{Gage pressure} = 102.24 - 13.16 = 89.08 \text{ lb}$$

EXAMPLE ILLUSTRATING THE SELECTION OF A COMPRESSOR

A compressor to supply air for the following conditions will be selected:

1. Tonnage to be broken—1000 tons of ore per drilling shift
2. Depth of shaft—1000 ft
3. Greatest distance of pipe underground from 1000-ft-level station—1000 ft
4. Receiver located at shaft house
5. Elevation of shaft collar, same as compressor house—5000 ft
6. Air pressure at drill farthest from receiver to be not less than 80 lb gage
7. One ton of ore in place equals 10 cu ft
8. Drilling time per shift—4½ hr
9. Steel changed every 24 in.

Desired:

1. Number of drills necessary to break tonnage
    - *a.* 50 per cent with stopers (2¾-in. bore)
    - *b.* 25 per cent with drifters (3½-in. bore)
    - *c.* 25 per cent with jackhammers (2⅝-in. bore)
2. Total cubic feet of free air at the given elevation required by each type of drill
3. Gage pressure of air at receiver
4. Size of compressor necessary
    - *a.* Diameter of low-pressure cylinder
    - *b.* Diameter of high-pressure cylinder
    - *c.* Stroke

- *d.* Rpm
- *e.* Horsepower (motor direct-connected)
  - (1) Theoretical
  - (2) Actual

Solution: Tonnage broken by each type of machine:

$50\% \times 1000 = 500$ tons with stopers
$25\% \times 1000 = 250$ tons with drifters
$25\% \times 1000 = 250$ tons with jackhammers

Stopers. Assuming that the stopers have a drilling rate of 6 in. per min, the depth of the holes is 6 ft, and the drilling time $4\frac{1}{2}$ hr, we find from Fig. 162 that a total of about 90 ft will be drilled per machine per shift. If 4 cu ft of rock are broken for each foot drilled, we have

$4 \times 90 = 360$ cu ft per machine
Tons per machine $= {}^{360}\!/_{10} = 36$
Machines required $= {}^{500}\!/_{36} = 14$ stopers

Drifters. Assuming the same conditions as for the stoper, and consulting Fig. 163, we find that 60 ft per shift per machine will be drilled.

$4 \times 60 = 240$ cu ft for each machine
Tons per machine $= {}^{240}\!/_{10} = 24$
Machines required $= {}^{250}\!/_{24} = 10.4$ or 11 drifters

Jackhammers. The drilling rate for the jackhammers is taken at 12 in. per min. The holes are 6 ft deep and the working time 5 hr per shift. From Fig. 164, the total footage is found to be about 100 ft. Assuming 2 cu ft per foot drilled are broken, we have

$2 \times 100 = 200$ cu ft for each jackhammer
Tons per machine $= {}^{200}\!/_{10} = 20$
Machines required $= {}^{250}\!/_{20} = 12.5$, or 13 jackhammers

Total Cubic Feet of Free Air Required. *Stopers.* At a pressure of 80 lb, one $2\frac{3}{4}$-in. stoper requires 104 cu ft of free air per minute (Table 53). Fourteen machines are required. From Table 54, the factor for 5000 ft for 15 machines is 11.12.

$104 \times 11.12 = 1156$ cu ft of free air per minute

Drifters. One $3\frac{1}{2}$-in. drifter requires 131 cu ft. The factor for 12 machines is 9.48.

$131 \times 9.48 = 1242$ cu ft of free air per minute

Jackhammers. One $2\frac{5}{8}$-in. jackhammer requires 104 cu ft. The factor for 12 machines is 9.48.

$104 \times 9.48 = 986$ cu ft of free air per minute

The total cubic feet of free air per minute to be delivered to the machines is $1156 + 1242 + 986 = 3384$.

As a factor of safety the volume of air required should be increased by 25 per cent.

Capacity of compressor is equal to $1.25 \times 3384 = 4230$, or 4200 cu ft of free air per minute to be compressed.

Gage Pressure of Air at the Receiver. The air pressure at the drill farthest from the receiver must be not less than 80 lb gage. Assuming that there are no leaks, the loss in pressure between the receiver and the drill will result from friction in the pipe. Selecting a 6-in. pipe, and a capacity of 3500 cu ft of free air per minute, we find on consulting friction loss tables that the friction loss is 5.27 lb per sq in. for each 1000 ft of pipe. The total length of pipe is 1000 ft on the level plus 1000 ft in the shaft or 2000 ft. The pressure drop is

$$^{2000}/_{1000} \times 5.27 = 10.54 \text{ lb}$$

The absolute pressure at the drill is $80 + 12.67 = 92.65$ lb per sq in. (4000 ft elevation).

The gage pressure at the receiver (elevation 5000 ft) is, from the formula given on page 400,

$$\begin{aligned} \log P_2 &= \log 92.67 - 0.0000157(5000 - 4000) \\ P_2 &= 89.38 \text{ lb per sq in. absolute.} \\ \text{Gage pressure} &= 89.38 - 12.2 = 77.18 \text{ lb} \end{aligned}$$

Actual gage pressure at the receiver, considering the friction loss, is

$$77.18 + 10.54 = 87.72, \text{ or say } 90 \text{ lb per sq in.}$$

Size of Compressor Necessary. For determining the compressor size we have the following information:

1. 4200 cu ft of free air per minute
2. 90-lb gage pressure (maximum required)
3. 16-in. stroke (assumed)
4. 300 rpm operating speed (assumed)
5. 2.5-in.-diameter piston rod (assumed)
6. Mechanical efficiency of compressor 85 per cent
7. Volumetric efficiency 85 per cent
8. Compressor double acting, two-stage
9. Various assumptions made by consulting manufacturers' catalogues

We wish to determine the diameter of the low-pressure and high-pressure cylinders and the theoretical and actual horsepower of the motor necessary to drive the compressor.

Diameter of Low-pressure Cylinder. At 300 rpm the number of strokes per minute $= 2 \times 300 = 600$.

The cubic feet of air taken into the cylinder at each stroke

$$= {}^{4200}/_{600} = 7.00$$

To this must be added the volume of the piston rod. As the stroke is 16 in., the volume is

$$\begin{aligned} 16 \times 2.5^2 \times \frac{\pi}{4} \times \frac{1}{1728} &= 0.045 \text{ cu ft} \\ V_1 &= 7.000 + 0.045 = 7.045 \text{ cu ft} \\ d_1 &= 47\sqrt{\frac{V_1}{L}} = 47 \times \sqrt{\frac{7.045}{16}} = 31.16 \text{ in.} \end{aligned}$$

The diameter of the low-pressure cylinder must be corrected for volumetric efficiency.

$$x^2 : d_1^2 = 100 : E$$

$$x = 31.16\sqrt{100/85} = 33.91, \text{ or } 34 \text{ in.}$$

Diameter of High-pressure Cylinder

$$d_2 = d_1\left(\frac{P_a}{P_2}\right)^{\frac{1}{4}} = 31.16\left(\frac{12.2}{90 + 12.2}\right)^{\frac{1}{4}} = 18.32, \text{ or } 18.5 \text{ in.}$$

We shall need a 34- × 18.5- × 16-in., 300-rpm compressor to deliver 4200 cu ft of free air per minute at 5000 ft elevation.

Horsepower Required

$$\begin{aligned}\text{Hp} &= 2\,\frac{144nV_aP_a}{33{,}000(n-1)}\left[\left(\frac{P_2}{P_a}\right)^{\frac{n-1}{2n}} - 1\right]\\ &= \frac{2 \times 144 \times 1.406 \times 4200 \times 12.2}{33{,}000 \times 0.406}\left[\left(\frac{102.2}{12.2}\right)^{\frac{1.406-1}{2\times 1.406}} - 1\right]\\ &= 555 \text{ (theoretical)}\end{aligned}$$

$$\text{Actual horsepower of motor} = \frac{555}{0.85} = 653$$

This is the net motor horsepower; its efficiency would have to be considered for the final selection of the motor. A synchronous motor would probably be used. The efficiency would be about 90 per cent. This would require a 726-hp motor. The nearest standard frame size would be chosen.

For the final selection of the compressor, the nearest commercial size would be selected.

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CHAPTER XI

MINE HAULAGE

Transportation of materials underground may be accomplished in a number of ways:

1. By hand
2. Transfer raises
3. Animal haulage
4. Mechanical haulage
5. Hoisting

A discussion of when and where these various methods may be most economically applied is not within the scope of this chapter. The reader is referred to the bibliography for guidance in making such a choice.[1]

We shall confine our attention entirely to the selection of an electrically operated locomotive, either storage battery or trolley, and the diesel locomotive. Much of the information presented can also be applied to other types of locomotives (gasoline and compressed air).

A few years ago some type of trolley locomotive was almost invariably chosen. The storage-battery locomotive had not yet reached that stage of perfection which made it suitable for underground haulage purposes. Its characteristics and points of application had not been carefully studied, so that many operators attempted to use it under conditions for which the machine was not adapted. In recent years surveys have been made of the possibilities of the storage-battery locomotive and manufacturers have designed and built them to meet conditions best suited to their uses. With this development the storage-battery locomotive has now become a very useful and necessary piece of equipment for most mines. Their operating cost per ton is, in many cases, less than the trolley type performing the same service and very seldom much higher. With few exceptions the trolley locomotive has but one point of superiority: the hauling of heavy loads through relatively great distances. From the standpoint of safety, the battery locomotive is more desirable than the trolley type.

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[1] YOUNG, G. J., "Elements of Mining," 4th ed., McGraw-Hill Book Company, Inc., New York.

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TROLLEY LOCOMOTIVES

There are three main objections to the trolley locomotive. It is limited to those headings where the trolley has been strung (by means of a reel and cable the range may be extended 300 to 500 ft); the rails must be well bonded for efficient operation; and because of the more or less exposed overhead trolley wire the system is dangerous from the standpoint of electrical shock to workmen and fire hazards and explosions resulting from arcing in gassy mines. A number of states have passed laws limiting the voltage to 250. This does not permit so efficient an operation as would a higher voltage. When 250 volts or less must be used, it becomes necessary that the proper size trolley wire and rails be carefully selected and that bonding and connections be well made; otherwise the voltage drop near the end of the line becomes so great that difficulty in operation may result. The voltage drop may be overcome by putting in a feeder alongside the trolley.

Trolley locomotives may be had in sizes ranging from 2.5 tons up. For ordinary underground work the range is 2.5 to 10 tons. The smaller ones within this range usually have one motor; whereas the larger have two or more motors. The speed at which the trolley locomotive is designed to operate is ordinarily about 6 to 10 mph. In some special cases of long hauls, when the track and other conditions are suitable, they may operate at a speed as high as 20 mph.[1]

The trolley locomotive receives greatest use on main-line haulage, with only a limited amount of service for gathering purposes.

STORAGE-BATTERY LOCOMOTIVES

As previously mentioned, the storage-battery locomotive is becoming quite widely used. This is especially true for gathering service and in gaseous mines. It is the only type of electric locomotive to receive a permissible rating from the U.S. Bureau of Mines.

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[1] ALTSHULER, H. I., The Choice of Suitable Trolley Locomotives for Metal-mine Haulage, *Eng. Mining J.*, **128**, 992 (1929).

The outstanding advantage of the storage-battery locomotive is its ability to travel anywhere that tracks are laid. This presumes, of course, that the size of the heading and the radius of the curves permit the passage of the machine. The moving parts have been, in most cases, so designed that a minimum amount of the battery capacity is required for overcoming friction in their operation. In the early days of the storage-battery locomotive this was not true in many cases.

The scope of operations is limited because of battery capacity. The range is to a certain extent enhanced by giving the battery a boosting charge at the lunch hour or by attaching a trailer carrying an extra battery.

The chief disadvantage is the necessity of installing charging equipment (usually a motor-generator set) and the depreciation of the batteries. Battery manufacturers have brought their product to such a stage of perfection that the last item is not nearly so important as formerly.

Storage-battery locomotives for mining purposes are made in sizes ranging from 1½ to 10 tons. The 1½-ton size takes the place of hand tramming.

The voltage depends on the number of cells and the way in which they are connected.

Storage-battery locomotives operate at a speed of usually not exceeding 3.5 to 4.0 mph. For estimating purposes, their speed is usually assumed to average 3 mph.

Storage-battery locomotives may be had in three different types:

1. Straight or plain type
2. Combination type
3. Permissible or flameproof type

Plain Type. Standard motors and controllers are used. On the smaller sizes one motor and on the larger sizes two motors are used. The battery is in a plain sheet-iron compartment, readily accessible for charging and changing. It is suitable for any line of work under nongaseous conditions.

Combination Type. This type operates in connection with a trolley. The motor is designed to operate on the mine voltage (usually 250 volts) and the battery voltage. Usually when the locomotive is operating from the trolley, the battery is charged by means of suitable resistance. Trains run at about 6 mph on trolley and 3 mph on battery. The combination locomotive permits both gathering and main-line haulage.

Permissible Type. The electrical equipment of this type has been approved by the U.S. Bureau of Mines. They are so built that no dangerous sparking is possible from the motor commutator, switches, fuse blocks, wiring, etc. A short circuit can produce no dangerous heat or flame. The battery is enclosed in a specially designed compartment.

Methods of Driving. There are four general ways of driving battery locomotives:

1. Chain drive
2. Worm-gear drive
3. Combination of worm-gear and chain drive
4. Spur-gear drive (single or double reduction)

Chain Drive. The drive is from the motor through a spur gear to a jackshaft; and from the jackshaft to each axle by means of roller chains. The chain operates on sprockets mounted on the jackshaft and axles.

Worm-gear Drive. Both axles are driven through worm gears and universal joints if a single motor is used. The motor shaft is connected to the worm gears from a single motor, and then to the worm gears by means of universal joints. If two motors are used, each axle is driven by means of a worm gear and connected directly to the motor. Some companies claim this drive is difficult to maintain in adjustment, and requires an expert to take up the wear in the gears.

Combination Worm-gear and Chain Drive. This type of drive is used mostly in small, short locomotives. One axle is driven through the worm gear, and the other axle is connected to the driven axle by means of sprockets and a roller chain.

Spur-gear Drive. The spur-gear drive may be single or double reduction. In storage-battery locomotives it is usually double. The gear arrangement may be overhung (gear outside of bearing support) or better, the gears are placed inside the bearing support. The automobile type of gears for double reduction is usually used. Double-reduction spur gears are more adaptable to higher motor speeds and are more easily adjusted. Single-reduction spur gears are used on most trolley locomotives.

The loss in efficiency for spur gearing is usually taken at 5 per cent for each reduction.

In storage-battery locomotives the gears and, for that matter, almost all other moving parts are mounted on ball bearings. This

TABLE 59. DATA ON STORAGE-BATTERY LOCOMOTIVES[1]

Wt. of locomotive, tons	Wheel base, in.	Track gage, in.	Rated drawbar pull, lb.	Speed at rated drawbar pull, m.p.h.	Volts, rated drawbar pull	Total hp.	Number of motors	Approximate capacity of battery required, kw.-hr.
1½	20	18–24	400	3.5	....	4¾	1	9
2	20	18	400	3.5	48	4¾	1	10
2½	24	18	500	3.5	80	6	1	19
3	34	18	800	3.5	80	9.5	2	19
4	36	18	1300	2.8	80	12	2	31
3	26	24	1000	2.8	80	10	2	17
1½	20	24	400	3.5	.....	4¾	1	9
2	24	24	500	3.5	80	6	1	12
3½	30	24	900	3.5	80	10	2	23
4½	34	24	1000	3.5	80	12	2	30
4½	36	24	1500	3.5	80	17	2	24
5	42	30	1500	3.5	80	17	2	31
7	42	36–48	2600	2.8–3.3	80–96	24–28	2	53
6	36–38	36	2600	2.8	80	24	2	37
10	38	48	3000	2.9	96	28	2	61

[1] Most of these machines may be obtained in any track gage and horsepower desired.

TABLE 60. DATA ON TROLLEY LOCOMOTIVES[1]

Wt. of locomotive, tons	Wheel base, in.	Track gage, in.	Rated drawbar pull running, lb.	Speed at rated drawbar pull, m.p.h.	Rated voltage, d.-c.	Number of motors	Hp. per motor	Diameter wheel, in.
4	45	48	2000	6.2	500	2	17.5	22
4	45	44	2000	5.8	250	2	17.5	20
6	42	44	3000	4.0	500	2	28	23
6	52	40	3000	5.8	500	2	28	26
6	42	48	3000	4.25	250	2	22	24
6	64	30	3000	6.5	250	2	30	33
6	52	42	3000	5.8	250	2	28	26
6	39	44	3000	5.8	250	2	28	26
6	44	24	3000	6.5	250	2	30	33
8	60	36	4000	6.5	250	2	43	30
8	46	48	4000	6.5	250	2	43	30
10	64	42	5000	7.1	250	2	51	33
10	72	33–36	5000	8.9	250	2	75	33
10	64	42	5000	7.9	250	2	60	33
13	68	38	6500	8.2	250	2	75	33
13	80	52	6500	7.3	250	2	90	36
15	76	42	7500	7.4	250	2	84	36
15	94	42	7500	7.1	250	3	50	33
15	80	32–42	7500	9.5	250	2	130	38

[1] Most of these machines may be obtained in any desired gage, voltage, horsepower, and wheel diameter. Data are for steel-tired wheels.

requires the least possible amount of the battery capacity to overcome internal friction in the machine.

Motors. The motors used in storage-battery locomotives are of the ball-bearing type and have a high efficiency. Commutation is especially designed to prevent flashing. The motors are of sufficient size to cause slipping of the wheels under full load without blowing fuses. They are designed to stand a temperature rise of 75°C in 1 hr above a surrounding (ambient) air temperature of 25°C. This is the sea-level rating. If the locomotive is to be used at an elevation above 3300 ft, the effect on the temperature rise should be investigated. This will be discussed later. These motors may be obtained in a wide range of voltages and capacities.

Tables 59 and 60 present data on battery[1] and trolley[2] locomotives.

TYPES OF STORAGE BATTERIES USED

There are two distinct types available:

1. Edison or nickel-iron-alkaline combination
2. Lead or acid type

Edison Type.[3] The active material, plates, and the electrolyte are contained in a welded steel container. It is impossible to remove the plates unless the container is forcibly opened. The negative plate or grid is made of iron oxide contained in a perforated steel tube. The positive plate or grid is also made of steel plates holding perforated tubes containing nickel hydrate and nickel in layers. The electrolyte is potassium hydroxide and water. The plates are separated by suitable insulating material. The normal voltage of the Edison cell is 1.8 volts.

The Edison cell is exceptionally rugged and will withstand most unusual abuse. In fact, cases are on record where these cells have been submerged under water (flooded mine) or have been in a mine fire, but after they were cleaned, the electrolyte replaced and the cells charged, were almost as efficient as they were before the mishap.[4]

The first cost of the Edison cell is greater than that of the lead cell. On the other hand, it has a longer guaranteed life and higher turn-in value.[5] These batteries have given service for as long as 10 years

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[1] Compiled from Atlas Car and Manufacturing Company Catalog 1245.

[2] Compiled from Westinghouse Electric and Manufacturing Company catalogues (spec. pub. 1774; 1841; and 1873).

[3] Edison Storage Battery Company: "Storage Battery Locomotive Manual," Orange, N. J., 1929.

[4] Fleming, *loc. cit.*

[5] Edison Storage Battery Company, *op. cit.*, p. 46.

TABLE 61. CAPACITY AND WEIGHT IN POUNDS OF SINGLE CELLS

No. of plates	Exide-Ironclad MVM 6-hr rate			Philco XVL 6-hr rate			Edison 5-hr rate			
	Amp-hr	Kw-hr	Wt.	Amp-hr	Kw-hr	Wt.	Type	Amp-hr	Kw-hr	Wt.
7	102	0.198	28	120	0.234	27	A4	150	0.180	17
9	136	0.264	36	160	0.312	35	A5	188	0.225	20
11	170	0.330	43	200	0.390	43	A6	225	0.270	22
13	204	0.396	51	240	0.468	51	A7	263	0.315	26
15	238	0.462	59	280	0.546	59	A8	300	0.360	31
17	272	0.528	68	320	0.624	67	A10	375	0.450	38
19	306	0.594	74	360	0.702	79	A12	450	0 540	47
21	340	0.660	82	400	0.780	86	A14	525	0.630	57
23	374	0.726	89	440	0.858	94	A16	600	0.720	63
25	408	0.792	97	480	0.936	100	C4	225	0.270	24
27	442	0.858	104	520	1.014	106	C5	281	0.338	29
29	476	0.924	112	560	1.092	113	C6	338	0.405	34
31	510	0.990	118	...	.....	...	C7	394	0.473	40
33	544	1.056	127	640	1.248	128	C8	450	0.540	46
37	612	1.188	147	...	.....	...	C10	563	0.675	61
	Exide-Ironclad TLM and TLAM			Philco XL			Edison			
9	200	0 386	48	220	0.428	54	C12	675	0.810	71
11	250	0 483	59	275	0.535	64	D4	300	0.360	33
13	300	0.580	69	330	0.642	74	D5	375	0.450	40
15	350	0.676	82	385	0.749	84	D6	450	0.540	46
17	400	0.773	92	440	0.856	94	D7	525	0.630	53
19	450	0.869	102	495	0.963	110	D8	600	0.720	60
21	500	0 966	113	550	1.070	120	D10	750	0 900	77
23	550	1 063	123	605	1.177	130	D12	900	1.080	92
25	600	1.159	133	660	1.284	140				
27	650	1.256	144	715	1.391	151				
29	700	1.352	154	770	1.498	162				
31	750	1.449	164							
33	800	1.546	176	880	1.712	183				
37	900	1.739	204							
41	1000	1.932	223							

with changing the electrolyte the only repair required.[1] Lead batteries have given service for as long as 113 months, with an average of probably about 65 months.[2]

The Edison cell costs about 1¾ times as much as the lead cell. The lead cell weighs about 1½ times as much as the Edison.

Lead Battery. The lead cell needs no discussion as its general make-up is known to all. For battery-locomotive work, the ordinary lead storage battery has been strengthened to withstand the unavoidable abuse imposed on it by locomotive service. The life has been very considerably extended by this reinforcing process. The Exide-Ironclad[3] and the Philco[4] are two examples of commonly used makes.

Table 61 gives data on three makes of storage battery cells used in locomotive service.[5]

DIESEL LOCOMOTIVES

In 1936 the results of a comprehensive survey on the use of diesel mine locomotives was published by the U.S. Bureau of Mines.[6] More recently additional information has become available.[7]

These various articles definitely indicate that, if certain precautions are observed, the use of the diesel locomotive is economical and safe for use in underground haulage.

In Great Britain, the following provisions are required by law:[8]

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[1] Fleming, *loc. cit.*

[2] The Electric Storage Battery Company, "Facts," p. 12, Philadelphia.

[3] The Electric Storage Battery Company, Philadelphia.

[4] Philco Corporation, Trenton, N. J.

[5] Exide-Iron: Electric Storage Battery Company, Philadelphia; Philco: Philco Corporation, Trenton, N. J.; Edison: Thomas A. Edison, Inc., West Orange, N. J.

[6] Rice, G. S., and F. E. Harris, Diesel Mine Locomotives—Development and Use in European Coal Mines, *U.S. Bur. Mines Reps. Invest.* 3320, November, 1936.

[7] Brusset, J. A., Diesel Locomotives in British Gaseous Coal Mines, *Can. Mining Met. Bull.*, June, 1946; pp. 245–255, or *Trans. Can. Inst. Mining Met.*, **XLIX**, 245–255, (1946).

Ruth, J. P., Application of the Diesel Engine to Mine Haulage, *Mines Mag.*, **29**, No. 11, 559, (1939).

Stiefel, F. W., Operation of Diesel Locomotives Underground, *Trans. AIMME*, **153**, 158, (1943).

Harris, W. B., Leonard Greenburg, and Gustäv Werner, Diesel Engines in Tunneling Operations, *Trans. AIMME*, **153**, 165, (1943).

Plumb, C. W., Diesel Locomotives for Underground Haulage, paper presented at the 1946 Metal Mining Convention and Exposition, Western Division, The American Mining Congress, Denver, Colorado, September, 1946, the Ruth Company, Denver, Colorado (manufacturers of diesel mine locomotives).

[8] Brusset, *op. cit.*, p. 246.

1. Stainless-steel plate flame traps must be used on inlet and exhaust. The plates are set edge to edge and must conform to certain specified dimensions so as to prevent flame getting into the atmosphere from any internal explosion that may take place. There is also an air filter on the intake side.
2. An exhaust gas conditioner must be installed, which consists of a stainless-steel box full of water and which is equipped with baffles through which the exhaust is passed in order to cool the gases and wash out noxious elements from the exhaust. An automatic device cuts off the fuel supply and applies the brakes should the water drop below a determined level.
3. No electric ignition system is allowed. For lighting, flameproof electrical equipment is required.
4. There must be adequate braking, including power braking, for the whole train when carrying a man or men.
5. In addition, devices are installed which prevent the injection of any proportion of fuel oil, in relation to air supply, which would cause the production of appreciable amounts of carbon monoxide in the exhaust gases. (It is stipulated that the air-fuel ratio must never drop below 25, which ensures that the carbon monoxide content of the air will not exceed 0.05 per cent.)

Harris, Greenburg, and Werner[1] of the New York State Department of Labor give the requirements of the state of New York for the design and maintenance; toxic concentration; testing; ventilation requirements; etc. Their summary and conclusions for trucks, bulldozers, and locomotives operated from shafts, tunnels open at both ends, and dead-end tunnels are as follows:

1. Diesel-powered machinery can be safely and satisfactorily used underground if careful adherence to certain strict regulations is maintained.
2. The New York State Labor Department has issued such regulations to govern the use of diesel engines in underground operations in New York State.
3. During all types of operating conditions, these regulations have been found to stand the test of practical operation and adequate control.
4. The section of requirements covers the design and maintenance of engines. The main points of this section are (*a*) 20: 1 minimum air-fuel ratio, (*b*) the exhaust gases must be cooled and scrubbed, and (*c*) before discharge the exhaust must be diluted at least 10 times.
5. The second section covers allowable limits of contamination. Careful correlation of all factors has shown that the control of carbon monoxide to the set limit of 0.002 per cent affords adequate control of all noxious elements.
6. The third section is devoted to requirements of ventilation, which have been found adequate to provide the required control. The governing factor in this section requires that a minimum of 10,000 cu ft per min of mechanical ventilation be maintained in any area for every engine that is or may be operated in that area.

According to Stiefel[2] the operating cost compares very favorably with trolley or storage-battery locomotives. In fact, on a section of the Delaware River Aqueduct the diesel locomotive proved to be the necessary choice.

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[1] HARRIS, GREENBERG, and WERNER, *loc. cit.*

[2] STIEFEL, *loc. cit.*

CONDITIONS AFFECTING THE SELECTION OF A LOCOMOTIVE

The selection of a locomotive to handle a specified daily tonnage ordinarily depends on the following considerations.

1. Track (condition and profile)
2. Resistance (friction) of trailing load and locomotive
3. Distance to be traveled per shift
4. Tonnage to be handled per shift
5. Capacity of battery to meet requirements
6. Time of trip

Track. To ensure efficient operation the trackage on which locomotives are called upon to travel should be kept in good condition. This is especially true for the battery type. In metal mines the gage is usually 30 in. or less and the width of the heading rather narrow as compared with coal mines. The roadbed should be well ballasted and the ties of proper size and spacing.

Rails. The proper weight of rail for locomotive use is given in Table 62.[1]

TABLE 62. WEIGHT OF TRACK RAILS FOR USE WITH FOUR-WHEEL LOCOMOTIVES

Weight of locomotive, tons	1½	2	2½	3	3½	4	5	6	7	8	9	10	13	15	20	25	30
Minimum rail (lb./yd.)	8	12	12	16	16	16	16	16	20	20	25	25	30	40	50	60	75
Recommended weight	12	16	20	20	20	25	25	25	30	30	30	30	40	50	60	70	80

Ties. The length of the ties should preferably be twice the gage;[2] or at least not less than the gage plus 24 in.[3] The thickness should be at least ¼ in. greater than the length of the spike,[4] and the width 1⅜ in. times the spike length.[4] (Length of spike is measured from the bottom of the head to the point.)

Table 63 shows recommended spike sizes for various weights of rails.[5] The spacing of the ties will depend on the condition of the roadbed, the nature of the ballast available, and the weight of rail used.

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[1] Atlas Car and Manufacturing Company, Catalog 1245, p. 93 (1932).

[2] PEELE, (ed.), *op. cit.*, Sec. 11, p. 15.

[3] Jeffrey Manufacturing Company, "Jeffrey Mine Handbook," p. 14, No. 505 (1931).

[4] PEELE, (ed.), *op. cit.*, Sec. 11, p. 15.

[5] *Ibid.*, Sec. 11, p. 16. (Abstracted from table.)

Curves. If a locomotive is to be used on a track already in place, it must have a wheel base (distance between centers of axles) that will permit it to pass the curve with the minimum radius in the system. If, on the other hand, the track may be placed to suit the wheel base, no curves with a minimum radius less than that suited for the locomotive should be used.

Table 64 gives data on the minimum radius of curve over which a locomotive will pass.[1]

TABLE 63. TRACK SPIKES

Size under head, in.	Number per 200-lb. keg	Suitable rail lb./yd.
2½ × 5⁄16	2230	8–12
2½ × 3⁄8	1650	12–16
3 × 3⁄8	1380	12–20
3½ × 3⁄8	1250	12–20
4 × 3⁄8	1025	16–25
3½ × 7⁄16	890	16–25
4 × 7⁄16	780	20–30
4½ × 7⁄16	690	20–30
4 × ½	605	25–35
4½ × ½	518	25–35
5 × 9⁄16	405	40–56
5½ × 9⁄16	360	45–90
6 × 9⁄16	320	50–100

Increase of Gage on Curves. Experience indicates that the gage of the track on curves should be increased 1⁄16 in. for each 2½ deg of curvature.[2]

Elevation of Outer Rail on Curve. Ordinarily the outer rail on mine tracks requires no superelevation on curves. The speed at which mine locomotives travel is seldom greater than 8 mph. On long hauls where the grade is in favor of the load, and a heavy train is being moved, it may become desirable to elevate the outer rail. Because of centrifugal force the train has an inclination to leave the track on curves. By elevating the outer rail this tendency is balanced. The amount of elevation is found from

$$e = \frac{gv^2}{32.2R}$$

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[1] Atlas Car and Manufacturing Company, *op. cit.*, p. 93.

[2] "Jeffrey Mining Handbook," p. 18.

where e = elevation of outer rail, in.
g = gage of track, in.
v = velocity of train, ft per sec
R = radius of curve, ft

TABLE 64. MINIMUM RADIUS OF CURVE, IN FEET, OVER WHICH LOCOMOTIVE WILL PASS

Wheel base, in.	Diameter of wheel, in.										
	14	16	18	20	22	24	26	28	30	33	36
18	6										
20	7	7									
22	7	8	8								
24	8	8	8	8							
26	8	8	8	9	9						
28	9	9	9	10	10						
30	10	10	10	11	11	11					
34	11	11	11	12	12	12	13				
36	12	12	12	13	13	13	14				
38	12	12	12	14	14	14	15	15			
40	13	13	13	14	14	14	16	16	16		
42	14	14	14	15	15	15	16	16	16		
44	15	15	15	16	16	16	17	17	17	20	
48	16	16	16	17	17	17	19	19	19	22	22
54	..	18	18	19	19	19	21	21	21	25	25
60	..	18	20	21	21	21	23	23	23	28	28
66	..	..	22	23	23	23	26	26	26	31	31
72	..	..	25	26	26	26	28	28	28	34	34
84	..	..	29	30	30	30	33	33	33	39	39
96		..	..	34	34	34	37	37	37	45	45
108		..	..	39	39	39	43	43	43	51	51
144	..		.	52	52	52	56	56	56	68	68

On grades the elevation of the outer rail on the curves should be reduced.[1] It is customary in cases of this kind to reduce the grade in order to compensate for the curve. This is done in such a way that the resistance of the corrected grade on the curve equals the resistance of the grade on the straight track. For mining work, the reduction in grade to compensate for curvature may be found from[2]

$$C = \frac{20b}{r}$$

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[1] PEELE, (ed.), *op. cit.*, Sec. 11, p. 18.
"Jeffrey Mine Handbook," p. 19.

[2] *Ibid.*, p. 20.

Where C = grade reduction, per cent
b = wheel base, ft
r = radius of the curve, ft

EXAMPLE

On straight track the grade equals 2 per cent. If the maximum wheel base is 4 ft and the radius of curvature equals 20 ft, then

$$C = \frac{20 \times 4}{20} = 4\%$$

The grade on the curve is

$$2 \times (100 - 4) = 1.92\%$$

Track Profile. A profile of the track over which the locomotive is expected to operate is necessary for selecting both battery and trolley locomotives, although for somewhat different reasons. In the case of either type we must know the steepest grade both plus and minus so that the proper weight of locomotive be chosen for starting and stopping the load. In addition, for the trolley locomotive (and in some instances battery locomotives), the various grades with the distances must be known to determine the heating capacity of the motor or motors in the locomotive. This subject will be discussed later.

In choosing the battery locomotive the grades and distances are necessary in order to determine the battery capacity. Sections of the profile less than 100 ft in length that have a variation in grade of ±0.75 per cent from that preceding or following are usually disregarded as a separate section. Such stretches are combined with one of the sections on either side of it.

Resistance. Resistance may be considered as coming from six sources.

1. Resistance of trailing load (bearing and track)
2. Rolling resistance of locomotive
3. Curve resistance
4. Grade resistance
5. Level drawbar pull
6. Acceleration and retardation

Trailing Load. The amount of resistance offered by the trailing load depends on the kind of bearings on the cars and the condition of the track. The resistance is expressed in pounds per ton of load hauled. For mine work it varies between 1 and 2 per cent of the gross load. Roller bearings are considered as having a resistance of 1 per cent (20 lb per ton), and plain bearings 1.5 per cent (30 lb per ton).[1]

For example, assuming a train of cars with plain bearings and weighing 60 tons, the rolling resistance would be

$$60 \times 30 = 1800 \text{ lb}$$

which is the force necessary to start the load on level track.

Rolling Resistance of Locomotive. The resistance considered here is that offered by the locomotive when the motor pinion is removed. It is usually taken at 20 lb per ton, although some manufacturers claim 15 lb is sufficient for their product.

Curve Resistance. The resistance due to curves is usually disregarded if at least the minimum radius of curvature has been used. In case a great many curves are to be traversed by a battery locomotive, the curve resistance might become very important in determining the battery capacity. Curve resistance depends on a variety of factors: track gage on the curve; wheel base; wheel diameter; speed; radius of curve; number of cars (load) on the curve; etc. If an ample radius for the curve is used, wheel base of the locomotive properly selected, and the gage increased on the curve, the resistance may be ignored. In case these conditions cannot be realized, the approximate resistance may be determined from the following:

Lewis[2] gives ½ to 1 lb per ton per degree of curve for that part of the train on the curve. The Westinghouse Company uses 0.8 lb per ton per degree for curve resistance.[3]

Grade Resistance. For the present type of work the grade is expressed in per cent instead of degrees. The grade resistance is the vertical component of the force acting in the plane of the grade. For 1 ton (2000 lb) it equals 2000 times the sine of the grade angle. The sine of small angles practically equals the tangent of small angles. For a 1 per cent grade the tangent equals 1 ÷ 100 = 0.01. The vertical component then equals 2000 × 0.01 = 20 lb for each per cent of grade. Up to 10 per cent we may consider the resistance as varying directly as the grade. Thus, a 4 per cent grade offers a resistance of

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[1] "Goodman Mining Handbook," p. 85.
"Jeffrey Mine Handbook," p. 45.
PEELE, (ed.), *op. cit.*, Sec. 11, p. 27.
"Edison Storage Battery Manual," p. 31.

[2] LEWIS, *op. cit.*, p. 359.

[3] Westinghouse Electric and Manufacturing Company, "Westinghouse Coal Mining Handbook," C.M. 5, p. 2.

$$4 \times 20 = 80 \text{ lb per ton}$$

Grade resistance must be considered for both trailing load and locomotive.

Level Drawbar Pull. Adhesion between the wheel tread of the locomotive and the steel rails is what permits the load to be started and kept in motion. This adhesive force depends on the material of which the tread is made and the condition of the rail's surface. Table 65[1] shows the percentage of adhesion for conditions usually met with. The factors are expressed in per cent of the listed weight of the locomotive.

TABLE 65. PER CENT OF ADHESION BETWEEN LOCOMOTIVE WHEELS AND LEVEL TRACK

Conditions	Wheels	
	Chilled cast iron	Steel tires or wheels
Dry rails, with sand	25	33
Dry rails, no sand	20	25
Wet rails	5–15	5–15

For estimating purposes, it is customary to use for level track 20 per cent for cast-iron treads and 25 per cent for steel. A careful study of conditions in the mine should be made before finally selecting the adhesive coefficient.

Acceleration and Retardation (Deceleration). Ordinarily the force to accelerate the train is neglected. If steep grades will be encountered, and if the locomotive may be required to start the train on such grades, acceleration should be investigated. From

$$F = ma = \frac{W}{g}a$$

where F = accelerating force, lb
W = weight, lb
a = acceleration, ft per sec^2
g = acceleration of gravity = 32.2

we can find the accelerating force. Expressing F in terms of pounds per ton with an acceleration of 1 mphps, we get

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[1] "Goodman Mining Handbook," p. 85.

$$1 \text{ mphps} = \frac{5280}{60 \times 60} = 1.46 \text{ ft per sec}^2$$

$$F = \frac{2000}{32.2} \times 1.46 = 90.7 \text{ lb}$$

In actual calculations 100 lb instead of 90.7 lb are used as the force necessary to give 1 ton an acceleration of 1 mphps. For mining purposes an acceleration of 0.1 to 0.2 mphps is usually satisfactory. Or, in other words, 10 to 20 lb per ton is required.

If acceleration or deceleration is considered, they must be applied to both the trailing load and the locomotive.

In underground haulage a deceleration of 0.1 mphps is usually sufficient. This will stop the train in

$$s = \frac{v^2}{2a} \text{ ft}$$

where s = distance, ft
v = velocity, ft per sec
a = deceleration, ft per sec^2

For a velocity of 3 mph, and a deceleration of 0.1 mphps,

$$s = \frac{\left(\frac{5280 \times 3}{3600}\right)^2}{2 \times \frac{0.1 \times 5280}{3600}} = 66 \text{ ft}$$

The distance varies as the square of the velocity. A train traveling with a velocity of 6 mph will be stopped in a distance of

$$(6/3)^2 \times 66 = 264 \text{ ft.}$$

Braking Effort. The weight of the locomotive selected for starting and hauling the load should be investigated for stopping the train. When the power is shut off and the train allowed to coast down grade, it is aided by gravity to the extent of 20 lb per ton for each per cent of grade. If plain bearing cars are used they retard the train with a resistance of 30 lb per ton. The tractive effort of a locomotive is 20 per cent of its weight for cast-iron tread and 25 per cent for steel tread. The braking effort is usually assumed to be 80 to 85 per cent of the tractive effort.[1]

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[1] LEWIS, *op. cit.*, p. 360.
"Edison Storage Battery Manual," p. 31.

EXAMPLE

A 5-ton locomotive with steel-tread wheels has been selected to haul a 40-ton trailing load up a 1 per cent grade. Is it large enough to top the train on a −2 per cent grade, with a deceleration of 0.1 mphps if plain bearings are used on the cars?

Trailing load:

Effect of gravity pulling train down grade (20 × 2)	40 lb per ton
Braking force required (100 × 0.1)	10 lb per ton
Total	50 lb per ton
Retarding force due to train resistance	−30 lb per ton
Net force to be overcome	20 lb per ton

Locomotive:

Effect of gravity pulling locomotive down grade (20 × 2)	40 lb per ton
Braking force required	10 lb per ton
Total	50 lb per ton
Retarding force due to locomotive resistance	−20 lb per ton
Net force to be overcome	30 lb per ton
Total net force to stop trailing load (20 × 40)	800 lb
Braking effort of locomotive (400 − 30)	370 lb per ton

Weight of locomotive = 800 ÷ 370 = 2.16 tons. The 5-ton locomotive is large enough. With steel treads the adhesive force is

$$2000 \times 25\% = 500 \text{ lb per ton}$$

Assuming that 80 per cent of this is effective in stopping the train, the braking force is 500 × 80% = 400 lb per ton.

Distance Traveled and Tonnage per Shift. These items will affect the size of locomotive and kilowatt-hour capacity of the battery selected. In case of the trolley locomotive, and to a minor extent the battery locomotive, they influence the heating capacity of the motor. Mine workings are usually small as regards width (height is usually the limiting factor so far as coal mines are concerned) so that it may be impossible to get a locomotive of sufficient weight, because of the narrow track gage, in which the motors do not heat beyond the permissible range. In such instances two locomotives in tandem are commonly used.

Time per Trip. The time per trip is very difficult to estimate or even to guess. If the ore is being drawn from chutes on a level especially designed for haulage purposes; chute gates designed for rapid loading; and cars and shaft pocket station properly designed for discharging the cars, the time for loading and unloading can be closely approximated. Delays due to block signals and dispatcher service must not be overlooked. The running time from loading point to ore pocket is found from the locomotive speed and the distance traveled.

$$\text{Running time, min} - \frac{\text{distance, ft}}{\text{mph} \times {}^{5280}/_{60}}$$

For lack of evidence to the contrary we may assume 1 min for loading and 1 min for unloading each car. This includes stopping, spotting, and starting the train. It must be understood that this is but a very rough estimate.

The time required in gathering and tramming operations is even more difficult to estimate. Probably 30 to 50 per cent of the time is required for loading, coupling, delays, etc. For example, if a gathering locomotive has a certain distance that it must travel to perform its service per 7-hr shift, we would assume that from 30 to 50 per cent of the time (2.1 to 3.5 hr) would be used for purposes other than actually transporting loaded cars to their destination or returning empties to the trammers. A consideration of this feature is necessary to determine the tonnage that will be handled per shift, and thus the number of locomotives required for the work.

HAULAGE CALCULATIONS[1]

Calculations for mine haulage are best given in the form of formulas or definitions. Such a plan will be followed. The data that follow apply in most cases to both trolley and battery locomotives. Table 66[1] gives the symbols used in the formulas to follow.

Weight of Locomotive Required. The weight given by the following formula is that required to haul a given trailing load on the level or up a grade.

$$L = \frac{W(F + 20G)}{20(A - G)}$$

Peele[2] and Altshuler[3] use

$$L = \frac{W(F + 20G)}{2000A - 20 - 20G}$$

When steel tires are used, the adhesion A equals 25 per cent, and

$$L = \frac{W(F + 20G)}{480 - 20G}$$

[1] "Goodman Mining Handbook," pp. 88 to 95.

[2] PEELE, *op. cit.*, Sec. 11, p. 36.

[3] ALTSHULER, *loc. cit.*

TABLE 66. SYMBOLS USED IN HAULAGE FORMULAS

Symbol	Item	Unit
A	Coefficient of level track adhesion—a function of L	Per cent
D	Drawbar pull required	Pounds
d	Level drawbar pull equivalent	Pounds
E	Gear efficiency	Per cent
e	Motor efficiency	Per cent
F	Resistance of trailing load due to friction—a function of W	Pounds per 2000-lb. ton
G	Grade rise in a level distance	Per cent
H	Power required at motor shaft	Horsepower
h	Power required at drawbar	Horsepower
K	Power input to motor	Kilowatts
L	Locomotive weight	Short tons
P	Locomotive drawbar pull capacity	Pounds
p	Added drawbar pull or push required to overcome inertia in accelerating or decelerating to or from a given speed in distance L	Pounds
S	Speed	Miles per hour
T	Tractive effort	Pounds
W	Weight of trailing load	Short tons
r	Constant (1 per cent) for rolling resistance of locomotive	Per cent
Y	Distance for starting and accelerating, or decelerating and stopping	Feet

This seems the most logical and will be used here. The formula is derived as follows:

$A \times 2000$ lb $\times L$ = adhesive force, pounds, that locomotive is capable of exerting.

$L(20 + 20G) + W(F + 20G)$ = tractive effort required to move the locomotive and load.

Equating these we get the expression given above for L, the locomotive weight.

When accelerating force must be considered,

$$L = \frac{W(F + 20G + 100a)}{480 - 20G - 100a}$$

where a = acceleration, mphps
= 0.1 to 0.2 mphps (for most purposes)

Drawbar Pull Required. The drawbar pull required to haul the trailing load on the level or up a grade is

$$D = W(F + 20G)$$

Drawbar-pull Capacity. The drawbar-pull capacity of a given locomotive is found from

$$P = 20L(A - G)$$

Equivalent Level-drawbar Pull. It is equal to the drawbar pull of the trailing load plus the effort required to lift the locomotive up the grade.

$$d = D + (20L \times G)$$

Tractive Effort. This is the effort required to haul the trailing load and the locomotive.

$$T = d + (20L \times r)$$

Drawbar Horsepower. This is given by

$$h = \frac{D \times S}{375}$$

where 1 hp = 375 lb-mph.

Motor Horsepower. This is found from

$$H = \frac{T \times S \times 100}{375 \times E}$$

For spur gears E is 95 per cent for single reduction and 90 per cent for double reduction. For worm gears E is 90 to 96 per cent.

Kilowatt Input to Locomotive

$$K = \frac{H \times 746 \times 100}{e \times 1000}$$

Added Drawbar Pull for Accelerating or Decelerating Load and Locomotive

$$p = \frac{2000(W + L) \times S^2}{29.92Y}$$

Rated Drawbar Pull. The rated drawbar pull of a locomotive depends on the adhesion between the wheel tread and the rail. For steel wheels and clean dry rails, it is 25 per cent of the locomotive weight; for example, a 6-ton locomotive with steel treads would have a rated drawbar pull of

$$2000 \times 6 \times 25\% = 3000 \text{ lb}$$

Capacity of the Battery. The method used for calculating the capacity of the battery is one of converting the total foot-pounds of work for a round trip into kilowatt-hours. This is then multiplied by the number of round trips to obtain the battery capacity.

$$1 \text{ kw-hr} = \frac{33{,}000 \times 60}{0.746} = 2{,}654{,}200 \text{ ft-lb}$$

In practice it is customary to consider an efficiency of about 66⅔ per cent between the battery and the wheels.[1]

For operating service,

$$1 \text{ kw-hr} = 2{,}654{,}200 \times 66\tfrac{2}{3}\% = 1{,}769{,}500 \text{ ft-lb}$$

This figure, by most users, is rounded off to become 1,760,000 ft-lb. It will be used as such in the present discussion.

The battery capacity for each section of the haul is found from

$$\text{Kw-hr} = \frac{T \times D(30 + 20G)}{1{,}760{,}000}$$

where T = total tons being hauled (including locomotive)
D = distance of each section, ft
G = grade, per cent

Some engineers determine the battery capacity for hauling the locomotive separately and use the rolling resistance of the locomotive (usually 15 to 20 lb per ton) instead of combining with the trailing load and using 30 lb per ton.

After the total capacity required of the battery to perform a shift's work had been found, it must be increased to allow for two contingencies that arise. The average voltage for the battery is based on a certain discharge rate, usually 6 hr for the lead type and 5 hr for the Edison. If for any reason a more rapid discharge takes place, the battery will, toward the end of the day's work, give a lower voltage than the average, which will reduce the duty of the locomotive. The factor of safety is also used since, as the battery approaches the end of its useful life, it will still give the performance in duty originally required. It is customary to increase the capacity of the battery 25 per cent to allow for the above conditions.

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[1] "Edison Storage Battery Manual," p. 34.

Electric Storage Battery Company, *Bull.* 195, p. 10.

Mancha Storage Battery Locomotive Company, "Selecting the Proper Locomotive," p. 7.

"Jeffrey Mining Handbook," p. 45.

Atlas Car and Manufacturing Company, Catalogue 1245, p. 94

To illustrate the application of the foregoing discussion, the following problem will be solved.

EXAMPLE

Given the following conditions to select a locomotive and battery to perform the work:

1. Haulage time = 6 hr
2. Tons to be handled per shift = 1200
3. Cars:
   - *a.* Weight = 3000 lb
   - *b.* Bearings = Plain
4. Weight of ore per car = 6000 lb
5. Cars per trip = 10
6. Steel-tired locomotive
7. Speed:
   - *a.* Average = 2.5 mph
   - *b.* Maximum = 3.5 mph

8. Length of haul

   - *a.* 600 ft of +1 per cent grade
   - *b.* 800 ft of −0.75 per cent grade
   - *c.* 200 ft of level track

9. Friction allowance

   - *a.* Locomotive = 20 lb per ton
   - *b.* Cars = 30 lb per ton
   - *c.* Grade = 20 lb per ton

10. Factor of safety for battery = 25 per cent
11. Acceleration 0.1 mphps

Find:

1. Weight of locomotive
2. Drawbar pull
3. Number of locomotives necessary
4. Number of round trips per locomotive per day
5. Total kilowatt-hour capacity of battery
6. Horsepower and input

Solution:

1. *Weight of Locomotive*

$$L = \frac{W(F + 20G + 100a)}{480 - 20G - 100a}$$

$$W = (3 + 1.5) \times 10 \text{ cars} = 45 \text{ tons}$$

Using for G the maximum grade condition of +1 per cent, and for a, 0.1 mphps

$$L = \frac{45(30 + 20 \times 1 + 100 \times 0.1)}{480 - 20 \times 1 - 100 \times 0.1} = 6.0 \text{ tons}$$

A 6-ton locomotive will also be large enough for braking effort.

2. *Drawbar Pull*

$$\begin{aligned} D &= W(F + 20G) \\ &= 45(30 + 20 \times 1) = 2250 \text{ lb} \end{aligned}$$

3. *Locomotives Necessary.* The total distance traveled per round trip is

$$2(600 + 800 + 200) = 3200 \text{ ft}$$

$$\text{Tons per trip} = 3 \times 10 = 30$$

$$\text{Trips per 6 hr} = {}^{1200}/_{30} = 40$$

With an average speed of 2.5 mph, the time to make a round trip is

$$\frac{D}{\text{Mph} \times {}^{5280}/_{60}} = \frac{3200}{2.5 \times {}^{5280}/_{60}} = 14.5 \text{ min}$$

Allowing ¾ min each per car for loading and dumping,

$$10 \times \tfrac{3}{4} + 10 \times \tfrac{3}{4} = 15 \text{ min}$$

Total time per round trip = 15 + 14.5 = 29.5, or 30 min

$$\text{Trips per locomotive} = \frac{6 \text{ hr} \times 60}{30} = 12$$

Tons per locomotive per shift = 12 × 3 × 10 = 360

Number of locomotives = $^{1200}/_{360}$ = 3.33, or 4

4. *Round Trips per Locomotive per Day.* Trips per 6 hr are

$$\frac{6 \times 60}{30} = 12$$

5. *Capacity of Battery.* The train operates under the following conditions:

A. Loaded train out

(1) 600 ft of +1 per cent grade
(2) 800 ft of −0.75 per cent grade
(3) 200 ft of level track

B. Empty train in

(1) 200 ft of level track
(2) 800 ft of +0.75 per cent grade
(3) 600 ft of −1 per cent grade

The kilowatt-hours for each section are found from

$$\text{Kw-hr} = \frac{T \times D(30 + 20G)}{1{,}760{,}000}$$

CONDITION A

$$T = (1.5 + 3) \times 10 + 6 = 51 \text{ tons}$$

Section 1

$$\text{Kw-hr} = \frac{51 \times 600(30 + 20 \times 1)}{1{,}760{,}000} = 0.870$$

Section 2

$$\text{Kw-hr} = \frac{51 \times 800[30 + 20 \times (-\tfrac{3}{4})]}{1{,}760{,}000} = 0.348$$

Section 3

$$\text{Kw-hr} = \frac{51 \times 200(30 + 20 \times 0)}{1{,}760{,}000} = 0.174$$

$$\text{Total kilowatt-hours for loaded train} = \overline{1.392}$$

CONDITION B

$$T = (1.5 + 0) \times 10 + 6 = 21 \text{ tons}$$

Section 1

$$\text{Kw-hr} = \frac{21 \times 200(30 + 20 \times 0)}{1{,}760{,}00} = 0.072$$

Section 2

$$\text{Kw-hr} = \frac{21 \times 800(30 + 20 \times \tfrac{3}{4})}{1{,}760{,}000} = 0.430$$

Section 3

$$\text{Kw-hr} = \frac{21 \times 600[30 + 20 \times (-1)]}{1{,}760{,}000} = 0.072$$

$$\text{Total kilowatt-hours for empty train} = \overline{0.574}$$

$$\text{Total kw-hr} = A + B = 1.392 + 0.574 = 1.966$$

The number of trips per day per locomotive are 12.

$$\text{Capacity of battery} = 1.966 \times 12 = 23.592 \text{ kw-hr}$$

Adding 25 per cent, a minimum battery of 23.592 × 1.25 = 29.5 kw-hr must be selected. Referring to Table 59, we find that this readily falls within the capacity of a 6-ton locomotive.

Selection of Battery. A common type of lead battery is one made up of 48 cells. With the lead battery this would give 2 × 48 = 96 volts. With the Edison type to get 96 volts requires 80 × 1.2 = 96. The number of plates and thus the battery can be selected from the following rule.

Exide-Iron Clad Cell. With the 6-hr rating, the capacity of each positive plate, provided that not more than one-sixth of the total work is done during the final hour, is as follows:

MVM cell	34 amp-hr
TLM and TLAM cells	50 amp-hr

Philco Cell. The same rule applies, except that the capacity is

XVL cell	40 amp-hr
XL cell	55 amp-hr

Edison Cell. The Edison cell is based on a 5-hr discharge rate. The average capacity per cell is

A type	37.5 amp-hr
C type	56.3 amp-hr
D type	75 amp-hr

To find the total ampere-hours required, divide the total watt-hours by the voltage. The number of positive plates is found by dividing the total ampere-hours by the capacity of one plate.

In the example, we found that a battery capacity of 29.5 kw-hr, or 29,500 watt-hours, was necessary. Using a 96-volt battery,

$$\text{Amp-hr} = \frac{29{,}500}{96} = 307$$

For an Exide-Iron MVM,

$$^{307}\!/_{34} = 9.03, \text{ or 10 positive plates}$$
$$\text{Total plates} = 2 \times 10 + 1 = 21$$

One-cell, MVM-21 plates have (Table 61) a capacity of 0.660 kw-hr. A battery made from 48 cells has

$$48 \times 0.660 = 31.7 \text{ kw-hr}$$

For the Philco XVL,

$$^{307}\!/_{40} = 7.7, \text{ or 8 positive plates}$$
$$\text{Total plates} = 2 \times 8 + 1 = 17$$

One-cell, XVL-17, has a capacity of 0.624 kw-hr. A battery made from 48 cells has 48 × 0.624 = 30.0 kw-hr.

For the Edison Type A cell,

$$\frac{307}{37.5} = 8.2$$

The nearest above this is the A-10, which has a capacity of 0.450 kw-hr. For 80 cells, the capacity is

$$80 \times 0.450 = 36.0 \text{ kw-hr}$$

Any one of the three batteries, 48-MVM-21, 48-XVL-17, or 80-A-10, would prove satisfactory.

Motor Horsepower Required. The level-drawbar equivalent is

$$d = D + (20L \times G) = 2250 + (20 \times 6 \times 1) = 2370 \text{ lb}$$
$$T = d + (20L \times r) = 2370 + (20 \times 6 \times 1) = 2490 \text{ lb}$$
$$\text{Motor horsepower} = H = \frac{T \times S \times 100}{375 \times E}$$

Using the maximum speed of 3.5 mph, and double reduction gearing with an efficiency of 90 per cent,

$$H = \frac{2490 \times 3.5 \times 100}{375 \times 90} = 25.6, \text{ or } 26 \text{ hp}$$

$$\text{Kilowatt input to the motor} = K = \frac{H \times 746 \times 100}{e \times 100}$$

Assuming an efficiency for the motor of 80 per cent,

$$K = \frac{26 \times 746 \times 100}{80 \times 1000} = 24.2 \text{ kw}$$

DUTY CYCLE FOR MINE-LOCOMOTIVE MOTOR

In selecting a locomotive for haulage purposes other than gathering it is important that the duty cycle for the proposed conditions be calculated. The horsepower rating for the locomotive will depend on the duty it must perform. Assuming that the locomotive weight is sufficient to handle the load under the most adverse conditions of starting, accelerating, stopping, etc., it must in addition have a continuous motor capacity which will not overheat during the period in which it is in service. The motor capacity required is determined by calculating a duty cycle for a round trip under operating conditions. For this purpose there must be available the profile of the route showing the grades, curves, and level stretches of track, and the characteristic curves of the motors in the proposed locomotive.

Characteristic Curves. The characteristic curves of the motors used in locomotives may be obtained from the manufacturers. Figure 165 shows the data for a 43-hp motor.[1] The application of the curves will be illustrated later by an example. These curves are determined for sea-level rating and a temperature rise of 75°C from an ambient temperature of 25°C. If the locomotive is to be used under conditions other than these, corrections must be applied to the temperature rating for sea level.

Effect of Altitude and Temperature on Motor Characteristics.[2] For altitudes other than sea level, the Standardization Rules of the AIEE give the following:[3]

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[1] Westinghouse Electric and Manufacturing Company.

[2] ALTSHULER, *op. cit.*, p. 994.

[3] PENDER, H., "Handbook for Electrical Engineers," 2d ed., p. 2090, John Wiley & Sons, Inc., New York, 1922.

Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 m (3300 ft). For machinery operating at an altitude of 1000 m or less, a test run at any altitude less than 1000 m is satisfactory, and no correction shall be applied to the observed temperature. Machines intended for operation at higher altitudes shall be regarded as special. It is recommended that when a machine is intended for service at altitudes above 1000 m (3300 ft), the permissible temperature rise at sea level shall be reduced by 1 per cent for each 100 m (300 ft) by which the altitude exceeds 1000 m.

For example, if the motor rating has been based on a 75°C rise at sea level, but is to be used at 1600 m (5280 ft), the correction would be

$$\frac{1600 - 1000}{100} = 6\%$$

Therefore a temperature rise at sea level of

$$75 - (75 \times 0.06) = 70.5°$$

would give an equivalent rise of 75° at the 1600-m altitude.

If the temperature conditions under which the locomotive operates are greater than 25°, a similar correction must be applied to the ambient temperature;[1] otherwise the ultimate temperature will exceed 100°. This may cause the insulation of the machine to break down; for example, if the locomotive is to operate under a temperature of 35°C, which is $35 - 25 = 10°$ higher than normal, the temperature-rise curve for the motor must be corrected to $75 - 10$, or 65° for rating the motor.

It may often happen that the location will call for a correction due to both altitude and ambient temperatures. For the illustration already used, the temperature-rise curve under which the motor would be rated would be

$$75 - 4.5 - 10 = 60.5°$$

The determination of the temperature-rise curve to meet given conditions will be illustrated later.

The calculation of the duty cycle for a locomotive motor is best illustrated by an example. The duty cycle for the following set of conditions will be investigated.

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[1] Altshuler, *op. cit.*, p. 994.

EXAMPLE

Given:

1. Type of service: Main line haulage
2. Average voltage = 250
3. Weight of empty car = 4000 lb
4. Weight of ore = 8000 lb
5. Cars per trip = 10
6. Net haulage time per day = 7 hr
7. Car bearings, roller = 20 lb per ton
8. Wheel tread = Steel
9. Wheel diameter = 30 in.
10. Tons per day (one shift) = 500
11. Acceleration = 0.2 mphps
12. Maximum speed = 10 mph
13. Elevation of haulage level above sea level = 6600 ft
14. Temperature in haulage drift = 30°C
15. Track profile (from loading chutes to skip pocket):
    - *a.* 500 ft of +0.5 per cent grade
    - *b.* 2500 ft of −0.5 per cent grade
    - *c.* 200 ft of −0.3 per cent grade, and 10-deg curve
    - *d.* 1500 ft of −0.6 per cent grade
    - *e.* 200 ft of level track

Weight of Locomotive

$$L = \frac{W(F + 20G + 100a)}{480 - 20G - 100a}$$

$$= \frac{10(2 + 4)(30 + 20 \times 0.5 + 100 \times 0.2)}{480 - 20 \times 0.5 - 100 \times 0.2} = 8 \text{ tons}$$

Average Speed

$$\text{Trips per day} = \frac{500}{4 \times 10} = 12.5, \text{ or } 13$$

$$\text{Time per trip} = \frac{7 \times 60}{13} = 32.2, \text{ or } 32 \text{ min}$$

Assuming 15 min for loading, unloading, etc., the running time per trip is

$$32 - 15 = 17 \text{ min}$$

$$\text{Average speed} = \frac{2 \times 4900}{17} = 577 \text{ fpm} = 6.5 \text{ mph}$$

As a trial, we shall select a standard 8-ton trolley locomotive, powered with two 43-hp motors. Figure 165 shows the characteristic curves for this motor.

In making the calculations for the duty cycle the following formulas are used:

Tractive Effort

$$T = LF_1 + WF + (L + W)20G + (L + W)cC + (L + W)100a$$

where T = total tractive effort, lb

F_1 = locomotive friction, lb per ton = 20 lb

W = weight of trailing load, tons
L = weight of locomotive, tons
G = per cent of grade
c = curvature of track, deg
C = curve resistance, lb per ton = 0.8 lb
a = rate of acceleration, mphps

$$a = \frac{T - LF_1 - WF - (L + W)20G - (L + W)cC}{100(L + W)}$$

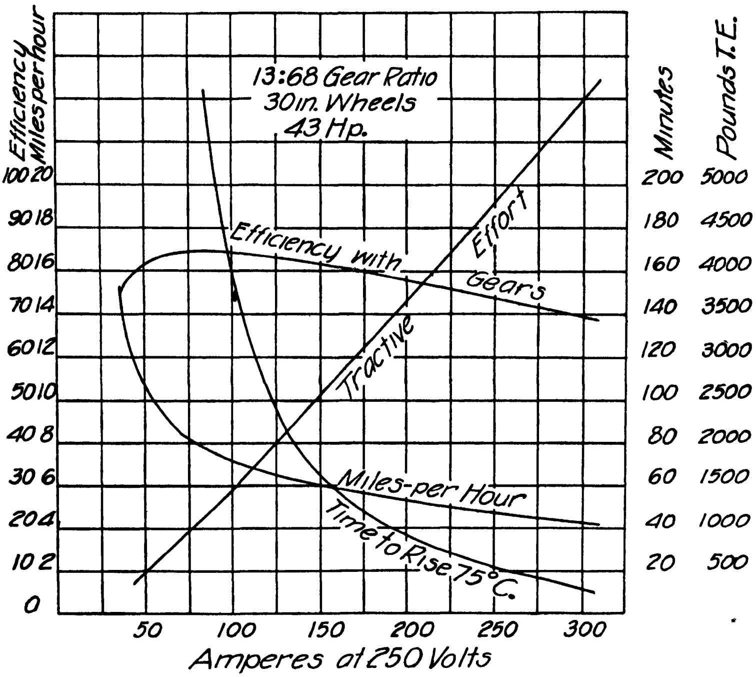

FIG. 165. Characteristic curves for a 43-hp motor. (*Westinghouse Electric and Manufacturing Company.*)

Speed-time-distance-acceleration Formulas. For a constant rate of acceleration,

$$t = \frac{v_1 - v_0}{a}$$

$$a = \frac{v_1 - v_0}{t}$$

$$D = \frac{v_1 + v_0}{2} \times t \times 1.46$$

$$t = \frac{D}{\frac{v_1 + v_0}{2} \times 1.46}$$

where t = time, sec
v_1 = speed at end of period, mph
v_0 = speed at beginning of period, mph
a = rate of acceleration, mphps
D = distance traveled, ft
1.46 = conversion factor (mph to fps)

The formula used for calculating the duty cycle is a special form of the rms method used for mine-hoist motors.[1]

$$I^2 = \frac{I_1^2 t_1 + I_2^2 t_2 + I_3^2 t_3 + \cdots}{t_E}$$

where I = the continuous current rating of the motor, amp
$t_1, t_2, \ldots$ = time required to traverse track sections, sec
$I_1, I_2, \ldots$ = motor current on track sections
t_E = total elapsed time for a forced-ventilated motor or totally enclosed motor
= total running time plus half the idle time for a self-ventilated motor

Section a

$$\begin{aligned} T &= 8 \times 20 + 60 \times 20 + (8 + 60) \times 20 \times 0.5 + (8 + 60) \times 100 \times 0.2 \\ &= 3400 \text{ lb per locomotive} \end{aligned}$$

As there are two motors, T per motor $= 3400/2 = 1700$ lb.
From Fig. 165, with $T = 1700$ lb,

$$\text{Current} = 112 \text{ amp}; \qquad \text{mph} = 6.9$$

$$t = \frac{6.9 - 0}{0.2} = 34.5 \text{ sec to reach 6.9 mph}$$

$$D = \frac{6.9 + 0}{2} \times 34.5 \times 1.46 = 174 \text{ ft}$$

The length of the section is 500 ft. There are left $500 - 174 = 326$ ft. We must find the tractive effort, time, and average amperes for this distance. To do so, the speed at the end of the run must be assumed and a cut-and-try method used. Assuming the speed at the end of the remaining part of section *a* to be 8.8 mph, we find on consulting Fig. 165, that

$$T \text{ per motor} = 800 \text{ lb}$$

For the locomotive, $T = 2 \times 800 = 1600$ lb.

$$T \text{ (average)} = \frac{3400 + 1600}{2} = 2500 \text{ lb}$$

$$T \text{ per motor} = 2500/2 = 1250 \text{ lb}$$

$$\text{Current} = 91 \text{ amp}; \qquad \text{mph} = 7.4$$

$$\begin{aligned} a &= \frac{2500 - 8 \times 20 - 60 \times 20 - (8 + 60) \times 20 \times 0.5}{100(8 + 60)} \\ &= \frac{2500 - 2040}{6800} = 0.0677 \text{ mphps} \end{aligned}$$

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[1] "Westinghouse Coal Mining Handbook," p. 6.

$$t = \frac{8.8 - 6.9}{0.0677} = 28.1 \text{ sec}$$

$$D = \frac{8.8 + 6.9}{2} \times 28.1 \times 1.46 = 323 \text{ ft}$$

This distance (323 ft) practically checks that remaining in the section.

Section b. Assuming the speed at 10 mph on this section,

$$T \text{ per locomotive} = 550 \times 2 = 1100 \text{ lb}$$

Average T on section a = 2500 lb.

$$a = \frac{\frac{2500 + 1100}{2} - 8 \times 20 - 60 \times 20 - (8 + 60) \times 20 \times -0.5}{100(8 + 60)}$$

$$= \frac{1800 - 680}{6800} = 0.165 \text{ mphps}$$

$$t = \frac{10 - 8.8}{0.165} = 7.3 \text{ sec}$$

$$D = \frac{10 + 8.8}{2} \times 7.3 \times 1.46 = 100 \text{ ft}$$

$$T \text{ per motor} = \frac{2500 + 1100}{2} \times \frac{1}{2} = 900 \text{ lb}$$

Current = 73 amp; mph = 8.3

Time for rest of section at 10 mph,

$$t = \frac{2500 - 100}{10 \times 1.46} = 164.5 \text{ sec}$$

$$T \text{ per motor} = {}^{1100}\!/_{2} = 550 \text{ lb}$$

Current = 55 amp; mph = 10

Section c. The speed on entering the section is 10 mph. Because of the 10-deg curve and the flattening of the grade to −0.3 per cent, the tractive effort must be increased by

$$(8 + 60) \times 20 \times -0.2 + (8 + 60) \times 10 \text{ deg} \times 0.8 = 816 \text{ lb}$$

or

$${}^{816}\!/_{2} = 408 \text{ lb per motor}$$

Therefore T per motor is 550 + 408 = 958 lb.

Current = 75 amp; mph = 8.2

$$t = \frac{200}{8.2 \times 1.46} = 16.7 \text{ sec}$$

Section d. Train enters at 8.2 mph and is to be speeded up to 10 mph.

$$a = \frac{\frac{1916 + 1100}{2} - 8 \times 20 - 60 \times 20 - (8 + 60) \times -0.6}{100(8 + 60)}$$

$$= \frac{1508 - 544}{6800} = 0.14 \text{ mphps}$$

Average T per motor $= {}^{1508}\!/_{2} = 754$ lb.

$$\text{Current} = 65 \text{ amp}; \qquad \text{mph} = 9.0$$

$$t = \frac{10 - 8.2}{0.14} = 12.8 \text{ sec}$$

$$D = \frac{10 + 8.2}{2} \times 12.8 \times 1.46 = 171 \text{ ft}$$

Remainder of section at 10 mph:

$$T \text{ per motor} = 550 \text{ lb}; \qquad \text{current} = 55 \text{ amp}$$

$$t = \frac{1500 - 171}{10 \times 1.46} = 91 \text{ sec}$$

Section e. The train enters the level section at 10 mph. It must stop in 200 ft. If permitted to coast to a standstill, D would be as follows:

When the constant speed is reached,

$$T = 8 \times 20 + 60 \times 20 + (8 + 60) \times 20 \times 0 = 1360 \text{ lb}$$

$$T \text{ per motor} = {}^{1360}\!/_{2} = 680 \text{ lb}; \qquad \text{mph} = 9.4.$$

$$\text{Average } T = \frac{1100 + 1360}{2} = 1230 \text{ lb}$$

$$a = \frac{1230 - 1360}{6800} = -0.0191 \text{ mphps}$$

$$t = \frac{9.4 - 10}{-0.0191} = 31.4 \text{ sec}$$

$$D = \frac{9.4 + 10}{2} \times 31.4 \times 1.46 = 445 \text{ ft}$$

As the section is but 200 ft long, the train must be brought to rest by means of the brakes. Current = 0 amp.

$$t = \frac{200}{\frac{10 + 0}{2} \times 1.46} = 27.4 \text{ sec}$$

Empty Train. *Section e.* There are 200 ft of level track. Accelerating at 0.2 mphps it will take

$$t^2 = \frac{2s}{a} \text{ sec to reach the end of the level section}$$

$$t^2 = \frac{2 \times 200}{1.46 \times 0.2} = 1369.8$$

$$t = 37 \text{ sec}$$

$$\text{Mph} = 0.2 \times 37 = 7.4$$

$$T \text{ per motor at } 7.4 \text{ mph} = 1330 \text{ lb}; \qquad \text{current} = 95 \text{ amp}$$

Section d. Acceleration to continue until speed is 10 mph.

$$T \text{ per locomotive} = 1100 \text{ lb}$$

$$\text{Average } T \text{ for } 7.4 \text{ to } 10 \text{ mph} = \frac{2660 + 1100}{2} = 1880 \text{ lb}$$

$$T \text{ per motor} = {}^{1880}\!/_{2} = 940 \text{ lb}; \qquad \text{current} = 75 \text{ amp}$$

$$a = \frac{1880 - 8 \times 20 - 20 \times 20 - (8 + 20) \times 20 \times 0.6}{100(8 + 20)}$$

$$= \frac{1880 - 896}{2800} = 0.351 \text{ mphps}$$

$$t = \frac{10 - 7.4}{0.351} = 7.4 \text{ sec}$$

$$D = \frac{10 + 7.4}{2} \times 7.4 \times 1.46 = 94 \text{ ft}$$

For the remainder of the section at 10 mph,

$$t = \frac{1500 - 94}{10 \times 1.46} = 96.3 \text{ sec}$$

Current = 55 amp

Section c. Because of the change in grade and the 10-deg curve the tractive effort will change. The change in grade decreases the tractive effort by

$$(8 + 20) \times 20 \times (0.6 - 0\ 3) = 168 \text{ lb.}$$

The curve increases it by $(8 + 20) \times 10 \times 0.8 = 224$ lb. The net change is

$$224 - 168 = +56 \text{ lb}$$

T per locomotive at 10 mph is 1100 lb.

T on curve = 1100 + 56 = 1156 lb
T per motor = $^{1156}\!/_2$ = 578 lb
Current = 56 amp; mph = 9.9

$$t = \frac{200}{9.9 \times 1.46} = 13.8 \text{ sec}$$

Section b. Train is to accelerate back to 10 mph. Average T per locomotive

$$\frac{1156 + 1100}{2} = 1128 \text{ lb}$$

T per motor = $^{1128}\!/_2$ = 564 lb; current = 55 amp

$$a = \frac{1128 - 8 \times 20 - 20 \times 20 - (8 + 20) \times 20 \times 0.5}{100(8 + 20)}$$

$$= 0.103 \text{ mphps}$$

$$t = \frac{10 - 9.9}{0.103} = 0.97 \text{ sec}$$

$$D = \frac{10 + 9.9}{2} \times 0.97 \times 1.46 = 14 \text{ ft}$$

For the remainder of the section at 10 mph,

$$t = \frac{2500 - 14}{10 \times 1.46} = 170.5 \text{ sec}; \qquad \text{current} = 55 \text{ amp}$$

Section a. Train must be stopped by the brakes.

$$t = \frac{500}{\frac{10 + 0}{2} \times 1.46} = 68.5 \text{ sec}; \qquad \text{current} = 0 \text{ amp}$$

Table 67 gives the tabulation of the data for the various sections. The time for loading was taken at 10 min. The average current on section *a* and one-half the loading time were taken as the effective conditions for loading.[1] Similarly for unloading the current is taken equivalent to that on section *e* (level) with the empty train and one-half the unloading time used.[1] The total time for a round trip is 32 min or 1920 sec. The total time required for running, loading, and unloading is 1227 sec. The difference (1920 − 1227 = 693 sec) is that required for switching, delays, etc. It is assumed that the current for this period is zero.[1] From

$$I^2 = \frac{I_1^2t_1 + I_2^2t_2 + I_3^2t_3 + \cdots}{t_E}$$

the continuous current consumption is found. This value, I, must not be greater than the continuous-current rating of the motors in the locomotive.

TABLE 67. SUMMARY OF DUTY-CYCLE DATA (ROLLER-BEARING CARS)

Section	T per motor, lb.	Distance, ft.	M.p.h.	Time, sec.	Current	Amperes^2 × time
Loaded:						
a. +0.5 per cent	1700	174	6.9	34.5	112	433,000
a	1250	323	7 4	28.1	91	232,500
b. −0 5 per cent	900	100	8.3	7.3	73	38,900
b.	550	2400	10.0	164.5	55	498,000
c. −0.3 per cent; 10-deg. curve	958	200	8 2	16 7	75	94,000
d. −0.6 per cent	754	171	9.0	12 8	65	54,100
d	550	1329	10.0	91.0	55	276,000
e Level	0	200	5.0	27.4	0	0
Total				382.3	. . .	1,626,500
Empty:						
e Level	1330	200	7.4	37.0	95	334,200
d. +0.6 per cent	940	94	8.2	7.4	75	41,600
d.	550	1406	10.0	96.3	55	291,500
c +0.3 per cent	578	200	9.9	13.8	56	43,300
b. +0.5 per cent	564	14	9 9	0.97	55	2,940
b.	550	2486	10.0	170.5	55	516,000
a. −0.5 per cent	0	500	5.0	68.5	0	0
Total				394.5		1,229,540
Loading				300.0	101	3,060,000
Unloading				150.0	95	1,353,000
Grand total				1226.8		7,269,040
Total time for trip = 32 min. × 60 sec.				1920.0		7,269,040

$$\text{Rms-amp} = \sqrt{\frac{7{,}269{,}040}{1920}} = 61.5$$

The continuous-current rating for mine-locomotive motors is based on 60 per cent and 80 per cent of normal voltage.[2] For the 43-hp motor under consideration, the continuous current for the 60 per cent normal voltage is

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[1] ALTSHULER, *op. cit.*, p. 994.

[2] Westinghouse Electric and Manufacturing Company, "Factory Testing of Electrical Apparatus" (I.B. 5400), p. 191.

$$(60\% \times 250 = 150 \text{ volts})$$

at 64 amp; and for the 80 per cent rating is

$$(80\% \times 250 = 200 \text{ volts}) \text{ at } 60 \text{ amp}^2$$

As the voltage will probably not drop below 200, the 60-amp rating will be used. This is based on a 75° rise at sea level and an ambient temperature of 25°C.

The haulage level on which the locomotive is to operate has an elevation of 6600 ft above sea level. Also the temperature of the level is 30°C. Before accepting the locomotive, the effect of this increase in ambient temperature and elevation on the continuous rating must be checked.

According to Altshuler[1] the following method is sufficiently accurate for this purpose. On page 445 it was stated that the 75-deg-rise curve for sea level must be corrected if the equipment is to be used at an altitude above 1000 m and that the permissible temperature must be decreased 1 per cent for each 100 m increase in altitude. The haulage level is at 6600 ft, or 2000 m. The correction is

$$\frac{2000 - 1000}{100} = 10\%$$

$$75° - 10\% \times 75 = 67.5°\text{C}$$

This must also be corrected for the new ambient temperature. This correction is $30° - 25° = 5°\text{C}$.

The temperature rise then becomes $67.5° - 5° = 62.5°\text{C}$.

For practical applications, the heating varies directly as the square of the current over the time it is applied. A temperature-rise curve for 62.5° must be constructed and the continuous rating of the motor for the new conditions must be determined from this. This curve may be constructed as follows:

From the 75°C curve in Fig. 165 points are chosen on the curve. For example, at 0.37 hr the current is 250 amp; at 0.6 hr it is 200 amp; at 1.00 hr it is 155 amp; at 1.5 hr it is 128 amp; at 2.0 hr it is 113 amp; at 2.5 hr it is 102 amp; at 3 hr it is 96 amp, at 3.5 hr it is 90 amp; and at 8 hr. it is 60 amp. By use of the formula $\sqrt{\text{hr} \times \text{amp}^2}$, we obtain values that are plotted against 75° in Fig. 166. From Fig. 166 we find the $\sqrt{\text{hr} \times \text{amp}^2}$ values for the 62.5° curve. These values, with the actual motor amperes, may then be solved for the time. A constant temperature (continuous rating) is assumed to occur after the motor has run continuously for 8 hr.

Table 68 gives the information for the current and the time chosen.

The amperes for the 75° curve are plotted against their equivalent time in the 62.5° curve. We now find that the 1-hr rating has dropped to 128 amp and the continuous rating, by interpolation from the curve, has decreased to about 52 amp. The duty cycle requires a rating of 61.5 amp; therefore, the locomotive selected is not large enough to perform the work satisfactorily unless auxiliary ventilation is used.

There are several other methods which may be used to find the continuous rating for the new temperature. The reliability of these methods is probably sufficient

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[1] ALTSHULER, *op. cit.*, p. 995.

[2] From manufacturer's data.

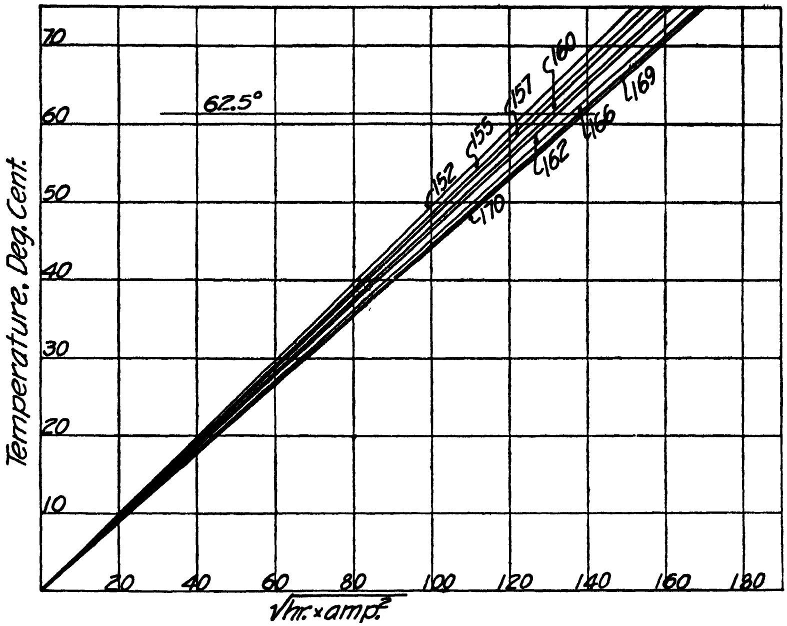

FIG. 166. Construction curve for obtaining values of time for any temperature rise at any given amperage. (*After Engineering Mining Journal.*)

when we consider the accuracy of the data (time, duty cycle, voltage drop, average current on sections, grades, etc.) used in arriving at the duty-cycle rating.

The 1-hr-rise rating at sea level is 155 amp, and the continuous-current capacity

TABLE 68. DATA FOR 62.5°-RISE CURVE

75° curve			62.5° curve		
Amperes	Hours	$\sqrt{\text{hr.} \times \text{amp.}^2}$	$\sqrt{\text{hr.} \times \text{amp.}^2}$	Hours	Minutes
250	0.37	152	126	0.25	15
200	0.6	155	129	0.42	24
155	1.0	155	126	0.66	40
128	1.5	157	128	1.00	60
113	2.0	160	131	1.35	81
102	2.5	162	132	1.67	100
96	3.0	166	136	2.01	121
90	3.5	169	138	2.33	140
60	8.0	170	139	5.37	322

For the 62.5° curve, the hours are found from

$$\text{Hr} \times \text{amp}^2 = 126, \text{ etc.}$$

$$\text{Hr} = \frac{126^2}{\text{motor amp}^2} = \frac{126^2}{250^2} = 0.25$$

at 200 volts is 60 amp. The continuous current is then

$$\frac{60}{155} \times 100 = 38.9 \text{ per cent of the 1-hr rating}$$

From data determined for the 62.5-deg curve the 1-hr rating is found to be 128 amp. The continuous current for the 62.5-deg curve is

$$128 \times 38.9 = 50.6, \text{ or } 51 \text{ amp}$$

which is about 1 amp less than by interpolating from the curve.

By this method we need only plot sufficient data to obtain the 1-hr rating under the desired temperature conditions.

A second procedure would be to determine the equation of the desired temperature curve from data obtained from the 75-deg curve between 1 hr and the time at which the temperature becomes constant. It may be safely assumed, for most motors, that a constant temperature probably occurs in not longer than 7 to 8 hr. Table 68 gives some of the data required for determining an equation which closely represents the 62.5-deg curve between the time limits chosen. Table 69 gives the information necessary for calculating the equation of the 62.5-deg curve.

It will be noted that no data below the 1-hr rating are taken. Such data would complicate the calculations and besides are not of particular use, as the information we desire is at the other end of the curve. On plotting the log amp as abscissa against log hr as ordinates,[1] we obtain practically a straight line. This indicates an equation of the order, $y = ax^b$. If b is negative, the curve is hyperbolic.[1]

TABLE 69. DATA FOR CALCULATING EQUATION OF 62.5°-RISE CURVE

Amperes	log amp.	Hours	log hr.
128	2.107	1.00	0.000
113	2.053	1.35	0.130
102	2.009	1.67	0.223
96	1.982	2.01	0.303
90	1.954	2.33	0.367
60	1.778	5.37	0.730

The constants a and b are determined by the method of averages.

Taking the logarithms of both members of $y = ax^b$, we get

$$\log y = \log a + b \log x$$

Substituting, $y = T$ (hours) and $x = A$ (amperes), the equation becomes

$$\log T = \log a + b \log A$$

Dividing the data in Table 69 into two groups of three sets each and adding, total log $T = 0.353$ when total log $A = 6.169$; and total $T = 1.400$ when total log $A = 5.714$. Then

$$\begin{array}{rcl} 0.353 & = & 3 \log a + 6.169b \\ 1.400 & = & 3 \log a + 5.714b \\ \hline -1.047 & = & 0 \quad + 0.455b \end{array}$$

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[1] LIPKA, J., "Graphical and Mechanical Computations," Part II, p. 128, John Wiley & Sons, Inc., New York, 1921.

and

$$b = -\frac{1.047}{0.455} = -2.30$$

$$\log a = \frac{0.353 + 6.169 \times 2.3}{3} = 4.847$$

$$a = 70{,}310$$

Therefore the equation of the 62.5° temperature curve is

$$T = aA^{b} = 70{,}310A^{-2.30} = \frac{70{,}310}{A^{2.30}}$$

Checking this by finding T when $A = 102$ amp,

$$T = \frac{70{,}310}{102^{2.30}} = \frac{70{,}310}{41{,}750} = 1.68 \text{ hr}$$

which practically checks the 1.67 hr from Table 68.

On the assumption that constant temperature (continuous rating) is reached in 8 hr, we now find the value of the continuous rating for the 62.5° curve.

$$A = \sqrt[2.3]{\frac{70{,}310}{T}} = \sqrt[2.3]{\frac{70{,}310}{8}} = 51.9 \text{ amp}$$

This value is almost the same as that obtained by the curve or percentage method. Preference would seem to lie with a method other than interpolation from the curve, because most of the characteristic curves obtainable give plotted data only up to about 200 min.

It will be cheaper to equip the motors with blowers than to increase the motor capacity for the locomotive. This is usually done by means of small Sirocco fans.[1] By so doing the continuous-current capacity is increased from 33⅓ to 50 per cent.[1] To construct a curve showing this, the 1-hr rating for the 62.5° curve is increased by 5 per cent and the continuous 62.5° rating by 33⅓ to 50 per cent. For the present case it will be increased 33⅓ per cent. The 1-hr rating now becomes

$$128 \times 1.05 = 134.4, \text{ or } 134 \text{ amp}$$

The continuous-current rating is

$$52 \times 1.33 = 69.2 \text{ or } 69 \text{ amp}$$

The 8-ton locomotive selected will, if equipped with blowers, perform the work satisfactorily at the altitude of 6600 ft. If the level on which the locomotive operates is dusty, blowers cannot be used; so larger motors must be installed.

When the above problem is solved for plain-bearing cars (30 lb per ton) it is found that a continuous current of 66 amp is required. This is somewhat higher than the sea-level rating. With blowers the locomotive would handle the work.

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[1] ALTSHULER, *op. cit.*, p. 995.

CHAPTER XII

POWER PLANT

For primary power a mining company may have four possibilities available from which to choose. In some cases all or part of them may be called into service.

The four sources are

1. Hydroelectric
2. Steam
3. Internal-combustion engines (usually some form of diesel)
4. Power company

A great many mining properties, large and small, and especially moderately sized ones, are so located that primary power must be generated at the property. For this purpose steam or diesel engines are usually selected.

In the early life of most mines, because of their isolated location, it is out of the question to purchase power from a public-service company. The cost of running the lines (poles, wire, labor, etc.) in to the property, a deposit to guarantee payment for minimum amount of power, the maintenance of the line in isolated regions, etc., prohibit the operator in the early development stage from buying power. He must, therefore, select the equipment which will prove the most economical, which usually depends on the location. In those camps located in the vicinity of oil-producing areas, the internal-combustion engine or oil-burning steam plant is ordinarily used. On the other hand, in timbered country or coal-producing areas, a steam plant may be selected. In other localities the development of hydroelectric power may prove desirable.

Because so many factors, many of them local, enter into the choice—economically—between the various possibilities, very little effort will be devoted to an operating- and installation-cost comparison. When transportation charges and fuel costs are reasonable, the diesel is probably the best primary source of power, mainly because of its higher efficiency. Under reasonable conditions electrical power has been produced for about 1 cent per kilowatt-hour (operating cost) by diesel engines.[1]

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[1] Tartt, P. B., Power for Mine and Mill, *Eng. Mining J.*, **129**, 2 (1930).
Huttl, J. B., Diesel Engines in Mining, *Eng. Mining J.*, **130**, 603 (1930).
Personal observations.

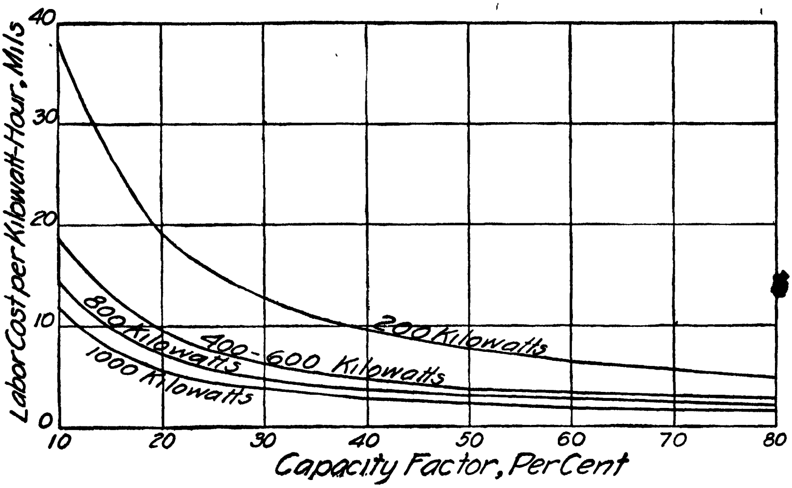

FIG. 167. Labor costs for oil- or gas-engine power plants. (*Engineering Mining Journal.*)

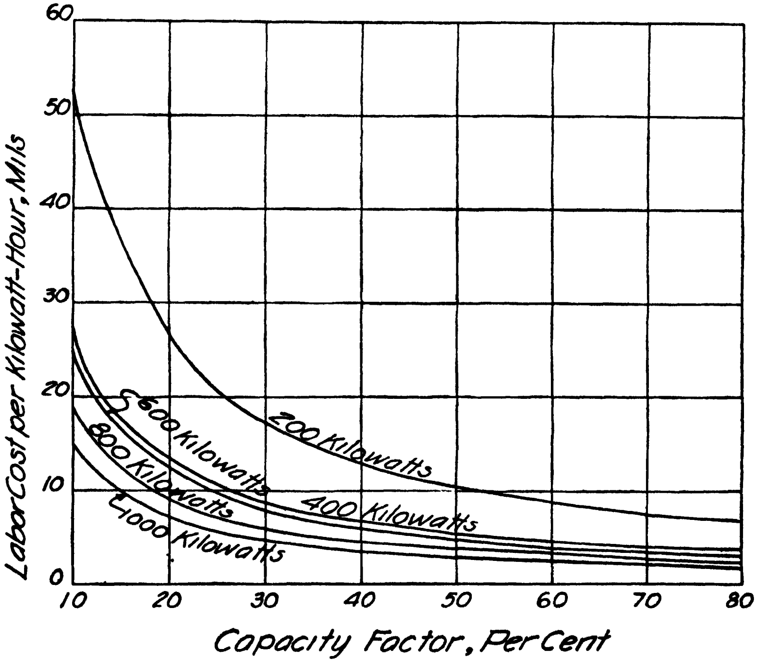

FIG. 168. Labor costs for steam-power plants. (*Engineering Mining Journal.*)

Economical Choice between Methods Available.[1] As an aid toward making the preliminary investigation in choosing the generating equipment, Figs. 167, 168, 169, 170, and 171 are presented. Figure 167 gives the labor cost per kilowatt-hour in mils for oil- or gas-engine power plants. Figure 168 gives the same information for steam plants. From Fig. 169 may be obtained fuel costs for oil engines and steam-power plants. Figure 170 gives solid-fuel costs for steam plants. Fixed charges for power plants are shown in Fig. 171.

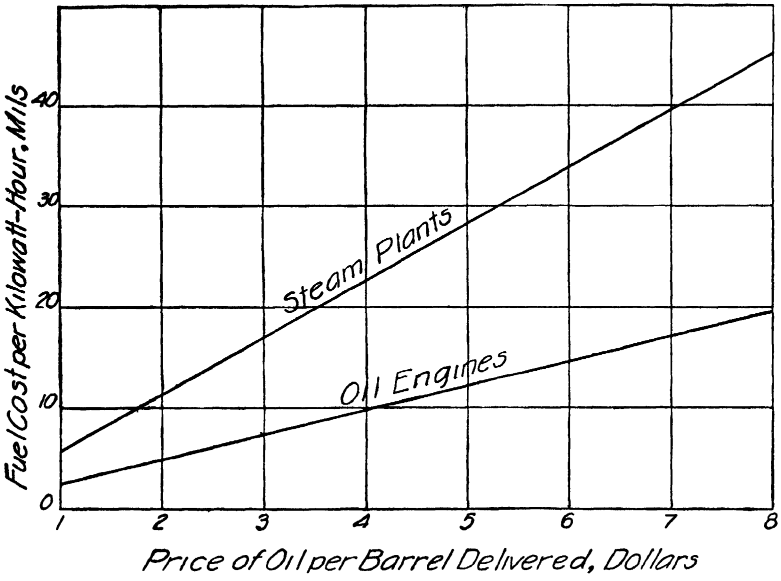

FIG. 169. Fuel costs for oil engines and steam-power plants. (*Engineering Mining Journal.*)

The capacity factor is the average annual percentage of the rated capacity that the plant delivers. The charges for labor, supplies, repairs, and maintenance for hydroelectric plants are about the same as for steam. For a preliminary investigation they may be assumed as being the same. The following table gives general operating costs.

	Steam plants, mils	Oil or gas engines, mils
Lubrication and supplies	1	2
Repairs and maintenance . . .	1	2

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[1] Abstracted from article by P. B. Tartt.

The following example illustrates the use of the figures.

Assume that a plant capacity of 500 kw is required, and the average annual capacity factor will be 40 per cent. An investigation shows that an adequate hydroelectric plant could be built for $250 per kilowatt of capacity. A transmission line costing about $15,000 would have to be built from the hydroelectric plant. A coal of 12,000 Btu calorific value is available at $2 per ton. Mexican crude oil of 15°Bé and about 3 per cent sulfur is available at $3 per barrel, and a good low-sulfur[1] diesel oil of 24°Bé, is available at $5 per barrel. By building a pipe line at a cost of $30,000, natural gas, at 25 cents per 1000 cubic feet and 1000 Btu fuel value, would be available. A power company offers the following typical rate:

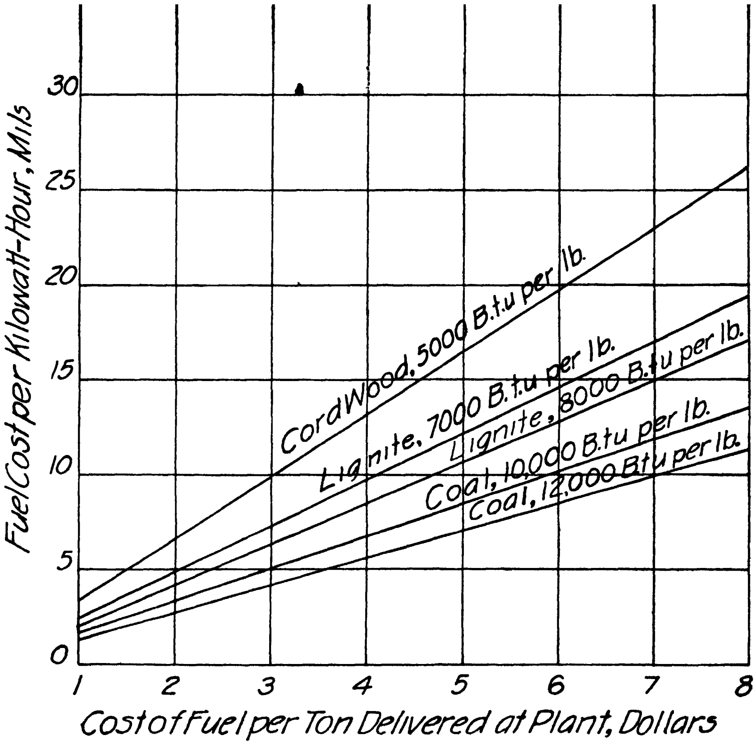

FIG. 170. Solid-fuel costs for steam-power plants. (*Engineering Mining Journal.*)

Demand charges:

$2 per kilowatt for first 50 kw of maximum demand per month
$1.50 per kilowatt for each kilowatt in excess of 50

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[1] Sulfur content in diesel oils is no longer specified. Except for intermittent service (a maximum of 2 per cent is suggested for such service), the sulfur content

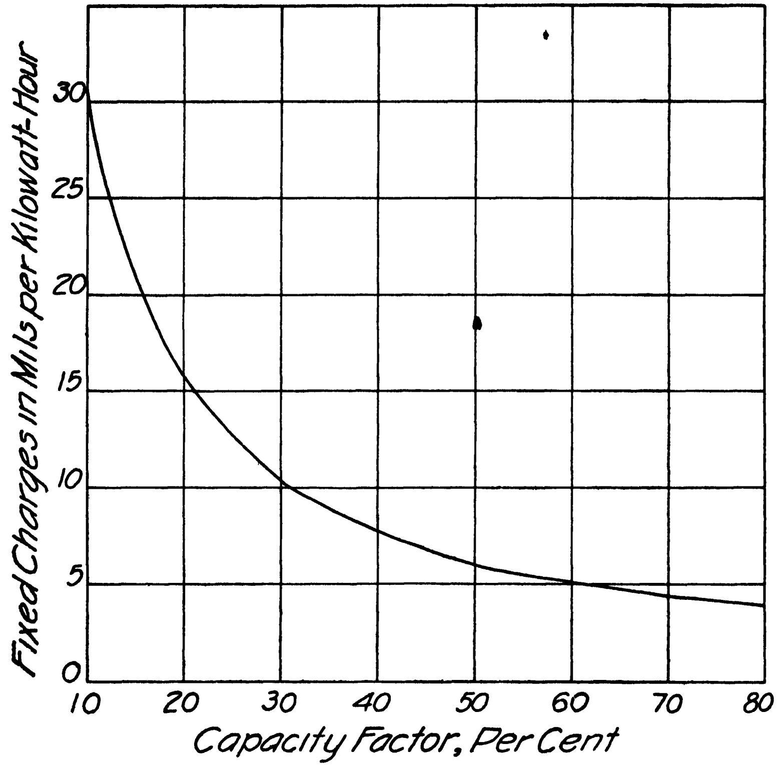

FIG. 171. Fixed charges for power plants in general. Based on 18 per cent of plant cost per annum to cover amortization of sum invested in plant; interest on investment; taxes and insurance. Plant cost assumed at $150 per kilowatt capacity. (*Engineering Mining Journal.*)

Energy charges:

4 cents per kilowatt-hour for first 100 kw-hr per month
3 cents per kilowatt-hour for next 1000 kw-hr per month
2 cents per kilowatt-hour for next 10,000 kw-hr per month
1.5 cents per kilowatt-hour for all in excess of 11,000 kw-hr per month

The estimated life of the mine is assumed at 10 years or better. What type of plant will be the most economical; or should power be purchased?

Hydroelectric Plant. The transmission line will add $30 per kilowatt to the estimated cost, making a total of 250 + 30 = $280 per kilowatt. The cost of energy per kilowatt-hour is

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is immaterial. See "Diesel Fuel Oil Classification," American Society for Testing Materials, 1934.

Lubrication and supplies	1.0 mil
Repairs and maintenance	1.0 mil
Labor and administration (Fig. 168)	6.3 mils
Fixed charges (Fig. 171) $(7.65 \times {}^{280}\!/_{150})$*	14.3 mils
Total cost per kilowatt-hour	22.6 mils

* Plant costs $250 per kilowatt. 15,000/500 = $30 per kilowatt for building transmission line. Total = 250 + 30 = $280.

Steam Plant. There are three fuels available: coal at $2 per ton; crude oil at $3 per barrel; and gas at 25 cents per 1000 cu ft. If gas is used, $60 per kilowatt must be added to the cost of the plant to cover the cost of the pipe line. This makes a total cost of $210 per kilowatt for the gas plant.

Fuel costs per kilowatt-hour are

Coal (Fig. 170)	2.8 mils
Mexican oil (Fig. 169)	16.9 mils
Gas*	8.5 mils

* One kilowatt-hour = 3412 Btu. The gas contains 1000 Btu per 1000 cu ft and costs 25 cents per 1000 cu ft. Therefore, 3412 cu ft are needed per kilowatt-hour.

$${}^{3412}\!/_{1000} \times \$0.25 \times 10 \text{ mils} = 8.53 \text{ mils per kw hr}$$

If a steam plant is selected, the cheapest fuel would be coal at 2.8 mils per kilowatt-hour. The total cost is

Fuel	2.8 mils
Lubrication and supplies	1.0 mil
Repairs and maintenance	1.0 mil
Labor and administration (Fig. 168)	6.3 mils
Fixed charges (Fig. 171)	7.7 mils
Total cost per kilowatt-hour	18.8 mils

Oil Engines.[1] Only one fuel is available: fuel oil at $5 per barrel.

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[1] The cost shown here is altogether too high. It is partly due to the high-cost low-sulfur oil specified. The following calculations (see page 482) show more nearly what the diesel cost would be. For simplicity in handling the calculations the 500 kw are converted to horsepower

$$500 \div 0.746 = 670 \text{ hp}$$

Taking the cost of the plant at $100 per brake horsepower [Morrison and Burdick ("Diesel Engineering Handbook," 7th ed., p. 27, Business Journals, Inc., New York, 1934), consider $75 ample to cover everything except real estate. The $100 assumed should cover extraordinary charges met with in making an installation in a mining camp], the cost of the plant is

$$670 \times \$100 = \$67{,}000$$

Fuel Oil. Except under the most extreme conditions fuel oil delivered at the plant should not exceed 8 cents per gallon. (At Boise, Idaho, during 1934, the price was 9¼ cents). For the present example, 8 cents per gallon will be assumed.

$$670 \text{ hp} \times 8760 \text{ hr} \times 40\% = 2{,}347{,}680 \text{ bhp-hr plant production per year}$$

Energy cost:

Fuel (Fig. 169)	12.3 mils
Lubrication and supplies	2.0 mils
Repairs and maintenance	2.0 mils
Labor and administration (Fig. 167)	4.8 mils
Fixed charges (Fig. 171)	7.7 mils
Total cost per kilowatt-hour	28.8 mils

Gas Engine. Only fuel available is gas at 25 cents per 1000 cu ft.
Energy cost:

Fuel	4.4 mils
Lubrication and supplies	2.0 mils
Repairs and maintenance	2.0 mils
Labor and administration (Fig. 167)	4.8 mils
Fixed charges (Fig. 171) ($7.65 \times {}^{210}/_{150}$)	10.7 mils
Total cost per kilowatt-hour	23.9 mils

Purchased Power. As the "demand" is usually based on the average maximum demand over a 15-min period, assume 80 per cent of the plant capacity and call the demand 400 kw instead of 500.

Demand charges per month:

50 kw at $2.00	$100
350 kw at 1.50	525
Total for 400 kw	$625

On a 30-day basis with 50 per cent load factor based on a 400-kw demand, which corresponds to the assumed 40 per cent capacity factor based on 500 kw, a total of $500 \times 0.40 \times 24 \times 30 = 144{,}000$ kw-hr will be used per month.

The fuel consumption of engines in a plant this size will be in the neighborhood of 0.43 lb per bhp-hr.

$$2{,}347{,}680 \times 0.43 \times \frac{1}{7.5} = 134{,}550 \text{ gal}$$
$$134{,}550 \times 8 \text{ cents} = \$10{,}764$$

Lubricating Oil. With fuel oil at the price assumed, lubricating oil will cost about 50 cents per gallon. The engines will deliver about 3000 bhp-hr per gallon.

$$2{,}347{,}680 \div 3000 = 691 \text{ gal}$$
$$691 \times 50 \text{ cents} = \$345$$

Labor. The plant is assumed to be available 365 days per year. In a plant this size, a chief engineer separate from the other labor is not necesary. He can operate one shift. We require two operators at $5 per day and one chief engineer-operator at $7.

$$365 \times 2 \text{ @ } \$5 + 365 \times \$7 = \$6205$$

Maintenance. The cost of the engines will be very close to $65 per brake horse-

Energy charges:

$0.04 × 100 = $ 4
0.03 × 1,000 = 30
0.02 × 10,000 = 200
0.015 × 132,900 = 1990
Total 144,000 kw-hr = $2224
Plus demand charges = 625
Total per month = $2849

$$\text{Average cost per kilowatt-hour} = \frac{2849 \times 1000 \text{ mils}}{144{,}000} = 19.8 \text{ mils}$$

The costs as determined are tabulated in the following table:

Plant	Cost per Kw-hr, Mils
Hydroelectric	22.6
Steam	18.8
Oil engine	28.8
Gas engine	23.9
Power company	19.8

From the above table we find, that so long as the price of coal does not rise, the steam plant would be the desirable installation.

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power (manufacturer's data). The maintenance charge is taken at 2 per cent of factory cost (see page 484).

$$2\% \times 65\% \times \$67{,}000 = \$871$$

Fixed Charges. The life of the mine was assumed at 10 years or better.

Interest on plant investment	6.00 per cent
Insurance and taxes	3.00 per cent
Redemption of capital, (interest on yearly sum set aside, 5 per cent compounded annually)	7.95 per cent
Total	16.95 per cent

$$\$67{,}000 \times 16.95\% = \$11.356$$

Total cost:

Fuel oil	$10,764
Lubricating oil	345
Labor	6,205
Maintenance	871
Fixed charges	11,356
Total	$29,541

Cost per kilowatt-hour:

$$\$29{,}541 \div (2{,}347{,}680 \times 0.746) = \$0.01685 = 16.85 \text{ mils}$$

On the basis of this cost the diesel plant would warrant a more detailed investigation along with the steam plant and power company.

DIESEL PLANT[1]

The first internal-combustion engine was probably built by Huygens in 1680. He used gunpowder as a source of power. In 1791, John Barber, an Englishman, designed an engine to operate on producer gas.

The first practical engine was built by Lenoir in 1860. The efficiency of these engines was extremely low (about 4 per cent), and their fuel consumption very high.

Otto, in 1866 and 1878, and Beau de Rochas, in 1862, proposed cycles which bear their names. Otto's proposal did not prove satisfactory and the cycle which today bears his name should be known as the "Beau de Rochas cycle." The events occurring in the modern four-stroke-cycle engine are indentical with those outlined by Beau de Rochas. In 1921, more than 500,000 hp. of the so-called Otto cycle engines were sold.

From the time of Otto to 1897, when Dr. Rudolf Diesel, a German engineer, brought out the engine that bears his name, many suggestions were advanced and improvements made in fuel injection, compression cycle, heat control, etc. The first American diesel was made in 1898.[2]

In 1932, there were about 4,500,000 hp of diesel engines installed in the United States.[3] This represented 38,453 engines. Of these 4,500,000 hp, almost 4,300,000 were still in actual use. Many of these engines had been in service for nearly 30 years, with only a nominal upkeep cost. This evidence should do much to allay the fears of those who feel that the diesel engine is short-lived. The manufacturers make all parts, subjected to wear, readily accessible and replaceable.

The industries in which diesel engines are used may, according to Morrison[3] be classified as follows:

Applications	Per Cent of Total	Applications	Per Cent of Total
General industry	21	Mines and quarries	3.4
Marine	20	Railroads	2.4
Petroleum	11.3	Irrigation	2.4
Municipal light	10.2	Ice plants	2.2
Public utility	7.8	Waterworks	2.2
Contractors	5.1	Export	2.0
Cotton gins	4.8	Government institutes	1.1
U.S. Navy	3.7	Buildings	0.4

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[1] Abstracted from STALEY, W. W., "Application of Diesel Engines to Small Mines," *Pamphlet* 42 (1935), Idaho Bureau of Mines and Geology. By permission of the Director of the Idaho Bureau of Mines and Geology.

[2] MORRISON, L. H., "American Diesel Engines," 1st ed., McGraw-Hill Book Company, Inc., New York.

[3] MORRISON, L. H., "Diesel Power," p. 272, June, 1933. Diesel Publications, New York.

By 1934 it was estimated that over 5,000,000 hp of oil engines would be installed. The field of use has increased rapidly. The reason for this rapid increase is probably due to two reasons: (1) their economy and (2), so far as the smaller (10 to 200 hp) units are concerned, the fact that highly trained engineers are unnecessary for their operation. Common sense, adherence to the manufacturer's instructions, and a knowledge of the ordinary gasoline engine is sufficient training for the small-plant operator. The idea has prevailed for years that the diesel engine was something mysterious, a contrivance which would do the bidding of only a chosen few. This notion, fortunately, is rapidly being dispelled. To the mine operator, both large and small, the diesel engine should prove a decided asset.

Of all the heat engines of present known application, the diesel engine is by far the most efficient; *i.e.*, it converts more of its heat energy into useful work. Steam turbines of more than 100,000 hp do not have the thermal efficiency of a 10-hp diesel engine.[1]

An important point in favor of the diesel engine is the fact that the efficiency of engines of but a few horsepower is but very little less than that of the largest engines. This is far from being true in the case of steam plants.

The advantages of diesel power are well summarized in the Fairbanks, Morse Company *Bulletin* previously quoted. They are

Low fuel cost
Maximum dependability
Long, useful life
Small operating force
Low maintenance cost
Delivers full power, instantly
No stand-by losses
Simplicity in design
No service or demand charges
High efficiency in all sizes
Plant can be enlarged without sacrifice of original investment or over-all economy
Large water supply unnecessary
No chimney, smoke, or ashes
No coal or ash-handling apparatus

Of the above items, it might be well to discuss two of them briefly. The advantages of the others are apparent.

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[1] "Economy of Diesel Engine Power," *Bull.* 3010, Fairbanks, Morse Company

Fuel Cost. The fuel cost will depend primarily on three things: cost delivered, heating value, and engine consumption per brake horsepower-hour. In most mining localities, especially those located at a distance from oil-producing centers, fuel oil is relatively high. In Idaho, it reaches a price in excess of 16 cents per gallon in one easily accessible locality. Under such conditions, coal, gasoline, or electrical energy will also generally be proportionately higher. The fuel-oil costs will usually account for a little less than one-half the total operating cost.

The heating value of the fuel oil depends on its grade and purity. Engines are usually rated on an oil containing at least 19,000 Btu per pound. Diesel fuel oil supplied by reputable oil companies is seldom less than this. A low Btu value will usually mean a higher fuel consumption. This may not in itself amount to much. However, the transportation charges will be the same for low-grade oils as for high-grade oils, and this is important for isolated districts. It behooves the purchaser to get the standard grade of oil.

The fuel consumption on most modern engines is in the neighborhood of 0.40 to 0.50 lb per bhp-hr. At full load, this is usually attainable and guaranteed by many manufacturers. The consumption increases slightly at less than full load.

Maintenance Cost. One of the points of controversy for years between the manufacturers of other prime movers and diesel-engine manufacturers has been maintenance cost. A survey by manufacturers of a large number of engines which have operated over a period of years indicates that a yearly maintenance charge of 2 per cent of the factory cost of the machinery, without regard to the annual output, is more than ample to cover repairs and upkeep.

It is interesting to note that the greater number of engines sold (1920 to 1932) ranged from 20 to 200 hp.

FUTURE OF DIESEL ENGINES

There are three things that might be considered as making an investment in diesel power risky.

1. Obsolescence
2. Engine wearing out
3. Exhaustion of oil supply

Obsolescence. For our purpose, obsolescence may be taken as meaning such a rapid change in design, construction, and efficiency of the engine that the investment in a few years would be worthless. To refute such a statement regarding the diesel engine, the argument is advanced that since its development, almost 50 years ago, the efficiency has been increased but very little. As a matter of fact, it is claimed that there is very little room left for increasing the present thermal efficiency beyond 35 per cent. From a structural and operating standpoint, there has been a great advancement. However, the fuel consumption of a great many of the older engines is but a few tenths of a pound more than the modern engine. In most cases, the older engine can be adapted with later improvements in combustion and fuel injection, etc., with but a small outlay. When it is all summed up, there is little to fear from obsolescence.

Engine Wearing Out. Diesel-engine manufacturers have compiled statistics to show that with ordinary care and attention there should be no trouble from worn parts for at least 10 years. The parts receiving the greatest abuse, with consequent wear, are the valves, pistons, and cylinder liners. Even though wear on these parts, in exceptional cases, becomes excessive, there is no great expense incurred in rectifying the trouble. The cost of completely overhauling an engine, *i.e.*, reboring cylinders and fitting new pistons, rebabbiting crankpin bearings, replacing piston-pin bearings and main bearings, replacing all wearing parts in fuel and control systems, etc. (in other words, completely rebuilding the engine), is but a very modest part of the original cost. This cost amounts to (including cost of parts, labor, and freight) about 20 per cent for large engines (800 to 1000 hp) to 40 per cent for small engines (25 to 200 hp). A yearly sinking fund (on the basis of 10 years) of 1.6 to 2.9 per cent of the original cost with accessories will provide for complete rebuilding.

Exhaustion of Oil Supplies. So far as mining men are concerned, it would seem superfluous to offer a defense against diesel-fuel supplies becoming depleted in the near future. Even when pumped oil is exhausted, that remaining (about 70 per cent) can be extracted by mining methods. The oil-shale reserves apparently approach the reserves of petroleum in size. In the last analysis, coal can be converted into a usable diesel fuel. Aside from the United States, other countries in the Western Hemisphere contain tremendous oil reserves. So far as the next century is concerned, the possible lack of fuel for diesel engines has caused far more apprehension than it deserves. Of the three items discussed, the lack of a fuel is the least to be feared. As a matter of fact, none of the three is of great importance.

The only real competition that the diesel engine has to fear is the production of cheap hydroelectric power. From the present outlook, it will be some years before outlying mining properties can expect to buy electric power at a reasonable cost. Investigations at the present time indicate that transmission and distribution costs are responsible for the major portion of the cost. The capital investment in hydroelectric plants is also very great.

TYPES OF DIESEL ENGINES

There are three types of diesel engines (so far as we are concerned).

1. Stationary
2. Marine
3. Automotive

Of these, we are primarily interested in the stationary engine. The application of the marine engine in the stationary field is very doubtful. If done at all, it should only be after very careful consideration and analysis of the proposed loading by an experienced diesel consultant.

The automotive diesels of the Waukeshas, caterpillar, and Hercules types (these companies also make small stationary engines), while primarily for truck and bus uses, may be mounted on a base and adapted to stationary work. They are relatively light in weight, and should prove advantageous for prospecting or development work (for example, for direct drive to compressors, generators, etc., of small capacity; or chain, gear, or direct drive to diamond drills.) Because of their light weight, they would cause a minimum of transportation trouble in mountainous regions.

Stationary Engines

It is with the small- to moderate-sized stationary engine that we are mainly interested. In a great many mining regions there is an almost total lack of year-round passable roads. The topography is rough and mountainous, which increases transportation costs enormously. Because of these features, a lightweight engine is very desirable. In many cases, it may be necessary to dismantle the engine. This requires simplicity in design and as few heavy parts as possible. These requirements are substantially met by the present-day engine ranging up to 180 to 200 hp. As will be shown later, it will seldom be necessary to require an engine greater than 200 hp. The load cycle of a mine and mill is usually such that two or more engines of small rating will prove more economical than one large engine.

The range in size of engines (this does not mean total power) for small mines and mills, and for prospecting and development work, may be confined between 10 and 200 hp. Where only the mine is being operated, one engine connected to a compressor or to a compressor and small generator may be sufficient. Or one engine may operate both a hoist and a compressor. Or they may have separate engines. The rating of such engines will, of course, depend on the number of drills operated, the tonnage of material hoisted, accessory equipment in the shops and the altitude of the engine. Ordinarily, 10 to 100 hp would suffice.

Where both the mine and mill are operating, the horsepower required will depend on the tonnage of ore and waste hoisted, the tonnage and method of treatment by the mill, and the altitude of the engine. The brake horsepower required for both mine and mill may be roughly estimated at 4 to 6 bhp per ton milled.

Two-cycle Engines. In the two-stroke cycle, the piston makes two movements, one in both directions to complete the cycle. This type of engine usually has no valves, and therefore none of the camshafts, lifters, gearing, etc., that usually accompany tappet valves. Naturally, this is a very important feature from the standpoint of maintenance. However, for high-speed engines (above 300 to 400 rpm) the combusted gases are not satisfactorily removed. The reason for this is as follows: Instead of valves the two-cycle engine has exhaust ports in the cylinders, which are uncovered by the piston shortly before it reaches the end of the exhaust stroke. As soon as the ports are uncovered, air at a pressure slightly greater than atmospheric is blown into the cylinders. This air is supposed to sweep out the burned gases. In engines having a speed greater than about 400 rpm, the removal of burned gases is not complete because the piston starts its return stroke before the gases are ejected. The two-cycle engine requires either crankcase compression or a separate compressor to furnish the scavenging air.

Briefly, the operation of the two-cycle engine is as follows: The two strokes are (1) the admission and compression stroke and (2) the power and exhaust stroke. When the exhaust ports are closed on the return stroke of the piston, the air remaining in the cylinder is compressed. Near the end of the return stroke the fuel oil is injected into the cylinder. Combustion and expansion then take place. The piston is forced downward. When the exhaust ports are uncovered, the cleaning air is blown through the cylinder. The compression stroke then begins. From this we see that admission and compression are done in one stroke, and expansion and exhaustion in one stroke—hence, the name, two-stroke cycle. The outstanding advantage is the lack of valve trouble and maintenance. One complete revolution of the crankshaft is made during the cycle. In some makes of engines, valves are used to admit scavenging air.

Four-cycle Engines. In the four-stroke-cycle engine, four strokes or two complete revolutions of the crankshaft are necessary to complete the cycle. Intake and exhaust valves are necessary. They are located in the engine head. Scavenging air is not used, except in special cases where the sea-level rating of the engine is desired at high altitudes (over 2000 to 3000 ft). The four strokes of the cycle are as follows: (1) admission stroke, in which air at atmospheric pressure is drawn into the cylinder; (2) the compression stroke, in which the admitted air is compressed, thus furnishing heat for the later ignition of the oil; (3) power stroke, the piston being repelled back from the compression stroke; (4) exhaust stroke, in which the piston returns from the power stroke, expelling the spent gases. Strokes one and two are accomplished in one revolution of the crankshaft, and strokes three and four by one revolution of the crankshaft. Thus, we have two complete revolutions of the crankshaft to give the four-stroke cycle.

The development in recent years seems to be toward the two-stroke cycle (two-cycle), airless (solid), injection engine. A better structural balance in the engine parts is claimed and fewer parts that cause trouble through wear and adjustment (valves, gears, etc.) are found in the two-cycle engine. Crankcase compression of air for scavenging has apparently been developed to practically a trouble-free point. As mentioned before, the two-cycle engine does not satisfactorily remove burned gases at speeds higher than 400 rpm.

From the standpoint of compression pressure, diesels may be considered as full diesels or simply diesels; and as semidiesels. The full diesel compresses the air to a pressure of 450 to 600 lb per sq in. with a temperature of 900 to 1400°F. The exhaust temperature, after passing the exhaust valve, is 400 to 750°F. Either air injection or solid (also known as "airless" and "mechanical") injection is used. In the air-injection type, the fuel oil is forced into the cylinder by means of air compressed to a pressure of about 1000 lb per sq in. In the solid-injection engine, the oil is forced into the cylinder through an atomizer by means of a fuel pump exerting a pressure of over 4000 lb per sq in. The solid-injection method appears to be rapidly superseding the other. About 6 to 8 per cent of the engine's rating is required to operate the three-stage air compressor needed in the air-injection engine.

The semidiesel operates at a considerable lower pressure (in the neighborhood of 300 lb per sq in.) than the full diesel. These engines are sometimes erroneously known as "solid-injection" engines. (Solid injection is not an exclusive feature of the semidiesel.)

From the economy standpoint, the semi-diesel consumes more fuel oil per brake horsepower-hour (at least 0.22 lb) than the diesel.[1] The lubricating oil consumption is much higher[1] (about 600 bhp-hr per gallon as against about 3000 for the diesel).

Morrison[1] states that the semidiesel is more economical in 100-hp sizes or less when the capacity-use factor is less than 70 per cent; *i.e.*, the engine should operate 70 per cent of the time fully loaded or 70 per cent loaded 100 per cent of the time based on a 10-hr day. Above 100 hp, the capacity-use factor should be 50 per cent. The first cost of the semidiesel is considerably less than the diesel. Because of its lower compression pressure, it need not be built nearly so strong and massive. From a transportation standpoint, this is of importance because it means a reduction in weight.

As has been stated, the use of the solid-injection engine is rapidly replacing the air injection. From the standpoint of fuel consumption, there is very little, if any, difference between it and the air injection (if anything, it is superior to air injection). For the size of engine we are considering, the solid-injection engine is undoubtedly better suited. It is much lighter (air compressor for charging air is not necessary), has fewer moving parts to keep in order, and the first cost is less. In over-all efficiency, the air-injection engine is less than the solid injection.

ACCESSORY EQUIPMENT

The nature of the accessory equipment required for the range of units under consideration will depend on the method of starting the engine and whether the operating equipment is driven by electric motors or directly by the diesel engine.

The small- to moderate-sized units (up to 200 hp) may be started by either a storage battery and electric motor or by compressed air. Practically all the recent models are electrically started. The principle is the same as for the ordinary automobile engine. A charging generator and motor are mounted on the diesel engine and a storage battery used to supply power. When compressed air is used as a starting medium, a small gasoline-engine-driven compressor with a storage tank is used. Other accessory equipment, among other items, will consist of the following:

1. Fuel-oil storage tank
2. Filter or some form of purifier or cleaner for lubricating oil

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[1] MORRISON, L. H.: "American Diesel Engines," 1st ed., p. 63.

3. Cooling-water thermometer
4. Pump to circulate cooling water
5. Pump to handle fuel oil
6. Muffler for exhaust
7. Switchboard if electrical energy is generated
8. In cold climates some arrangement of utilizing waste exhaust heat to warm the building

One of the accessories requiring special mentioning is the cooling system. No definite data can be given here concerning the proper cooling-water temperature, as it depends on the size and design of the engine. The manufacturer's recommendations must be followed. The most satisfactory method for small engines is by means of a constant-pressure pump, as compared with a valve operated from a water main or standpipe. Many of the small- to moderate-sized engines are now built with a radiator and fan similar to the ordinary automobile engine. This ensures a good temperature control, especially where only intermittent attention is given to the engine.

Diesel engines are equipped with replaceable liners for the cylinders. When the output of the engine is seriously reduced, because of cylinder wear, the liner is quickly and easily replaced. All moving parts are as readily accessible. This ability to replace worn parts contributes to a relatively high resale value. The replacement of cylinder liners, valves, piston rings (in extreme cases, pistons), bearings, etc., can be done at a comparatively small outlay considering the first cost of the machine (20 to 40 per cent of original cost with accessories on large to small engines, respectively). After such replacements are made, the efficiency and output of the diesel engine are slightly, if any, less than when new.

Fuel Oil. For satisfactory operation, a good grade of fuel oil should be used. The engine manufacturer's rating is usually based on a fuel oil having a heat value of 19,000 Btu per lb.

The following table gives a typical analysis of the Standard Oil Company of California, Standard Diesel Oil 27 plus. It is well within the limits recommended by the American Society for Testing Materials in their publication "Diesel-fuel-oil Classification" (1934).

Gravity, API = 27 plus
Specific gravity at 60°F = 0.8927 (minimum)
Pounds per gallon = 7.434 (minimum)
Flash point (Pensky Martens) = 150° to 200°F
Viscosity at 100°F (Saybolt-Universal) = 40 to 50 sec
Pour point = 0°F (maximum)

Water and sediment = 0.10 per cent (maximum)
Conradson carbon residue = 0.15 per cent (maximum)
Distillation temperature:
  10 per cent point = 475°F (maximum)
  90 per cent point = 685°F (maximum)
Btu per pound = 19,000 plus

The marketing guarantee on this oil is that its API gravity at 60°F will never be lower than 27°.

Lubricating Oil. The use of a high-grade lubricating oil cannot be too highly emphasized. Maintenance and trouble-free operation depend to a very large extent on successful lubrication. The lubrication of the diesel presents no more difficulty than other types of prime movers.

It is beyond our scope to undertake a discussion of diesel lubrication. It is sufficient to stress the point that the best grade of oil recommended by the established oil companies be used. The use of an oil filter or purifier will do much toward lowering lubricating oil costs.

Capacity Factor. Ordinarily, an engine should be so selected that it will operate at slightly below the manufacturer's continuous full-load rating. This permits momentary overloads without undue injury to, or stalling of, the engine. In the selection of an engine for mining purposes, it is probably better to base the selection on approximately its three-quarter load rating (75 to 80 per cent). The reason for this is the difficulty in choosing exactly the right size of mining and concentrating equipment to handle a given tonnage. The nature of underground conditions and ore and waste (hard, wet, sticky, fine, coarse) contribute to this uncertainty. Some parts of the equipment selected may prove capable of operating at well over their rating. Other parts, because of the physical condition of the material treated, may fall below their rating. An excess of available power would thus permit a better operating adjustment. Also, the starting of hoists and ball mills usually takes about 100 per cent overload of their normal running requirements. Some of this may be taken up by a clutch, but the engine must be capable of supplying a large part of the overload.

The fuel-oil consumption per brake horsepower-hour of the diesel is but slightly greater at three-quarter load than at full load. It is apparent that the excess stand-by power more than compensates for the slight increase in fuel costs.

APPLICATION TO MINING AND CONCENTRATING EQUIPMENT

Diesel-engine power can be applied to drive any type of equipment where steam, water, or electrical power may be used. During recent years it has been adapted to power shovels, drag lines, excavators, dredges, trains, ships, etc., where formerly steam or electricity was used exclusively. The development of suitable clutches, chain, gear, and V-belt drives, have furthered the applications. Belt drive and direct connection to the crankshaft are very common and much used. The development of relatively high-speed engines (formerly the speed was in the neighborhood of 80 to 400 rpm; engines may now be had with a speed in excess of 2000 rpm, and for units of 200 hp or less, 1200 rpm is not unusual) has probably contributed to their application to both direct and indirect drives.

Practically any sort of machine usually found applicable to the mineral industry, with the exception of underground equipment (such as pumps and fans), may be driven by diesel engines without the medium of a generator.

The connection to the engine may be direct (clutch or coupling to crankshaft) or indirect (belt and pulley, gears, chain and sprocket, V belt, with or without clutch). Equipment such as hoists and ball mills, because of their overload requirement, should be connected through a clutch. Crushers, because of their liability to plugging up (large pieces of ore or waste, hammer heads, picks, drill steel, etc.) should be belt- or V-belt-connected. Slippage of the belt will prevent damage to crusher or engine. This also would apply to rolls. Line shafting for operating machine shop, drill-sharpening equipment, or the odds and ends of small apparatus around the mine or mill may be easily rigged up.

The following list indicates some of the possible applications of the diesel engine to mining and crushing machinery. The use of a clutch is recommended with all of them.

1. Chain drive for ball mill
2. Belt drive for reciprocating pump
3. Belt drive for centrifugal pump
4. Belt drive for generator
5. Belt drive for two-stage compressor (or single-stage compressor)
6. Gear drive for hoist drum
7. Direct connection to generator (exciter sometimes belt-driven)
8. Direct connection to compressor
9. Direct connection to centrifugal pump
10. Direct connection to compressor and belted to generator
11. Dual operations, either generator or compressor by simply changing belt
12. Belt connection to ventilating fan

13. Belt connection to gyratory crusher
14. Belt connection to jaw crusher
15. Direct connection to generator and compressor
16. Belt connection to ball mill (not especially recommended)
17. Direct connection to hoist
18. Belted to line shafting
19. V-belt drive to any of above

It should be mentioned that direct connection to the crankshaft of the engine can be made only when the driven machine operates at the same revolutions per minute as the engine. Most mining and concentrating equipment operates at relatively slow speeds. In the case of electric generators, the higher their speed, the lower their cost. Centrifugal pumps, depending on their size and capacity, operate at high speeds (1000 to 2000 rpm) and therefore will usually have to be indirectly connected.

In selecting the type of drive, the loss in efficiency between the diesel crankshaft and the shaft of the driven equipment must not be overlooked. For estimating purposes, the following figures are within safe limits:

Belt-connected. It is customary to assume a loss of about 10 per cent between the engine and the driven pulley when belting is used. The speed, load, length of belt, and the use of belt tighteners must, of course, be considered. The figure given is probably the minimum.

Gear Drive. A loss of 5 per cent for each reduction or increase is safe for gearing. Thus, if two changes in speed (necessitating three gears) were desired, a loss of 10 per cent would be assumed.

Chain Drive. Same figure as for gear drives is used.

V-belt Drive. Somewhat less than for gears.

Direct Connection. The consideration of loss of power when the engine is directly connected is necessary only in the case of a generator. The loss here is not entirely a mechanical one but is mostly a generator (electrical) loss. For most generators (with the exception of very small ones) an efficiency of 90 per cent is a safe figure. (This is slightly low, but is on the safe side.) For other directly connected equipment (fans, pumps, compressors, crushers, etc.), the manufacturer's efficiency for the machine must be used. It is difficult to generalize on these figures because they vary with the size and capacity. With the exception of pumps and fans, a loss of 15 per cent will usually be sufficient for estimating purposes. Centrifugal pumps range from about 60 to over 90 per cent in efficiency, depending on their size and duty. The larger the pump, the higher its efficiency.

Effect of Altitude. The manufacturer's rating (in brake horsepower) is given for sea-level conditions. Most mining properties are located at elevations considerably above sea level. Because, with increasing altitudes above sea level, the density (weight per cubic foot) of air decreases, the oxygen available for combustion decreases. This means that less oil is burned and consequently there is a loss in power. This decrease in the sea-level rating is about 3 per cent for each 1000 ft above sea level. A similar loss occurs for compressor ratings and steam plants. In addition to the effect on combustion, heat radiation (cooling of engine) is lowered, so that the regulation of cooling water and possible overheating of the engine must be closely watched. Figure 172 shows a curve giving the loss in power with an increase in altitude.[1]

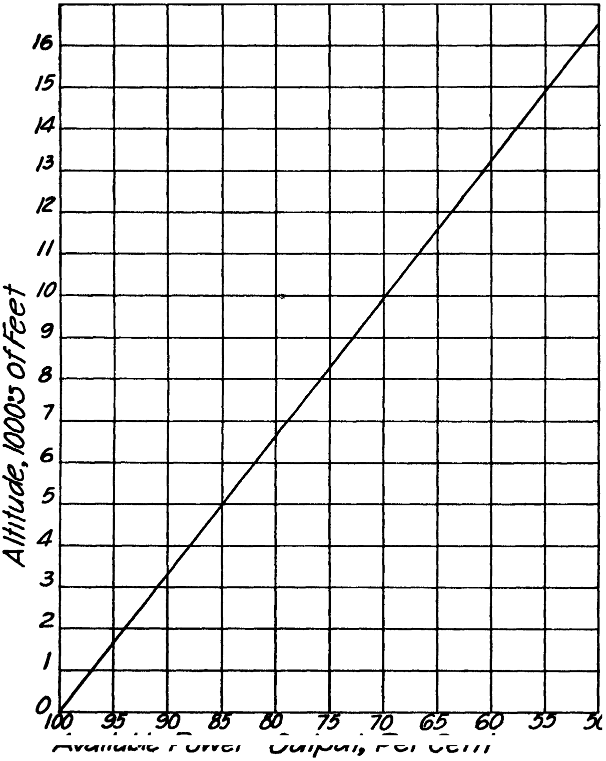

FIG. 172. Effect of altitude on power output for diesel engines. (*Morrison, "American Diesel Engines."*)

A loss of power because of altitude may be overcome by providing a scavenging pump that compresses air to slightly above sea-level pressure (14.7 lb per sq in.) for the two-cycle engines. For the four-cycle type a compressor furnishes air for the air-intake valve. In the past, a number of engines have been equipped to supply compressed air to the cylinders. It is now generally thought that the better way is to increase the engine size rather than to equip the plant with auxiliary compressors.

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[1] MORRISON, L. H., "American Diesel Engines," 1st ed., p. 537.

SELECTION OF POWER PLANT FOR A SMALL MINE AND MILL

To illustrate the selection of diesel power for small-scale mining, the following example will be investigated.

Known Requirements:

1. Elevation of mine above sea level = 4000 ft
2. Ore to be hoisted per day = 50 tons
3. Waste to be hoisted per day = 25 tons
4. Shifts that mine operates per day = 1 shift of 8 hr
5. Shifts mill operates per day = 3
6. Days worked per year = 300
7. Tonnage treated by mill per 24 hr = 50
8. Life of mine (for purpose of redeeming power-plant investment) = 10
9. Type of driving power for equipment = Electric motors
10. Rated (motor) horsepower of equipment
    - *a.* Mine:
        - (1) Hoist = 15 hp
        - (2) Compressor, two-stage = 50 hp
        - (3) Lights, machine shop, etc. = 20 hp
    - *b.* Mill:
        - (1) Crusher = 15 hp
        - (2) Rolls = 25 hp
        - (3) Ball mill = 50 hp
        - (4) Flotation = 12 hp
        - (5) Classifier = 2 hp
        - (6) Filter = 5 hp
        - (7) Lights, etc. = 25 hp

Desired:

1. Horsepower of diesel engine or engines
2. Probable cost
    - *a.* Per brake horsepower-hour
    - *b.* Per kilowatt-hour

To arrive at the plant horsepower required, the known data are listed in the accompanying table. This table shows at just what time of day it is desirable for each piece of equipment to be operated.

It will be seen from the preceding table that, if the coarse crushing is done after the mine shuts down (after 4 P.M.), the maximum power

TIME AT WHICH EQUIPMENT IS RUNNING

Period

Equipment	8 A.M	9	10	11	12	1 P.M.	2	3	4	5	6	7	8	9	10	11	12	1 A.M.	2	3	4	5	6	7
	Horsepower required																							
Hoist	15	15	15	15	15	15	15	15	15															
Compressor	50	50	50	50	50	50	50	50	50															
Lights, etc	20	20	20	20	20	20	20	20	20															
Total mine	85	85	85	85	85	85	85	85	85															
Crushers	...	...	...	...	...	...	...	...	...	15	15	15												
Rolls	...	...	...	...	...	...	...	...	..	25	25	25												
Ball mill	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
Flotation	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
Classifier	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Filter	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
Lights, etc	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
Total mill	94	94	94	94	94	94	94	94	94	134	134	134	94	94	94	94	94	94	94	94	94	94	94	94
Grand total	179	179	179	179	179	179	179	179	179	134	134	134	94	94	94	94	94	94	94	94	94	94	94	94

required will not exceed 180 hp. If, for any reason, it is necessary that the mine work two shifts, the crushing can be done after 1 A.M., so that the same power balance can be maintained. From this tabulation it is indicated that two engines would be desirable, one supplying about 85 motor hp, the other about 100 motor hp (the 85-hp engine will operate when crushing is being done, thus supplying the 134 motor-hp requirement). The total required is 179 motor hp.

This may be split between two engines of 80 and 100 hp, respectively. The question may be raised that the 80-hp engine would not handle the mine load alone if the larger engine were not running (in other words, if the mill were shut down). In answer to this it may be said that if the mill were down, it would be very unlikely that the compressor would maintain full-load rating as constantly, because less drilling would be done; also, a hoist does not draw continuously its full rating. Other mine equipment would operate at correspondingly reduced capacity. However, in case the mine did operate at maximum output, the larger engine could be run instead of the smaller one. It would operate at about three-quarters load. Its efficiency and oil consumption would differ but slightly from the smaller engine.

The horsepower of the engines selected is the motor horsepower. That is, it is the manufacturer's rating on the name plates. This rating must be converted to brake horsepower. As the generated power is to be electrical energy, the generators will be directly connected to the engine crankshafts. The efficiency of the generator is 90 per cent.

$$\begin{aligned} \text{Bhp} &= 80 \div 90\% = 90 \text{ hp} \\ &= 100 \div 90\% = 111 \text{ hp} \end{aligned}$$

From Fig. 172 the altitude equivalent of the sea-level rating is 87 per cent. The brake horsepower must be increased as follows:

$$\begin{aligned} 90 \div 87\% &= 103 \text{ hp} \\ 111 \div 87\% &= 127 \text{ hp} \end{aligned}$$

Choosing the nearest engine rating (engine to be based on continuous rating, not normal rating; if manufacturer's rating is for maximum output, select engine on the basis of 50 per cent of engine, *i.e.*, maximum, rating), we find that a 100-hp and a 140-hp engine will supply the requirement. This is the full-load rating with a maximum overload factor of safety of 25 per cent. This provides for a short period of time (0.5 to 1 hr)

$$100 \times 125\% + 140 \times 125\% = 300 \text{ hp maximum output}$$

This will take care of such a contingency as starting the ball mill if crusher and rolls are running when the mine is also operating. Or the 140-hp engine will handle the starting of the ball mill if the concentrating equipment is on.

It will be noted that two shifts could be worked by the mine without increasing the power-plant capacity. No allowance is made in the calculation of costs for mine operations being reduced during the lunch hour. This is, to some extent, offset by the fact that the hoist and compressor are held available ½ hr after the shift has ended for blowing out heading, etc. The mill would not shut down during the lunch hour. The rated capacity of the jaw crusher and rolls selected should readily provide for reducing 50 tons in 3 hr. If they do not, no overloading will occur by keeping them on longer. However, storage bins of such capacity must be provided that it will never be necessary to crush when the mine is operating, if the rest of the mill is running.

Probable Operating Cost of Engines Selected. In making the following estimation, two items entering into the cost cannot be determined with any degree of accuracy. They are the transportation costs of the fuel oil and equipment from the railhead to the mine, and the cost of constructing the powerhouse if a separate building is provided for the engines. With the engine horsepower selected, a separate powerhouse would not be absolutely necessary. Extra space in the mill building could be provided. A building of the size required would probably cost in the neighborhood of $1000 to $2000. Its cost will be ignored. Because even a rough estimation of the cost of transporting the oil cannot be made, the delivery price at the railhead will be used. The installed cost of the engines and accessories will be taken at $100 per brake horsepower. The actual factory price of the engines will probably not exceed $65 per brake horsepower with accessories. The extra $35 should take care of transportation, installation, generating equipment, etc.

PROBABLE OPERATING COST

Total horsepower installed = 100 + 140 = 240

Operating time = 300 days per year

Fuel oil at 8 cents per gallon	$7,216
Lubricating oil at 50 cents per gallon	225
Labor	4,500
Maintenance—2 per cent at 65 per cent of installed cost	312
Fixed charges—16.95 per cent	4,068
Total operating cost	$16,321

Brake horsepower-hour output at 64.7 per cent annual capacity	1,353,000
Kilowatt-hour output at 64.7 per cent annual capacity	1,009,338
Total cost per brake horsepower-hour	$0.01206
Total cost per kilowatt-hour	0.01617
Cost without fixed charges per brake horsepower-hour	0.00906
Cost without fixed charges per kilowatt-hour	0.01213
Fuel and lubricating oil charges per brake horse-power-hour	0.0055
Fuel and lubricating oil charges per kilowatt-hour	0.0073
Brake horsepower-hours per gallon of fuel oil	15.0
Kilowatt-hours per gallon of fuel oil	11.2

The various items in the above table were arrived at in the following manner:

Fuel Oil. The total of brake horsepower-hours generated is found as follows:

$$\text{Bhp-hr} = 100 \times 11.5 \times 300 + 140 \times 24 \times 300$$
$$= 345{,}000 + 1{,}008{,}000 = 1{,}353{,}000$$

This is arrived at on the basis of the 100-hp engine running 11.5 hr per day, and the 140-hp engine running 24 hr per day, both 300 days per year.

The fuel-oil consumption, using a standard diesel 27 plus fuel, is taken at 0.50 lb per bhp-hr. Actually it will probably be slightly less than this. The weight of 1 gal of fuel oil is taken at 7.5 lb.

Total gallons consumed per year is

$$1{,}353{,}000 \times 0.5 \times \frac{1}{7.5} = 90{,}200 \text{ gal}$$

$$\text{Cost} = 8 \text{ cents} \times 90{,}200 = \$7216$$

Lubrication. With a good grade of lubricating oil and an oil filter or purifier, 3000 bhp-hr per gal is a safe assumption. (Even though one-half this figure were used, the cost per brake horsepower-hour would be but very little greater.)

$$1{,}353{,}000 \div 3000 = 451 \text{ gal}$$
$$451 \times 50 \text{ cents} = \$225.50$$

Labor. Up to 700 hp, one man per shift can easily operate a diesel plant. No chief engineer (as an extra man) is required for the 240-hp plant under consideration. One man per shift is more than ample. If the power plant were located close to or in the mill building, it would probably be possible to do without an attendant on the midnight shift and let the mill operator supervise the power-plant operations. One man for each shift is assumed in the present example.

$$\text{Labor cost} = 3 \text{ shifts at } \$5 \times 300 = \$4500$$

Maintenance. As stated previously, companies supplying power other than diesel do not agree with diesel manufacturers on the use of the yearly 2 per cent maintenance figure. An investigation of some hundreds of operating plants has shown that 2 per cent of the factory cost is ample.[1] If this were increased to twice or even three times this amount, the total operating cost per brake horsepower-hour would be but slightly increased. Competitive salesmen contend that the maintenance should be based on the plant production (in our case, 1,353,000 bhp-hr per year). Experience has shown that the 2 per cent figure based on factory cost of equipment is sufficient.

The installed cost of the two engines was assumed at $100 per brake horsepower. It was also stated that $35 of this was allowed to cover cost of accessories other than those included with the engines, transportation, and installation. The factory price is then $65 per brake horsepower.

The total cost of engines with oil tank, switchboard, generators, lubricating-oil filter, batteries, etc., installed is

$$240 \times \$100 = \$24{,}000$$

Only 65 per cent of this is the factory price.

Maintenance cost = 24,000 × 65% × 2% = $312.

Fixed Charges. The fixed charges will consist of interest on the power-plant investment, insurance, taxes, and redemption of the plant investment.

A customary figure for interest on the plant investment is 6 per cent per annum.

Insurances and taxes, for want of a more exact figure, are taken at 1.5 per cent each.

The redemption or amortization of the capital invested in the power plant depends on the life assigned to the equipment. In our case, it depends on the life of the mine. It has been very definitely shown that diesel engines are good for at least 30 years as there are engines in service that have operated over that period. Their life is undoubtedly much longer. In the case of a mining property, the machinery investment should be written off during the life of the property, if the mine has a shorter life than the equipment. Some allowance probably should be made for the resale value of the diesels. However, as this value is impossible to determine, it is left out of the calculations. An idea may be had of their resale value when we consider that engines used in the Texas oil fields have been resold in the northwest at about 40 per cent of their original value, and this after they had been in use 5 years or more.

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[1] ADKINS and BACON, "Applications, Economies, and Operation of Modern Diesel Engines," *Nat. Engr.*, May and June, 1926.

In amortizing capital, various interest rates may be obtained. Tables may be had giving such charges over a long period of years at rates as high as 8 per cent compound interest. The following table gives a few such values for interest rates ordinarily obtained, and the life in years commonly assigned to mineral properties.[1] Very few mines can show at the start of operations an assured life of more than two to five years. A great many properties start operating with no more than one to two years of ore blocked out (many even less than this).

REDEMPTION TABLE

Amount to Be Set Aside Each Year for Redemption of Capital Outlay, Per Cent

Years	Interest rate			
	3 per cent	4 per cent	5 per cent	6 per cent
1	100.00	100.00	100.00	100.00
2	49.26	49.02	48.78	48.54
3	32.35	32.04	31.72	31.41
4	23.90	23.55	23.20	22.86
5	18.84	18.46	18.10	17.74
10	8.72	8.33	7.95	7.59
15	5.38	4.99	4.63	4.30
20	3.72	3.36	3.02	2.72

It will be seen from the above table that the longer the life of the mine or equipment, the less becomes the annual redemption charges and thus the fixed charges. If a definite and assured resale value can be assigned to the equipment, this amount can be deducted from the total costs. The annual fixed charges will in consequence be reduced because the capital investment is less. The percentage to set aside will not be changed.

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[1] For complete tables, see Parks, "Examination and Valuation of Mineral Properties," 3d ed., Addison-Wesley Press, Inc., Cambridge, Mass., (1949).

In the problem under consideration the redemption must take place in 10 years as that is the life of the mine. The interest return on investing the yearly sum set aside is taken at 5 per cent compounded annually. Entering the table at 10 years and 5 per cent interest, we find that the amount to be set aside and invested at 5 per cent is 7.95 per cent of the total power-plant investment.

The total interest for the fixed charges is

7.95 per cent	for redemption of capital
6.00 per cent	for capital invested
1.50 per cent	for taxes
1.50 per cent	for insurance
16.95 per cent	for fixed charges

Fixed charges = $24,000 × 16.95% = $4068 yearly

Annual Plant Output. The annual plant capacity is the percentage of actual operating time compared to the maximum possible output.

In one year there are 24 hr × 365 days = 8760 hr

If a power plant of 240 hp operated continuously for one year, there would be produced

240 × 8760 = 2,102,400 bhp-hr

The plant actually operated 1,353,000 bhp-hr.

The annual output is, therefore,

$$\frac{(100 \times 11.5 \times 300 + 140 \times 24 \times 300) \times 100}{(100 + 140) \times 8760} = 64.7\%$$

The annual output in kilowatt-hours is

1,353,000 × 0.746 = 1,009,338 kw-hr

From the cost table, it is found that power may be generated at a probable cost of 1.21 cents per brake horsepower-hour. As stated previously, this does not include fuel-oil transportation charges from the railhead to the mine, or the cost of constructing the building. If we assume the cost of the oil delivered to the mine to be 12 cents per gallon and the cost of constructing the building $1500, the cost per brake horsepower-hour becomes 1.49 cents.

STEAM PLANT

It is not desirable to go into the theory of thermodynamics as a basis for the short discussion to follow. For purposes of designing steam-operated machinery, such a dissertation would be very necessary. The object here is to present such information and methods as will aid the selection of equipment already properly designed from a mechanical and thermodynamical standpoint. For our purpose we shall consider the following subjects:

1. Properties of steam
2. Boilers
3. Steam engines.

PROPERTIES OF STEAM

A short outline of some of the fundamentals involved in the production and use of steam is necessary, so that the use of steam tables for solving boiler problems will be understood. This information will be given more or less in the form of definitions.

Dry Saturated Steam. Steam which exists at the temperature of the boiling point and which contains no water in suspension is said to be dry saturated steam.

Wet Steam. Saturated steam that contains entrained moisture is called "wet" steam.

Superheated Steam. Superheated steam is steam at a higher temperature than the boiling point corresponding to the pressure.

Heat of the Liquid. The heat necessary to raise 1 lb of water from 32°F to the temperature of the boiling point is called the "heat" of the liquid. It is expressed as follows:

$$h = c(t - 32)$$

where h = the heat of the liquid
c = the specific heat = one for all practical purposes
t = temperature of the boiling point

Accurate values of h may be obtained from steam tables.

Latent Heat of Steam. The heat necessary to convert 1 lb of water at the temperature of the boiling point into steam at the same temperature is called "latent" heat. From experimental work it is found to become less as the pressure increases. This diminishing effect is about 0.695 heat units for each degree the temperature of the boiling point is increased.

$$L = 1072.6 - 0.695(t - 32)$$

where L = latent heat of steam
t = temperature of boiling point
1072.6 = latent heat of evaporation at 32°F

Total Heat of Steam. The heat required to change 1 lb of water at 32°F into 1 lb of steam at the temperature of the boiling point is called the "total heat" of steam.

$$H = h + L$$

or approximately by

$$H = 1072.6 + 0.305(t - 32)$$

where H = total heat of steam
t = temperature of boiling point

For very accurate work, H should be obtained from steam tables.

The steam tables that have been mentioned will not be reproduced here in their entirety, as they are too lengthy. Values for pressures commonly used are given in Table 70[1]. Complete tables will be found in engineers' handbooks.

TABLE 70. PROPERTIES OF SATURATED STEAM

Abs. press., lb. per sq. in.	Temp., °F.	Heat of liquid	Latent heat of evap.	Total heat of steam	Sp. vol., cu. ft. per lb.	Density, lb. per cu. ft.
p	t	h	L	H	v	$1/v$
0.0886	32	0	1072.6	1072.6	3301.0	0 000303
14.7	212	180.1	970.4	1150.4	26.79	0 03733
95	324.1	294.5	890.7	1185.2	4.65	0 2152
114.7	337.9	308.8	879.8	1188.6	3.888	0.2572
140	353.1	324.5	867.6	1192.1	3.219	0.3107
164	365.6	337.6	857.2	1194.8	2.768	0.3613
214	387.6	360.6	838.4	1199.0	2.147	0.4660

BOILERS

Boilers may be classified in one of three ways, or by combining these.

1. By the path taken by the fire

   a. Fire-tube boilers are those where the hot gases pass through the tubes, the water being on the outside of the tubes. For installations where it is difficult to get boiler water low in solids and sulfates, this is probably the best type. It is easier cleaned and fouling around the tubes does not lower the efficiency so much as would the deposition of sediment in the tubes.

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[1] ALLEN and BURSLEY, "Heat Engines," 2d ed., p. 42, McGraw-Hill Book Company, Inc., New York.

b. Water-tube boilers are those where the water passes through the tubes, the gases circulating around the tubes.

2. By the position of the fire

   a. Externally fired. The fire is completely outside the boiler; usually is confined in a brick chamber. Used mostly for stationary plants.
   b. Internally fired. The fire is inclosed in the steel shell of the boiler. Used on locomotives, marine engines, etc.

3. Vertical or horizontal boilers. This refers to the position of the tubes.

There is essentially no difference in the efficiency of the various types. The internally fired boiler has a greater first cost per horsepower than the stationary external type.

Horsepower Rating of Boilers. The term "horsepower" as applied to boilers has no definite meaning, as it does when applied to other types of machines. The steam produced by a boiler depends on the heating surface in the boiler. Experience has shown that the best results are obtained (for the ordinary boiler) when not more than 3 lb of water per hour are evaporated for each square foot of heating surface. This gives the most economical results at nominal rating. Most mining installations can operate at about 150 per cent normal rating.

Boiler specifications usually give the number of square feet of heating surface and the number of pounds of water to be evaporated under given conditions. The American Society of Mechanical Engineers have defined the term "boiler horsepower" for rating boilers as follows:[1]

> "A boiler horsepower is 34.5 lb of water evaporated per hour from and at 212 deg into dry and saturated steam."[2]

It must be remembered that there is no connection whatsoever between boiler horsepower and engine horsepower.

To determine the size of boiler required for a given set of conditions we must work back from the total engine horsepower necessary and find the total heating surface required to produce the desired amount of steam.

Total Heating Surface. The heating surface of a boiler is that surface which has water on one side and hot gases on the other. Superheating surface has steam on one side and hot gases on the other.

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[1] ALLEN and BURSLEY, *op. cit.*, p. 98.

[2] An additional and much used definition for boiler horsepower is 33,479 Btu per hour, which is equal to 34.5 × 970.4.

Grate Surface. The proportion of grate surface to heating surface depends on the draft and nature of the fuel to be used. In small boilers of light draft it is customary to allow 1 sq ft of grate for each 20 to 30 sq ft of heating surface. In large boilers the difference is much greater, varying from 1 to 50 to 1 to 70. In locomotives it may go up to 1 to 100.

Table 71[1] gives some information along this line.

TABLE 71. APPROXIMATE PROPORTION OF HEATING AND GRATE SURFACE PER BOILER HORSEPOWER

Type of boiler	Sq ft of heating surface per hp.	Coal per sq. ft. of heating surface per hour	Relative rapidity of steaming	Heating surface per sq. ft. of grate	Lb. of coal per sq. ft. of grate per hour	Lb. of water per pound of coal
Water tube	10–12	0.3	1.00	35–50	12–20	9–12
Cylindrical tubular	14–16	0.25	0.60	25–35	10–15	8–11
Vertical tube	15–20	0.25	0.60	25–30	10–15	8–10
Locomotive	12–16	0.275	0.55	50–100	20–40	8–11
Flue	8–12	0.40	0.25	20–25	10–20	8–10
Plain cylindrical	6–10	0.50	0.20	15–20	15–25	7–9

Rate of Combustion. The rate at which fuel is consumed depends on the kind of fuel and the draft. With anthracite coal and a draft not exceeding 0.5 in. of water pressure the consumption varies from 12 to 15 lb per sq ft of grate surface per hour. With bituminous coal the quantity is 15 to 20 lb.

The air opening in the grate usually does not exceed 50 per cent of the total grate area. With the better grades of coal less opening is required.

Heating Surface per Boiler Horsepower. *Water-tube Boiler.* It is customary to allow 10 sq ft of heating surface for each boiler horsepower.

Fire-tube Boiler. For fire-tube boilers it is safe to allow 12 sq ft of heating surface per boiler horsepower.

EXAMPLE[2]

A 48- by 12-in. return flue fire-tube boiler has thirty 4-in. tubes. It evaporates 1400 lb of water per hour from a feed temperature of 120°F into steam at 100 lb gage. What per cent of its rating is the boiler developing?

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[1] PEELE, R. (ed.), "Mining Engineers' Handbook," 2d ed., p. 2220, John Wiley & Sons, Inc., New York.

[2] ALLEN and BURSLEY, *op. cit.*, p. 102.

Solution: First the total heating surface must be found. This can be done from the following rule:[1]

"The heating surface is equal to two-thirds of the cylindrical surface of the shell, plus the internal area of all the tubes, plus two-thirds the area of both tube sheets, minus twice the combined external cross-sectional area of all the tubes, all expressed in square feet."

$$\text{Heating surface of cylindrical part of shell} = \tfrac{2}{3} \times \pi \times 4 \times 12 = 100.6 \text{ sq ft}$$

$$\text{Heating surface of tubes} = 30 \times \pi \times \frac{3.75}{12} \times 12 = 352.5 \text{ sq ft}$$

Heating surface of tube sheets

$$= 2[\tfrac{2}{3}(\pi \times 2 \times 2) - 30\pi \times \tfrac{1}{6} \times \tfrac{1}{6}] = \underline{11.5 \text{ sq ft}}$$

$$\text{Total heating surface} = 464.5 \text{ sq ft}$$

For this type of boiler the rated horsepower is

$$\frac{464.5}{12} = 38.7 \text{ boiler hp}$$

We have assumed 12 sq ft of heating surface as producing 1 boiler hp. The actual horsepower that would be developed is now found.

Heat required to evaporate 1400 lb of water per hour is

$$1400[1188.6 - (120 - 32)] = 1{,}514{,}000 \text{ Btu}$$

where 1188.6 Btu is the total heat of steam from a temperature of 32°F. Because the feed water has a temperature of 120°F, as much heat is not required as if the water temperature were 32°F.

The equivalent evaporation must be found next. The following definition explains equivalent evaporation.[2]

"Economy may be expressed as the number of pounds of equivalent evaporation from and at 212°F per pound of combustible burned. By equivalent evaporation is meant the number of pounds of water that would be evaporated from a feed temperature of 212°F into dry and saturated steam at 212°F by the expenditure of the same amount of heat as is actually used in evaporating the water under given conditions."[3]

The latent heat of evaporation at 212°F is 970.4 Btu. In our problem the equivalent evaporation if 1,541,000 Btu are expended is

$$\frac{1{,}541{,}000}{970.4} = 1588 \text{ lb of water per hour}$$

From the definition of boiler horsepower

$$\frac{1588}{34.5} = 46 \text{ boiler hp actually developed}$$

$$\frac{46}{38.7} \times 100 = 119\%$$

The boiler is developing a 19 per cent overload.

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[1] *Ibid.*, p. 99.

[2] ALLEN and BURSLEY, *op. cit.*, p. 100.

[3] This assumes 100 per cent steam with no entrained moisture.

EXAMPLE[1]

A boiler evaporated 8.23 lb of water per pound of coal fired. Feed temperature is 120°F; steam pressure, 100 lb gage. Coal fired contains 5 per cent ash and has a heating value of 12,800 Btu per pound. Twelve per cent of coal fired is taken from the ash pit in the form of ash and refuse. Coal contains 2 per cent of moisture.[2]

1. Find the efficiency of the boiler, furnace, and grates combined.
2. Find the efficiency of the boiler and furnace.

Solution: *Part a.* Heat necessary to evaporate 1 lb of water is

$$1188.6 - (120 - 32) = 1100.6 \text{ Btu}$$

$$\text{Water evaporated per pound of dry coal fired} = \frac{8.23}{1.00 - 0.02} = 8.4 \text{ lb}$$

$$\text{Heat utilized per pound of dry coal fired} = 8.4 \times 1100.6 = 9245 \text{ Btu}$$

Efficiency of boiler, furnace, and grate combined is

$$\frac{\text{Heat utilized per pound of dry coal fired}}{\text{Heating value of 1 lb of dry coal}} = \frac{9245}{12{,}800} = 72.23\%$$

Part b

$$\text{Heating value of 1 lb of combustible} = \frac{12{,}800}{1.00 - 0.06} = 13{,}474 \text{ Btu}$$

Water evaporated per pound of combustible burned is

$$\frac{8.23}{1.00 - (0.02 + 0.12)} = 9.57 \text{ lb}$$

$$\text{Heat utilized per lb of combustible burned} = 9.57 \times 1100.6 = 10{,}533 \text{ Btu}$$

Efficiency of boiler and furnace is

$$\frac{\text{Heat utilized per lb of combustible burned}}{\text{Heating value of 1 lb of combustible}} = \frac{10{,}533}{13{,}474} = 78.17\%$$

Efficiency of Boilers.[3] The combined boiler and furnace efficiency is 60 to 80 per cent. The average is about 70 per cent. This average is approached in large, well-supervised plants, but in small plants the efficiency is about 50 per cent.

STEAM ENGINES

Steam engines may be simple or compound. The simple engine has but one expansion cylinder, whereas the compound has two or more expansion cylinders. The compound type usually refers to a two-cylinder engine. In a broad way it covers the entire field.

Steam engines may be classified in a number of ways. They are practically always double-acting, being single-acting only in special types. A classification suitable for our purpose would be as follows:

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[1] ALLEN and BURSLEY, *op. cit.*, p. 103.

[2] Quality of steam assumed at 100 per cent.

[3] PEELE, (ed.), *op. cit.*, p. 2220.

1. Noncondensing
2. Condensing
3. Throttling
4. Automatic
5. Simple
6. Compound
7. Triple expansion
8. Quadruple expansion

Any one engine may, of course, embrace a number of the above characteristics.

A noncondensing engine is one where the steam exhausts into the atmosphere and is wasted. In the condensing type the exhaust is condensed to water in a cooling chamber. In throttling engines the speed is controlled by means of a valve placed in the steam pipe. This valve regulates the pressure of the steam entering the engine. The pressure in the automatic engine is kept constant by means of a governor that regulates the point of cutoff, or the amount of steam entering the cylinder.

Theoretical Horsepower of the Steam Engine. The theoretical horsepower is based on the following assumptions:

1. There is no clearance for the piston between the end of the cylinder, nor is there clearance for the valve chambers.
2. Full pressure of steam is maintained during admission.
3. Cutoff and release occur instantly.
4. Engine acts without compression.

Figure 173 shows the theoretical indicator card. The curve *bc* is assumed as being a rectangular hyperbola, in which

$$pv = \text{a constant}$$

Let Pressure at point of cutoff $b = p_1$
Volume at point of cutoff $= v_1$
Pressure at point $d = p_2$
Volume at point $d = v_2$

The area of the work, then, is represented by

$$abcde = oabg + gbcf - oedf$$

but the area $oabg = p_1v_1$

$$\text{Area } gbcf = \int_{v_1}^{v_2} p\,dv$$

$$\text{Area } oedf = p_2v_2$$

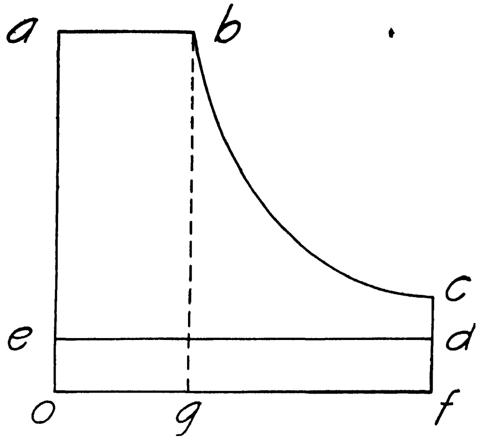

FIG. 173. Theoretical indicator card for steam engine.

Substituting these values, the work area becomes

$$abcde = p_1v_1 + \int_{v_1}^{v_2} p\,dv - p_2v_2$$

As v_1 and v_2 are the volumes before and after expansion, the ratio of expansion is

$$r = \frac{v_2}{v_1}$$

And as the curve *bc* is a rectangular hyperbola,

$$pv = p_1v_1$$

or

$$p = \frac{p_1v_1}{v}$$

and the work area becomes

$$abcde = p_1v_1 + p_1v_1\int_{v_1}^{v_2}\frac{dv}{v} - p_2v_2$$
$$= p_1v_1\left(1 + \int_{v_1}^{v_2}\frac{dv}{v}\right) - p_2v_2$$

Integrating and substituting $r = v_2/v_1$

$$abcde = p_1v_1(1 + \ln r) - p_2v_2{}^*$$

The average pressure or mean effective pressure (mep) is found by dividing *abcde* by the length of the card, which is v_2.

---

*ln represents base *e* logarithms.

$$\text{Mep} = \frac{p_1v_1(1 + \ln r) - p_2v_2}{v_2} = p_1\frac{1 + \ln r}{r} - p_2$$

In practice the four assumptions made cannot be realized. The actual mep is less than the theoretical. We must multiply the theoretical mep by a diagram factor, or efficiency factor called *e*.

The actual mep is, therefore,

$$\text{Mep} = e\left[\frac{p_1(1 + \ln r)}{r} - p_2\right]$$

The factor *e* has values ranging from 70 to 90 per cent.

Indicated Horsepower (*ihp*). The indicated horsepower of a steam engine is obtained from the following formula. It is the horsepower up to the crankshaft.

$$\text{Ihp} = \frac{2plan}{33{,}000} \quad \text{for a double-acting engine}$$

where p = mean effective pressure, lb per sq in.
l = length of stroke, ft
a = area of cylinder cross section, sq in.
n = rpm

EXAMPLE

An 8- by 12-in. double-acting engine runs at 250 rpm; cutoff, ¼ stroke; steam pressure is 100 lb gage; back pressure is 2 lb absolute. Card factor is 85 per cent. What is the rated horsepower? What is indicated horsepower if the back pressure is atmospheric (sea level)?

Solution:

$$r = \frac{v_2}{v_1} = \frac{1}{\frac{1}{4}} = 4$$

$$\text{Mep} = 0.85\left[\frac{100 + 14.7}{4}(1 + \ln 4) - 2\right]$$
$$= 0.85[28.7(1 + 1.39) - 2] = 56.6$$

Cross-sectional area of the cylinder:

$$a = \pi\frac{d^2}{4} = \pi \times \frac{8^2}{4} = 50.3 \text{ sq in.}$$

$$\text{Ihp} = \frac{2plan}{33{,}000} = \frac{2 \times 56.6 \times \frac{12}{12} \times 50.3 \times 250}{33{,}000} = 43.3$$

When the back pressure is atmospheric (14.7 lb),

$$\text{Mep} = 0.85[28.7(1 + 1.39) - 14.7] = 45.8 \text{ lb}$$

$$\text{Ihp} = \frac{2 \times 45.8 \times \frac{12}{12} \times 50.3 \times 250}{33{,}000} = 35.1$$

Figure 174 shows an actual indicator card for a simple steam engine.[1]

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[1] Allen and Bursley, *op. cit.*, p. 141.

Explanation of Fig. 174. The Y axis represents pressure and the X axis volume. The steam enters along the line CDE, the pressure at D being slightly lower than boiler pressure. At E, known as the "point of cutoff," the valve closes. The steam expands from E to F along the expansion line EF. At F, called the "point of release," the valve opens, and from F to H exhausting occurs. At H the exhaust valve closes and the remaining steam is compressed along the line HC to point C. At C steam again enters the cylinder. If the engine is double-acting, a similar action takes place on the other end. If the piston rod does not extend clear through the cylinder the head-end card will be slightly different from the crank end (shown in Fig. 174) because of the volume of the piston rod.

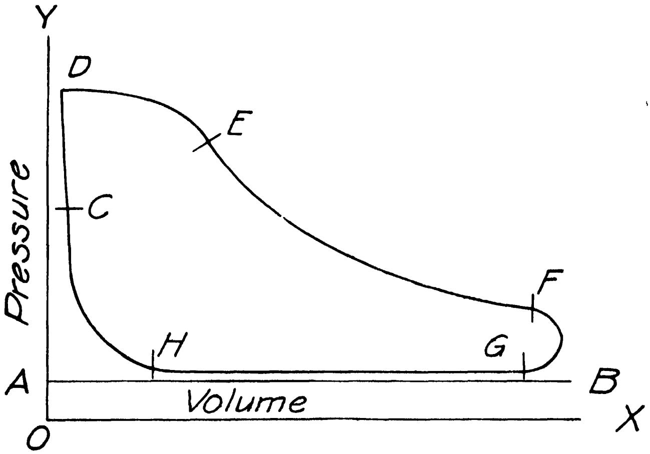

FIG. 174. Actual indicator card for steam engine. (*Allen and Bursley, "Heat Engines."*)

Horsepower of a Compound Engine. To determine the horsepower of a compound engine we must know the following:

1. Absolute initial pressure
2. Total number of expansions of the steam
3. Number of strokes per minute
4. Length of stroke
5. Diameter of high-pressure cylinder
6. Diameter of low-pressure cylinder

The horsepower is determined as though there were but one cylinder, and that one the size of the low-pressure cylinder, and the total expansion of the steam took place in that cylinder. As with compressors, the cylinders are proportioned so that there is an equivalent amount of work done in each.

The indicated-horsepower formula is

$$\text{Ihp} = \frac{2plan}{33{,}000}$$

$$p = e\left[\frac{p_1}{r}(1 + \ln r) - p_2\right]$$

$$r = \frac{\pi \dfrac{D^2}{4}}{x\pi \dfrac{d^2}{4}} = \frac{D^2}{xd^2}$$

where D = diameter of low-pressure cylinder
d = diameter of high-pressure cylinder
a = area of low-pressure cylinder
l = stroke, ft
p = mep
n = rpm
x = ratio of expansion for whole engine
= cutoff in high-pressure cylinder
e = card factor
p_1 = initial pressure of steam entering engine
p_2 = pressure of exhaust

EXAMPLE

An 8- by 16- by 12-in. engine runs at 300 rpm. The initial steam pressure is 137 lb gage; cutoff in high-pressure cylinder is ¼ stroke. A barometer reads 26.5 in. of mercury. In the condenser chamber it reads 22.4 in. Card factor is 70 per cent. What ihp does the engine develop?

Solution:

Atmospheric pressure = 26.5 × 0.491 = 13.0 lb
Exhaust pressure = (26.5 − 22.4) × 0.491 = 2.0 lb absolute

$$\text{Ratio of expansion} = \frac{D^2}{xd^2} = \frac{16 \times 16}{\frac{1}{4} \times 8 \times 8} = 16$$

$$\begin{aligned} p &= 0.70\left[\frac{137 + 13}{16}(1 + \ln 16) - 2\right] \\ &= 0.70[9.37(1 + 2.77) - 2] = 23.4 \text{ lb} \end{aligned}$$

$$\text{Area of low-pressure cylinder} = A = \pi \frac{16^2}{4} = 201 \text{ sq in.}$$

$$\text{Rated ihp} = \frac{2 \times 23.4 \times {}^{12}\!/_{12} \times 201 \times 300}{33{,}000} = 84.5$$

STEAM CONSUMPTION

Steam consumption is usually determined in one of three ways.

1. Catching and weighing the steam condensed from the exhaust over a given period of time. This is very difficult to do.

2. Accurately measuring the amount of feed water going to the boiler. The test should last at least 10 hr and the engine should run under uniform conditions.

3. Clayton's relation between n and x_c from logarithmic diagram is used.

Table 72 gives a general idea as to the steam consumption for various types of steam engines.[1] For estimating purposes, the values contained therein are probably accurate enough.

TABLE 72. AVERAGE STEAM CONSUMPTION OF VARIOUS TYPES OF ENGINES

Type of engine	Steam, lb. per i.hp.-hr.	Gage pressure, lb.
High-speed, simple, single-valve, noncondensing	30–35	80–115
High-speed, simple, single-valve, condensing	22–27	80–115
High-speed, simple, multivalve, noncondensing	25–30	80–115
Low-speed, simple, multivalve, noncondensing	22–23	125
Low-speed, simple, multivalve, condensing	16–20	80–100
High-speed, compound, noncondensing	24–26	110–150
High-speed, compound, condensing	19–21	110–150
Low-speed, compound, noncondensing	20–22	110–150
Low-speed, compound, condensing	14–18	125–150

By means of Table 72 the approximate amount of water necessary to operate a given sized steam engine per day may be obtained. From this the water supply for the plant is calculated. This probably represents about 70 to 80 per cent of the water actually needed. The consumption in Table 72 is based on indicated horsepower.

BRAKE HORSEPOWER

Up to the present we have dealt with the indicated horsepower of the steam engine. The horsepower actually delivered at the flywheel is the brake horsepower (bhp), and is always less than the indicated horsepower. The amount of loss depends on the construction of the engine. It usually amounts to about 15 per cent; but in well-built engines it may be no greater than 10 per cent. The mechanical efficiency is the brake horsepower divided by the indicated horsepower, the result being expressed in percentage.

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[1] PEELE (ed.), *op. cit.*, p. 2211.

To determine the power actually delivered to the flywheel, use is made of a device known as the "Prony brake," or similar piece of apparatus. The horsepower thus obtained is called the "brake horsepower. Figure 175 shows a setup for using the Prony brake.

The iron strap *CD* is lined with wooden blocks, or other suitable brake lining, as is also the lever arm *EF*. The two wheels *E* and *E'* are used to tighten the friction blocks. As the straps are tightened, the arm *EF* turns in the direction of rotation. It exerts a pressure on the column *FB*, which in turn causes the scales to record the amount of pressure. The force exerted by the arm and column without rotation of the flywheel must be known. It is simply their weight in position on the scales. This weight is subtracted from the total reading shown by the scales. The rpm is taken at the instant the reading is made. The arm *EF* must be in a horizontal position when readings are taken.

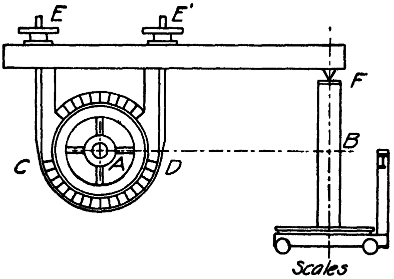

FIG. 175. Prony-brake connection.

The brake horsepower is given by

$$\text{Bhp} = \frac{2\pi lwn}{33{,}000}$$

where $l = AB$ = horizontal length of brake arm, ft
w = net weight obtained from scales, lb
n = rpm of shaft.

EXAMPLE

An 11- by 12-in. engine, running at 300 rpm gave 68.5 ihp. A Prony brake with an arm 5 ft long was attached to the engine. The gross weight on the scales was 238 lb. The weight of the lever and post was 40 lb. What are the brake horsepower and the mechanical efficiency?

$$\text{Bhp} = \frac{2\pi \times 5 \times (238 - 40) \times 300}{33{,}000} = 56.6$$

$$E = \frac{56.6}{68.5} \times 100 = 82.5\%$$

SELECTION OF A STEAM PLANT

The following example will illustrate the use of the foregoing discussion for selecting a moderately sized plant.

EXAMPLE

A plant requires 550 actual horsepower for operating the hoist, compressor, pumps, lights, etc. A steam plant to furnish steam in sufficient quantity to operate a compound engine, which in turn runs a generator, is to be used.

The engine selected is a Corliss compound condensing engine, 27- by 35- by 48-in., running at 80 rpm. It operates at 125 lb gage; the cutoff is ⅓; and the card factor is 0.80. The temperature of the feed water is 120°F. Mechanical efficiency is 85 per cent.

The plant will be located at an elevation of 4000 ft above sea level. The mean barometric pressure at this elevation is 25.76 in. The barometer reading in the condenser chamber must approximate 21.6 in.

The fuel available for operating the plant is coal containing 12,000 Btu per pound. The fuel contains: 4 per cent ash and 3 per cent moisture. Of the total fuel fired, 10 per cent passes through the grates in the form of ash and unburned coal. The efficiency of the boiler, furnace, and grates is 70 per cent.

The condenser recovers 75 per cent of the steam and returns it to the system.

The efficiency between the engine and the generator, which is belt-driven, is 80 per cent.

Determine:

1. If the steam engine selected is large enough.
2. Boiler horsepower necessary to operate engine.
3. Heating surface of boiler required.
4. Coal consumption per hour.
5. Gallons of water per day to operate plant.
6. Gallons of water per day to make up loss.

Solution: We first find the actual horsepower that must be delivered by the steam engine.

$$550 \text{ hp} = 550 \times 0.75 = 412.5 \text{ net kw to be supplied by the generator}$$

Assuming the generator efficiency at 90 per cent,

$$412.5 \div 0.9 = 458.4, \text{ or a 475-kw generator is needed}$$

For exciting the generator there will be needed an auxiliary generator (exciter) with a capacity of about 5 per cent of the large one.

$$5\% \times 475 = 23.75, \text{ or say 25 kw}$$

The total kilowatts needed = 475 + 25 = 500 kw to be supplied by the steam plant. In terms of horsepower this is 500 ÷ 0.75 = 666 hp.

Therefore, a steam engine that will supply 666 hp at the generator is necessary. As the system has an efficiency of 80 per cent between the engine and generator, the brake horsepower must be

$$\text{Bhp} = 666 \div 0.8 = 832$$

Part a (*Engine*)

$$r = \frac{D^2}{xd^2} = \frac{35 \times 35}{\frac{1}{3} \times 25 \times 25} = 5.04$$

The absolute pressure at 4000 ft (Table 50) is 12.67 lb.
The back pressure = (25.7 − 21.6) × 0.491 = 2.0 lb absolute.

$$\begin{aligned}\text{Mep} &= 0.8\left[\frac{125 + 12.67}{5.04}(1 + \ln 5.04) - 2\right] \\ &= 0.8[27.3(1 + 1.62) - 2] = 55.7 \text{ lb}\end{aligned}$$

$$\text{Area of low-pressure cylinder} = \pi\frac{35^2}{4} = 962 \text{ sq in}$$

$$\text{Rated ihp} = \frac{2 \times 55.7 \times {}^{48}\!/_{12} \times 962 \times 80}{33{,}000} = 1040$$

As the mechanical efficiency is 85 per cent, the brake horsepower is

$$\text{Bhp} = 1040 \times 0.85 = 885$$

which is 885 − 832 = 53 hp more than needed. The engine is satisfactory

Part b (*Boiler Horsepower*). From Table 72, we find the probable steam consumption to be 14 to 18 lb per ihp-hr. Using 16 lb, the total water required per hour is

$$16 \times 1040 = 16{,}650 \text{ lb} = 1980 \text{ gal}$$

The heat required to evaporate this amount of water is

$$\begin{aligned}&\text{Lb of water [total heat of steam} - \text{(temp. of feed water} - 32°)] \\ &\qquad = 16{,}650[1192 - (120 - 32)] = 18{,}400{,}000 \text{ Btu}\end{aligned}$$

$$\begin{aligned}\text{Equivalent evaporation} &= \frac{\text{Btu}}{\text{total heat of evaporation at } 212°} \\ &= \frac{18{,}400{,}000}{970.4} = 18{,}860 \text{ lb of water}\end{aligned}$$

One boiler horsepower = 34.5 lb of water evaporated per hour from and at 212°.

$$\text{Boiler hp} = \frac{18{,}860}{34.5} = 547^*$$

Part c (*Heating Surface*). For mining purposes it is probably best to choose a fire-tube boiler (if soft water is cheaply obtained the water-tube boiler may be selected). For the fire-tube boiler 12 sq ft of heating surface is required for each boiler horsepower.

$$\text{Heating surface} = 12 \times 547 = 6564 \text{ sq ft}$$

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* A 500-hp boiler would be sufficient. It would readily stand a 150 per cent overload.

Part d (Coal Consumption). To evaporate 16,650 lb of water requires 18,400,000 Btu.

$$\text{Coal consumed} = \frac{18{,}400{,}000}{12{,}000 \times [1.00 - (0.03 + 0.1)]} = 1762 \text{ lb per hr}$$
$$= 1762 \div 2000 = 0.88 \text{ ton per hr}$$

As the efficiency of the boiler and grates is 70 per cent, the actual fuel consumption is

$$0.88 \div 0.70 = 1.26 \text{ tons per hr}$$

Part e (Water to Operate Plant). Seventy-five per cent of the steam generated is saved.

$$\text{Total water per day for plant} = \frac{1980 \times 24}{0.75} = 63{,}300 \text{ gal for minimum operating}$$

conditions.

Part f (Water to Be Supplied). 63,300 × 25 per cent = 15,825 gal to be supplied each day to make up loss.

Instead of designing the plant on the basis of one engine, it would be better to select two engines. The load on the day shift is usually much heavier than at other periods. One engine should be of such capacity that it would be capable of handling the minimum peak. This would permit shutting down part of the plant during the slack portion of the day (24 hr). The boiler plant would also be designed for such a selection of equipment. It is generally considered best to not generate all the power required by only one piece of equipment; nor is it good practice to drive everything from one motor.

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CHAPTER XIII

MINE VENTILATION

The discussion to follow is in no sense to be considered comprehensive. Mine ventilation is too broad a subject to be thoroughly covered in the limited space available. The moderately deep mine ordinarily gives rise to no complicated ventilating problems. With the constant deepening of mines—which in practically all instances means an increase in temperature and humidity—mine ventilation, or air conditioning as it is now being termed, becomes a decided factor for continued operation. It has been estimated[1] that, in the Witwatersrand area, for every 1°F (200 ft of depth) the temperature is lowered, £20,000,000 of gold is made available. In the United States the Coeur d'Alene, Butte, and some Arizona districts are faced with the difficulty of combating increasing temperatures. Dust conditions can be overcome to a large extent by using wet machines, rock dusting, and guniting.

The pages to follow will be confined entirely to a brief discussion of the application of mine ventilation to those mines where more or less special problems do not arise.

The need for ventilation usually arises because of the physiological effect of bad air on the workman. In some special cases a controlled ventilation is desirable from the standpoint of possible mine fires. In the deeper mines having extensive workings, and especially those having numerous blind headings, natural ventilation becomes extremely difficult. In order to better the cooling effect and to supply fresh air, the mine operator must depend on some form of power ventilation.

Before the ventilation of a mine can be placed on a satisfactory basis, a variety of factors, each more or less peculiar to the mine under consideration, must be considered. The dimensions of the openings; present and future extent of drifts, raises, cross cuts, shafts, etc.; temperature increment with depth; wetness of workings; working force maintained underground; fire hazard; type of mining method; and possible change in noxious-gas constituents of the mine air (in metal mines usually sulfur dioxide from oxidation of sulfides, powder fumes, and gases resulting from decaying timber) may be mentioned.

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[1] Reducing Temperature and Humidity in Deep Mines, *Mining Met.*, p. 329, August, 1935.

For our purpose, ventilation may be considered as being accomplished in one or both of two ways: power ventilation and natural ventilation.

By *power ventilation* is meant the use of fans or blowers. Fans may be of the pressure type or the exhausting type. Mechanically these may be centrifugal fans or propeller fans. The centrifugal type may be single-inlet or double-inlet fans. For secondary ventilation (blind headings or workings which are offshoots from the main ventilating drifts and which do not form a continuous circuit) high-speed compressed-air-operated blowers, centrifugal fans operated by air motors, and air jets, all in connection with metal or canvas tubing, may be used. Air from the compressed-air line is commonly used.

Before a fan can be selected for a particular job, a decision must be reached as to the quantity of air desired. Directly influencing this will be the working temperature desired, relative humidity, dust and gas dilution necessary, and the number of workmen (and in some cases also animals) to be served. It is at once apparent that each mine, and in many instances different parts of the same mine, require individual investigation.

Resistance of Mine Workings. The resistance to the passage of air through mine workings depends essentially on the irregularity or sinuousness of the drifts; the amount of timbering; the nature of the wall rock; and the size of the opening. The friction factor, or coefficient of friction, as it is variously known, is represented in ventilation formulas by K. Table 73[1] gives comprehensive values of K. Values for K may also be found experimentally from the formula

$$P = \frac{KSv^2}{a}$$

or from

$$P = \frac{KSq^2}{a^3}$$

where P = pressure, lb per sq ft
a = cross-sectional area of airway, sq ft
S = rubbing surface of airway, sq ft
= lo

---

[1] McElroy and Richardson, Friction Factors for Metal Mine Airways, *U.S. Bur. Mines, Rep. Invest.* 2663 (1925).

l = length of airway, ft
o = perimeter of airway, ft
K = friction factor
v = velocity of air flowing through airway, ft per min
q = quantity of air flowing through airway, cu ft per min
$= av$

The velocity may be found by use of an anemometer or calculated from

$$v = 1097.4 \sqrt{\frac{i}{w}} \quad \text{ft per min}$$

where i = water gage, in.
w = weight of 1 cu ft of air at given temperature, pressure, and saturation

Temperature and Relative Humidity. The temperature may be expressed either as dry-bulb or wet-bulb. The dry-bulb gives the ordinary temperature. The wet-bulb temperature is obtained from a thermometer, the bulb of which is kept moist by means of a wick. The evaporation of the moisture from the wick causes a lowering of the temperature. By means of the dry- and wet-bulb temperatures, the relative humidity is calculated or obtained from hygrometric tables.[1] The relative humidity is indicated by the difference between the wet- and dry-bulb readings. Air for a given temperature, *i.e.*, completely saturated with moisture, has a humidity of 100 per cent. In such an atmosphere work is impossible. The wet-bulb temperature should not exceed 80°F for ordinary work. For hard continuous work it must be less. The relative humidity should not fall below 60 to 65 per cent. Nor, on the other hand, should it approach too closely to 100 per cent.

Gas and Dust Dilution. The amount of harmful gases or dust permitted in mine air is usually specified by state laws. Their control requires periodic chemical or physical examination, which is beyond the scope of this chapter. In metal mines, a gaseous condition is usually due to powder fumes, with sulfur dioxide occasionally causing trouble. Some long water works tunnels have encountered methane gas. A rough and ready rule for diluting the gases would be never to let "gas"

---

[1] For such tables, see the following:

HADDOCK, M. H., "Mine Ventilation and Ventilators," J. B. Lippincott Company, Philadelphia.

WALSH, J. J., "Physics and Chemistry of Mining and Mine Ventilation." 2d ed., D. van Nostrand Company, Inc., New York.

WEEKS, W. S., "Ventilation of Mines," 1st ed., McGraw-Hill Book Company, Inc., New York.

TABLE 73. FRICTION FACTORS FOR MINE AIRWAYS

Type of airway	Irregularities of surface, areas and alignment	Values of $10^{10} \times K$*											
		Straight			Sinuous or curved								
					Slightly			Moderately			High degree		
		Clean	Slightly obstructed	Moderately obstructed	Clean	Slightly obstructed	Moderately obstructed	Clean	Slightly obstructed	Moderately obstructed	Clean	Slightly obstructed	Moderately obstructed
Smooth-lined	Minimum.....	10	15	25	20	25	35	25	30	40	35	40	50
	Average.......	15	20	30	25	30	40	30	35	45	40	45	55
	Maximum.....	20	25	35	30	35	45	45	40	50	45	50	60
Sedimentary rock	Minimum.....	30	35	45	40	45	55	45	50	60	55	60	70
	Average	55	60	70	65	70	80	70	75	85	80	85	95
	Maximum.....	70	75	85	80	85	95	85	90	100	95	100	100
Timbering	Minimum.....	80	85	95	90	95	105	95	100	110	105	110	120
	Average	95	100	110	105	110	120	110	115	125	120	125	135
	Maximum.....	105	110	120	115	120	130	120	125	135	130	135	145
Igneous rock	Minimum	90	95	105	100	105	115	105	110	120	115	120	130
	Average.	145	150	160	155	160	165	160	165	175	170	175	195
	Maximum ...	195	200	210	205	210	220	210	215	225	220	225	235

Note. Values underscored are recommended as good average values for theoretical computations. All values of K are for air weighing 0.075 lb. per cu ft.

* 10 in table represents $K = 0.0000000010$, 100 in table represents $K = 0.0000000100$.

be present in amounts that will cause headache to the workman. It should be understood, however, that death may result before such a symptom is noticed. This is especially true if carbon monoxide is present. So far as dust is concerned, and especially silica dust, the concentration should be kept below 2,000,000 particles (10μ and less in size) per cubic foot of air.[1]

Quantity of Air Required. It is almost impossible to give any definite rule that will, offhand, estimate the amount of air to be circulated. For metal mines, the quantity per man is usually reckoned at about 50 cu ft per min per man. In coal mines, this runs as high as 200 to 300 cu ft per min per man. The nature and constancy of the labor performed directly influence the consumption. The size and resistance of the airways, the underground temperature, and the relative humidity must all be considered. For animals, about 200 cu ft per min is usually taken.

POWER VENTILATION

Before a fan can be selected, the total resistance must be determined, and the volume of air to be circulated and its pressure decided upon and the mine characteristic curve plotted.

Pressure. The ventilating pressure is usually given in inches of water gage, i. Inches of water gage is the column of water i in. high that is equivalent to a given pressure P in pounds per square foot.

$$P = wh \qquad \text{or} \qquad P = \frac{62.4}{12} \times i = 5.2i$$

and

$$i = \frac{P}{5.2} \text{ in.}$$

where w equals the weight of 1 cu ft of water = 62.4 lb.

By substituting in other formulas,

$$i = \frac{KSv^2}{5.2a} = \frac{KSq^2}{5.2a^3} \text{ in.}$$

EXAMPLE

What pressure is necessary to force 60,000 cu ft of air through an untimbered, fairly clean, straight drift in igneous rock? The drift is 8 by 10 ft. It is 3000 ft long.

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[1] CUMMINGS, D. E., Good Practice in Combating Dust Hazards Associated with Mining Operations, *AIMME Tech. Pub.* 638 (1935).

From Table 73,

$$K = 0.0000000195$$
$$a = 8 \times 10 = 80 \text{ sq ft}$$
$$S = 2(8 + 10) \times 3000 = 108{,}000 \text{ sq ft}$$
$$q = av \quad \text{or} \quad v = \frac{q}{a} = 60{,}000 \div 80 = 750 \text{ ft per min}$$
$$P = \frac{KSv^2}{a} = \frac{0.0000000195 \times 108{,}000 \times 750^2}{80} = 14.8 \text{ lb per sq ft}$$

or

$$i = \frac{P}{5.2} = 14.8 \div 5.2 = 2.84 \text{ in.}$$

ECONOMIC SIZE OF AIRWAYS

In most mines very little thought is given to the size of openings to be used for passing the air. In a great many cases, and in metal mines especially, development and prospecting drifts are later used for mining or extraction purposes. They are driven just large enough to conduct the work through. When it later becomes necessary to use them for ventilation, they cause a very considerable increase in the power required because of their relatively small size. For a large part of the underground workings there is no help for this situation. However, when it is planned or suspected that headings will later be used as airways, in addition to other uses, their size should be carefully considered before they are driven. This can quite frequently be done when planning long air raises or shafts, which will be used solely as airways. The following table illustrates the rapid increase in power necessary to force air through openings of various cross-sectional areas. If the friction factor and the length are constant (and we are justified in making this assumption), the power will vary as the perimeters and the cube of the areas.

That is,

$$\frac{l_2}{l_1}\left(\frac{a_1}{a_2}\right)^3$$

or for the 9 × 8 drift, $^{34}\!/_{40} \times (^{100}\!/_{72})^3 = 2.28$.

It will take 2.28 times as much power to pass the same quantity of air through the 9 by 8 drift, as through the 10 by 10 drift.

Size of airway, ft.	Perimeter, ft.	Area, sq. ft.	Power, relative
10 × 10	40	100	1.00
9 × 8	34	72	2.28
8 × 7	30	56	4.27
7 × 6	26	42	8.77
6 × 5	22	30	20.40

The importance of driving large airways will be realized after an inspection of the above table. A medium should be struck between the cost of driving the airway and the future ventilating costs.

TYPES OF FANS

There are two types of fans in general use: the centrifugal and the propeller or axial flow. In recent years the propeller fan has received considerable attention and is replacing the centrifugal in many mine-ventilation installations. It has many features that suggest its adaptability to most mine problems.

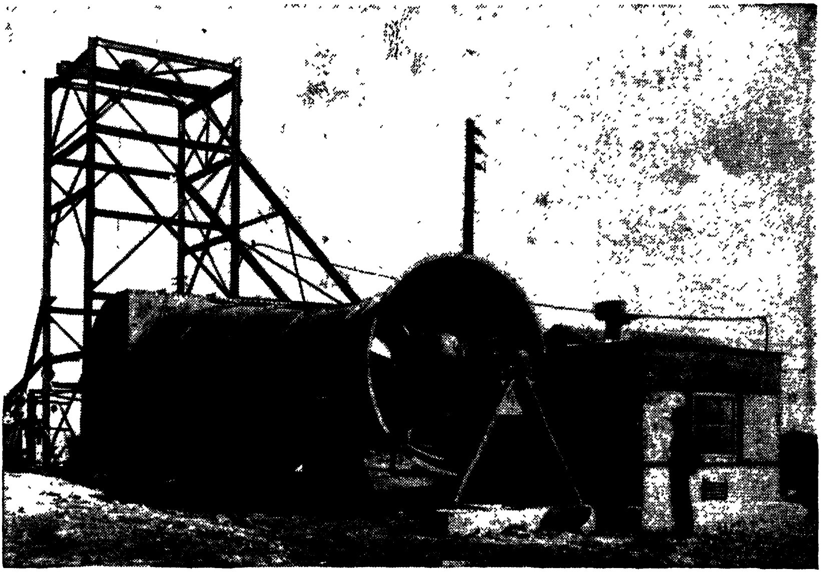

FIG. 176. Princeton Mining Company, Princeton, Ind.

Figures 176, 177, 178, and 179 show widely used types of propeller fans.[1] The Aerodyne Junior may be had in sizes ranging from 3 to 10 ft in diameter; 5000 to 200,000 cu ft per min capacity; ¼ to 4 in. of water-gage pressure (single-stage up to 2 in.; two-stage up to 4 in.); and one or two stages. Aerodyne fans are available in sizes that range from 5000 to 600,000 cu ft per min; ½ to 17 in. of static pressure (up to 8 in. on single-stage; and to 17 in. on the double-stage); and seven possible blade positions. The ability to adjust the position (pitch) of the blades, permits a maximum efficiency over a wide change in the mine characteristic. Blades may be temporarily removed in the Aerodyne Junior as an aid toward meeting the changing requirements of a mine.

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[1] By permission of the Jeffrey Manufacturing Company, Columbus, Ohio.

An outstanding feature of the propeller type of fan is the ease with which the air current can be reversed. It is only necessary to reverse the rotation of the fan. The efficiency is somewhat lower than for the normal rotation. With the centrifugal fan, a system of doors must be used when reversing the direction of flow. This requires a more elaborate installation.

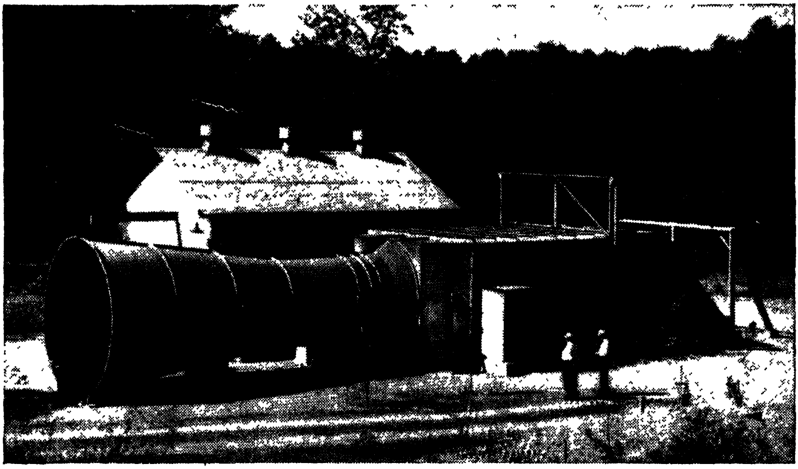

Fig. 177. Tennessee Coal, Iron and Railroad Company, Birmingham, Ala.

Fig. 178. Underground fan at International Salt Company, Retsof, N. Y.

When selecting a fan, it is important to keep in mind the effect on the motor of a sudden change in mine resistance. A nonoverloading type of fan is the safest. Figure 180[1] illustrates this term. At 100 per cent rated volume the horsepower is 100 per cent rated horsepower. If there is a sudden cave in the mine so that the volume passing is reduced, the power can only increase an insignificant amount. Thus, there would be no serious effect on the motor. On the other hand, if the delivered volume suddenly increases, the power drops off slightly, which will not endanger the motor.

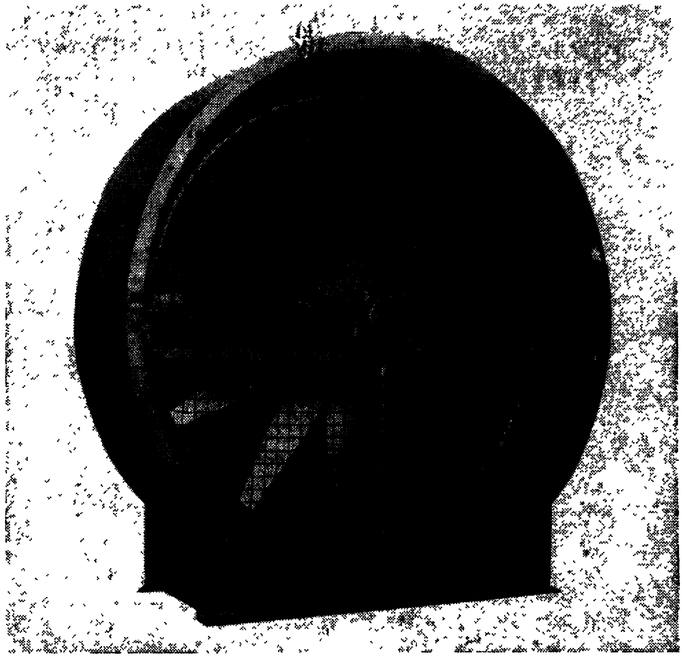

FIG. 179. Aerodyne Junior fan. (*The Jeffrey Manufacturing Company.*)

The main fan may be installed on the surface or underground. It may be primarily a pressure (blowing) or exhaust fan. Provision is nearly always made for reversing the direction of the current. Figure 186[2] shows the general way in which a centrifugal fan may reverse the direction of flow. The fan may be connected to a shaft, tunnel, or drift.

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[1] By permission of the Jeffrey Manufacturing Company, Columbus, Ohio.

[2] MONTGOMERY, W. J., Theory and Practice of Mine Ventilation, Fig. 91, p. 231, The Jeffrey Manufacturing Company, Columbus, Ohio, 1936.

The fan shown in Fig. 186 is exhausting, the duty for which it was primarily designed. By moving the doors *A* in the direction of the arrows, door *C* (in the chimney) as indicated and opening doors *B* to the outside air the fan will blow air into the mine. Its efficiency will suffer somewhat because the connection of the mine to the fan was designed for exhausting.

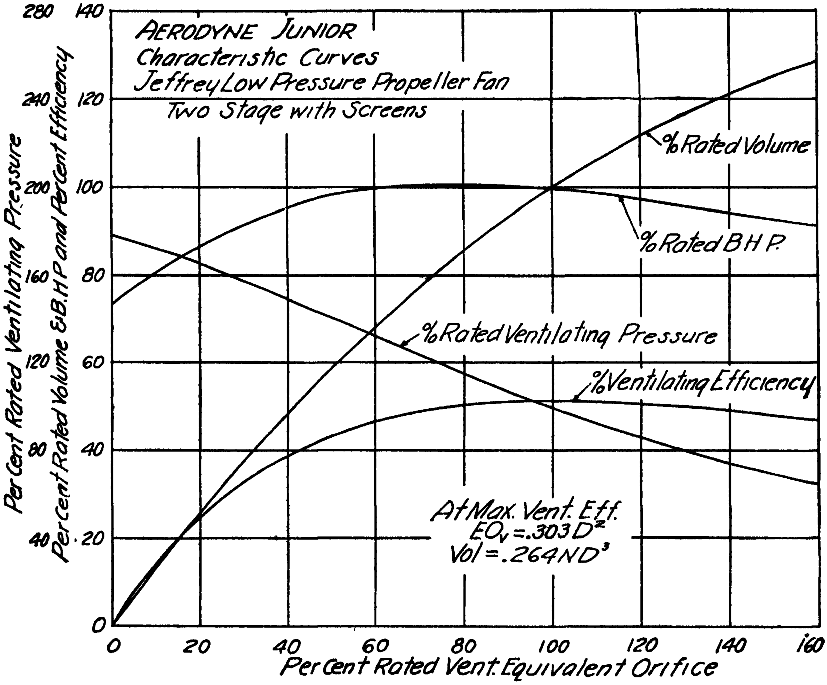

FIG. 180. Nonoverloading fan curve. (*The Jeffrey Manufacturing Company.*)

All sizes of the Aerodyne series of fans are symmetrical. The following relationships, for practical purposes, hold.

1. Pressure varies directly as number of stages (Aerodyne Junior: maximum, 2 in. for single-stage, 4 in. for two-stage; Aerodyne: maximum, 8 in. for single-stage, 17 in. for two-stage).
2. Pressure varies approximately as number of blades (for Aerodyne Junior).
3. Volume varies as the square of the diameter with the same specific volume and pressure.

There are several other fan laws that should be kept in mind when changing the speed of a fan is contemplated.

1. The volume is constant regardless of the density.
2. Pressure varies directly as the density.
3. Horsepower input to the fan varies directly as the density.
4. Volume varies directly as the speed (rpm).
5. Pressure varies directly as the square of the speed-change ratio.
6. Horsepower input varies directly as the cube of the speed-change ratio.
7. With the exception of the rather small fans, the efficiency remains practically constant with a change of speed.

The ventilation of a mine will depend on the mine characteristic (resistance) and the effect of natural ventilation. Many mines may depend on natural ventilation to a considerable depth and extent of underground workings.

Natural ventilation results when there is a difference in pressure between the air in the mine and the outside air. This pressure difference is caused by a difference in the temperature between the mine air and atmospheric air. When the temperatures are nearly equal, the ventilation is sluggish and the direction of flow may reverse itself several times during a 24-hr period. It may even cease altogether at certain periods. There is a decided seasonal control of the direction and velocity. If there are underground conditions (temperature, humidity, gas, dust, and fire hazard) that make a definite volume of air and direction of flow necessary, then natural ventilation may become dangerous. Its effect on the mechanical ventilation must be decided before choosing the fan. If not, the fan may become overloaded opposing the natural air current, and the desired volume and pressure not obtained.

Before beginning the mine-ventilation survey, an accurate map of the mine workings must be available. Judicious splitting of the air; parallel or series combinations of drifts, raises, stopes, levels, etc., all may result in a materially lower fan pressure than would offhand be suspected. The dimensions (cross-sectional and length) of these airways must be known with a reasonable degree of accuracy. A difficult decision to make is assigning a value to K in the pressure-drop formula, $P = \frac{KSq^2}{a^3}$. Table 73 would cover most cases.

When the determination of pressure differences may be used instead of resorting to the pressure-drop formula, they should be made with either the Pitot tube and inclined manometer or the Paulin altimeter.[1]

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[1] MONTGOMERY, W. J., *ibid.*, p. 150.

The American Paulin System, Inc., 1220 Maple Ave., Los Angles, California.

Seasonal measurements of wet- and dry-bulb temperatures, both outside and inside the mine, are necessary to determine the density of the air columns causing natural ventilation. If there is a considerable difference in elevation (more than 1000 ft) between the outside and mine air columns, then the air density must be carefully averaged. It may be necessary to reduce the total air column to smaller individual columns to obtain a more accurate figure for the density.[1]

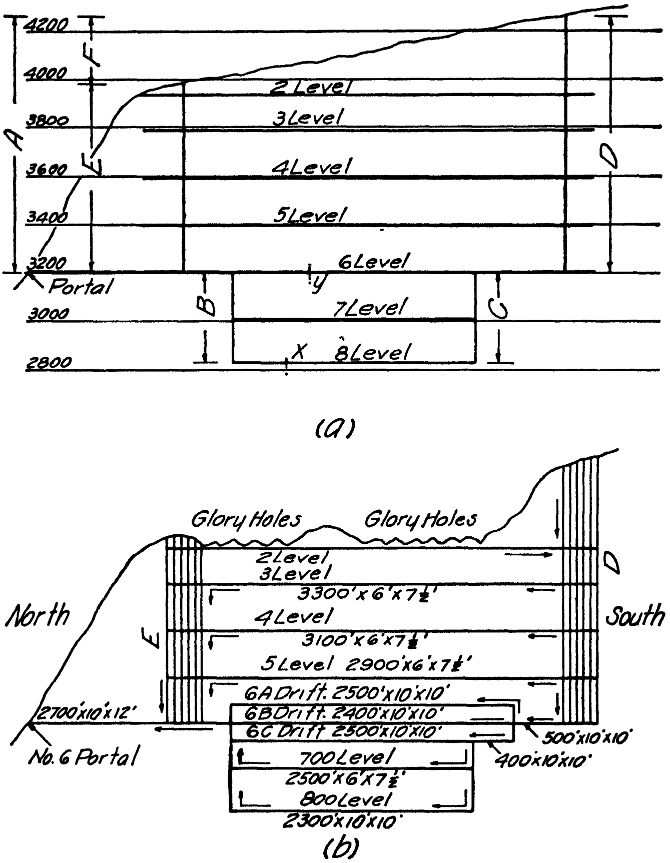

FIG. 181. Vertical projection of mine workings for ventilation. (*The Canadian Institute of Mining and Metallurgy.*)

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[1] WEEKS, W. S., Ventilation of Mines, p. 82, McGraw-Hill Book Company, Inc., New York.

HADDOCK, M. H., Mine Ventilation and Ventilators, p. 148, J. B. Lippincott Company, Philadelphia.

The following example will illustrate the general procedure for selecting a fan.[1]

Figure 181*a* shows a vertical projection of the mine workings for determining the natural ventilation pressure. Table 74 summarizes the various data. From the table it is seen that in the summer the direction of flow is in at *D* and out the portal; in the winter, with the door at the portal closed, air flows in at *F* and out at *D*. If the portal door is left open, the air flows in at the portal, with a very considerable velocity. An exhaust fan, if placed near the portal, would have to overcome 2.48 in. of water gage during the winter. Bad icing conditions would also result.

TABLE 74. NATURAL VENTILATION DATA

Air column	Height, ft	Pressure, Hg	Avg. air density, lb per cu ft	Temp., °F		Pressure, lb per sq ft	Pressure diff., lb per sq ft	In., water gage
				Dry-bulb	Wet-bulb			
Summer:								
D	1050	26.12	0.0677	52	51	71.085		
A	1050	26.12	0.0629	85	70	66.045		
							5.040	0.97
Winter:								
F	270	25 74	0 0800	30		21 600		
E	780	26.30	0 0692	44	43	53 976		
F + *E*	1050		0.1492			75 576		
D	1050	26 12	0.0677	52	51	71.085		
							4.491	0 865
No. 6 Portal:								
A	1050			−30		84 00		
D	1050					71.085		
							12.915	2 48

Column *D* is in the mine.
Column *A* is atmospheric column.
Column *F* is atmospheric column.
Column *E* is in the mine.

Mine Characteristic. To estimate the quantity of air that will flow because of the natural ventilation pressure, the mine characteristic must be found. When plotted, it is a curve showing the pressure-volume relationship.

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[1] TOUGH, W. J., Mine Ventilation Calculations, *Can. Mining Met. Bull.*, July, 1942, pp. 350–378, or *Trans. Can. Inst. Mining Met.*, **XLV**, 350–378 (1942). (Abstracted by permission.)

Figure 181*b* is a sketch of the mine giving the necessary dimensions of the workings. Sufficient study must be devoted to the arrangement of the mine workings in order to obtain the most economical parallel and/or series combinations.

From

$$P = \frac{KSv^2}{a} = \frac{KSq^2}{a^3} \quad \text{lb per sq ft}$$

the pressure drop at key points is found.

To simplify the tedious arithmetical work, it is customary to make certain changes in the above formula.

The members K, S, and a for any one airway remain constant. Then P varies as q^2. Combining the constants and representing them by R,

$$R = \frac{KS}{a^3} \qquad \text{and} \qquad P = Rq^2$$

For airways that are in series, the total resistance is

$$R = R_1 + R_2 + R_3 + \cdots$$

If in parallel, the total resistance is

$$\frac{1}{\sqrt{R}} + \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_3}} + \cdots$$

and since $R = KS/a^3$, then

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{\frac{KS}{a^3}}} = a\sqrt{\frac{a}{KS}}$$

The combination of the various drifts, raises, etc., will be as follows (see Fig. 181*b*):

Levels 8 and 7, and part of 6*C* drift, are in parallel. This total resistance is in series with 400 ft of 6*C* drift.

Next, drifts 6*A*, 6*B*, and 6*C* are in parallel (6*C* is represented by the total resistance found from levels 8 and 7, and 6*C* previously determined). This combination is in series with the 500 ft of 10 by 10 drift at the south end. We now have the total resistance of 6, 7, and 8 levels. Number 3, 4, 5, and 6 levels are in parallel. To this total parallel resistance must be added, in series, the 2700 ft to No. 6 portal. This will give the total mine resistance.

No allowance is made for No. 2 level, because it consists of numerous glory holes connecting with the surface, or for raises *D* and *E*. These openings are all very large in cross section and assumed to have a negligible resistance.

TABLE 75. DIMENSIONS OF MINE WORKINGS

Level	Drift	Length, ft	Cross section, ft	Perimeter, ft	S, sq ft	A, sq ft
3		3300	6 × 7½	27	89,000	45
4		3100	6 × 7½	27	83,700	45
5		2900	6 × 7½	27	78,300	45
6	*A*	2500	10 × 10	40	100,000	100
	B	2400	10 × 10	40	96,000	100
	C	2500	10 × 10	40	100,000	100
6	.	2700	10 × 12	44	118,800	120
		500	10 × 10	40	20,000	100
	. .	400	10 × 10	40	16,000	100
7	. .	2500	6 × 7½	27	67,500	45
8	.	2300	10 × 10	40	92,000	100

Table 75 shows a convenient method for assembling the data before starting the final calculations.

Considerable reflection should be given to the selection of a value or values for K. It may be possible to use the same K for all of the workings; or, because of a big difference between the conditions in the various headings, different values for K may be required (see Table 73). Tabulated values of K are usually for air at 0.075 lb per cu ft. It is desirable to change these standard values to their equivalent at the average mine-air density. For this example, $K = 1.65 \times 10^{-8}$ for air at 0.075 lb per cu ft. This is corrected to air at 3540 ft elevation and 49°F dry-bulb and 48°F wet-bulb; the density equals 0.0684 lb per cu ft.

$$K = 1.65 \times 10^{-8} \times \frac{0.0684}{0.075} = 1.50 \times 10^{-8}$$

This value for K is assumed constant for the entire mine.

Case 1. Resistance of 6*C*, 7, and 8 is parallel.

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_{6C}}} + \frac{1}{\sqrt{R_7}} + \frac{1}{\sqrt{R_8}}$$

$$= a_{6C}\sqrt{\frac{a_{6C}}{KS_{6C}}} + a_7\sqrt{\frac{a_7}{KS_7}} + a_8\sqrt{\frac{a_8}{KS_8}}$$

$$\frac{1}{\sqrt{R}} = 100\sqrt{\frac{100}{1.5 \times 10^{-8} \times 10^5}} + 45\sqrt{\frac{45}{1.5 \times 10^{-8} \times 6.75 \times 10^4}}$$

$$+ 100\sqrt{\frac{100}{1.5 \times 10^{-8} \times 9.2 \times 10^4}}$$

$$= 6.236 \times 10^4$$

$$R = 2.58 \times 10^{-10}$$

Case 2. To $\frac{1}{\sqrt{R}}$ for 6*C*, 7, and 8 is added, in series, the resistance of 400 ft of 6*C*.

$$R = \frac{KS}{a^3} = \frac{1.5 \times 10^{-8} \times 16{,}000}{100^3} = 2.4 \times 10^{-10}$$

Total R for $6C$ drift is

$$R_{6C} = 2.4 \times 10^{-10} + 2.58 \times 10^{-10} = 4.98 \times 10^{-10}$$

Case 3. Resistance of $6A$, $6B$, and $6C$ is in parallel.

$$\begin{aligned}
\frac{1}{\sqrt{R}} &= \frac{1}{\sqrt{R_{6A}}} + \frac{1}{\sqrt{R_{6B}}} + \frac{1}{\sqrt{R_{6C}}} \\
&= 100\sqrt{\frac{100}{1.5 \times 10^{-8} \times 10^5}} + 100\sqrt{\frac{100}{1.5 \times 10^{-8} \times 9.6 \times 10^4}} \\
&\qquad + \sqrt{\frac{1}{4.98 \times 10^{-10}}} \\
&= 2.58 \times 10^4 + 2.63 \times 10^4 + 4.48 \times 10^4 \\
&= 9.69 \times 10^4 \\
R &= 1.06 \times 10^{-10}
\end{aligned}$$

Case 4. To the resistance found in Case 3 must be added, in series, the resistance of 500 ft connecting 6 level with raise D.

$$R = \frac{KS}{a^3} = \frac{1.5 \times 10^{-8} \times 20{,}000}{100^3} = 3.0 \times 10^{-10}$$

Total R for 6, 7, and 8 levels is equal to

$$3.0 \times 10^{-10} + 1.06 \times 10^{-10} = 4.06 \times 10^{-10}$$

Case 5. Resistance of 3, 4, 5, and 6 levels is in parallel.

$$\begin{aligned}
\frac{1}{\sqrt{R}} &= \frac{1}{\sqrt{R_3}} + \frac{1}{\sqrt{R_4}} + \frac{1}{\sqrt{R_5}} + \frac{1}{\sqrt{R_6}} \\
&= 45\sqrt{\frac{45}{1.5 \times 10^{-8} \times 8.9 \times 10^4}} + 45\sqrt{\frac{45}{1.5 \times 10^{-8} \times 8.37 \times 10^4}} \\
&\qquad + 45\sqrt{\frac{45}{1.5 \times 10^{-8} \times 7.83 \times 10^4}} + \sqrt{\frac{1}{4.06 \times 10^{-10}}} \\
&= 0.826 \times 10^4 + 0.850 \times 10^4 + 0.881 \times 10^4 + 4.96 \times 10^4 \\
&= 7.517 \times 10^4 \\
R &= 1.77 \times 10^{-10}
\end{aligned}$$

Case 6. In this case, the total mine resistance is considered. To the resistance of the mine up to this point must be added, in series, the airway from the raise E to the No. 6 portal.

$$\begin{aligned}
R &= \frac{KS}{a^3} = \frac{1.5 \times 10^{-8} \times 118{,}800}{120^3} = 10.3 \times 10^{-10} \\
\text{Total resistance} &= (10.3 + 1.77) \times 10^{-10} \\
&= 12.07 \times 10^{-10}
\end{aligned}$$

Mine Characteristic Curve. By substituting values, from 0 to any amount desired, for q in $P = Rq^2$, and converting to inches of water gage, data for constructing the mine characteristic curve are obtained.

$$i = \frac{Rq^2}{5.2} \quad \text{in. of water gage}$$

When $q = 0$,

$$i = 0$$
$$q = 10{,}000$$
$$i = \frac{12.07 \times 10^{-10} \times 10{,}000^2}{5.2} = 0.0232 \text{ in.}$$

Other values for i are obtained in a like manner.

Figure 182 shows the characteristic curve. On the same figure is plotted the natural ventilation pressures for the different temperature combinations. These are straight lines. Where they intersect the mine characteristic, the volume at the particular pressure is indicated; for example, the summer line gives about 65,000 cu ft per min.

The preceeding calculations indicate what may be expected from natural ventilation with different temperature changes.

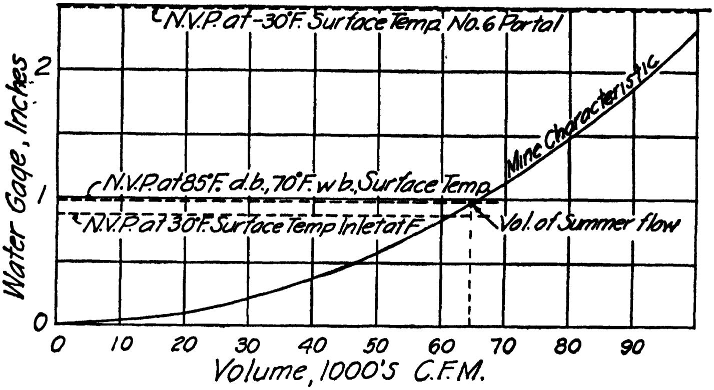

FIG. 182. Mine characteristic curve. (*The Canadian Institute of Mining and Metallurgy.*)

Selection of a Fan. Before the fan may be decided upon a number of decisions must be made. It is beyond the purpose of this book to enter into an exhaustive discussion on this subject. Books on mine ventilation and air conditioning and other literature should be consulted. Among the many factors that have influence may be mentioned the following:

1. The location of the fan and whether primarily blowing or exhausting
2. The coursing or flow of air to the fan and away from the fan (combinations in series and parallel of mine passageways)
3. The minimum volume of air that will meet the requirements
4. Ventilating pressure
5. Series or parallel operation of fans
6. Temperature, humidity, possible fire control, and dust and gas dilution

The last named is apt to become very important in deep mines (those approaching 5000 ft) and in some shallower depth mines. Ventilating or air conditioning hot, humid mines requires the application of thermodynamics to the investigation.[1]

Continuing with the illustrative problem, a fan will be selected for the conditions shown in Fig. 183. This is a diagrammatic portion of the south end of the mine shown in Fig. 181. A minimum volume of 60,000 cu ft per min will be assumed to be sufficient. To the resistance offered by this volume of air must be added the effect of natural ventilation with or against the fan.

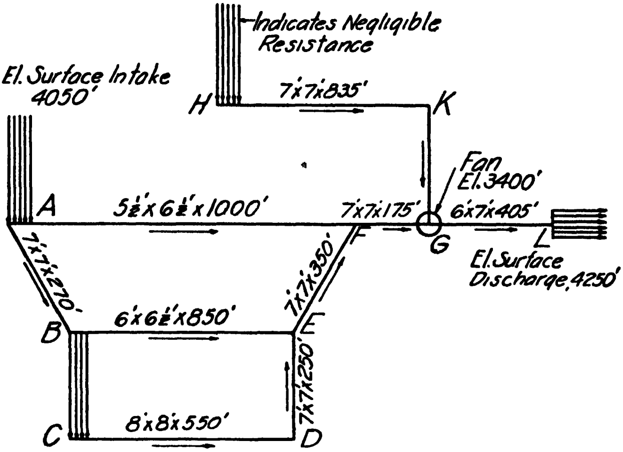

FIG. 183. Airflow for fan. (*The Canadian Institute of Mining and Metallurgy.*)

There are two methods by which the mine characteristic may be found. One of these has already been illustrated; the other involves considerably more work, although the result is the same with both methods. For the second method, textbooks on mine ventilation should be consulted.

An explanation of Fig. 183 is as follows:

1. Airway *BCDE* is in parallel with airway *BE*. The resultant of these two resistances, plus the series resistance of *AB* and *EF*, is in parallel with *AF*.

---

[1] WATERFILL, R. W., Air Conditioning Deep Mines, *AIMME Tech. Pub.* 206 (1929).

CARRIER, W. H., Air Cooling in the Gold Mines on the Rand, *AIMME Tech. Pub.* 970, *Mining Technol.* **2,** No. 5 (1938).

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2. The resultant of (1), plus the series resistance of *FG*, is in parallel with *HKG*.
3. The resultant of all the airways leading to the fan inlet, plus the series resistance *GL*, is the total resistance to the flow of air.
4. An adjustment for the effect of the natural ventilation must be made.
5. The fan, it will be noted, is exhausting.
6. The raises shown are of such size that their resistance is considered negligible.

Table 76 shows a convenient tabulation of the data.

As previously found, $K = 1.5 \times 10^{-8}$.

$$P = Rq^2 \qquad R = \frac{KS}{a^3} \qquad \frac{1}{\sqrt{R}} = a\sqrt{\frac{a}{KS}}$$

For series flow,

$$R = R_1 + R_2 + R_3 + \cdots$$

TABLE 76. DATA FOR CALCULATING MINE RESISTANCE

Airway	Cross section, ft	Perimeter, ft	Length, ft	Rubbing surface, sq ft	$(\text{Area})^3$
BCDE					
CD	8 × 8	32	550	17,600	262,144
DE	7 × 7	28	250	7,000	117,650
BE	6 × 6½	25	850	21,250	59,320
AB + EF	7 × 7	28	640	17,920	117,650
AF	5½ × 6½	24	1000	24,000	45,700
FG	7 × 7	28	175	4,900	117,650
HKG	7 × 7	28	835	23,380	117,650
GL	6 × 7	26	405	10,530	74,088

For parallel flow,

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_2}} + \cdots$$

Case 1. Total series *R* of *BC*, *CD*, and *DE*; *BC* is, because of its size, negligible. For *CD* and *DE*,

$$R = 1.5 \times 10^{-8}\left(\frac{17{,}600}{262{,}144} + \frac{7000}{117{,}650}\right) = 1.9 \times 10^{-9}$$

Case 2. *R* for *BE* and *BCDE* is in parallel.

$$\begin{aligned}\frac{1}{\sqrt{R}} &= 39\sqrt{\frac{39}{1.5 \times 10^{-8} \times 2.125 \times 10^4}} + \sqrt{\frac{1}{1.9 \times 10^{-9}}}\\ &= 1.37 \times 10^4 + 2.3 \times 10^4 = 3.67 \times 10^4\\ R &= 7.5 \times 10^{-10}\end{aligned}$$

Case 3. *AB* and *EF*, in series, are added to case 2.

$$\begin{aligned}R &= 7.5 \times 10^{-10} + \frac{1.5 \times 10^{-8} \times 17{,}920}{117{,}650}\\ &= 7.5 \times 10^{-10} + 2.28 \times 10^{-9} = 3.03 \times 10^{-9}\end{aligned}$$

Case 4. *R* of *ABCDEF* is in parallel with *AF*.

$$\begin{aligned}\frac{1}{\sqrt{R}} &= \frac{1}{\sqrt{3.03 \times 10^{-9}}} + 35.75\sqrt{\frac{35.75}{1.5 \times 10^{-8} \times 2.4 \times 10^{4}}}\\ &= 1.82 \times 10^{4} + 1.126 \times 10^{4} = 2.946 \times 10^{4}\\ R &= 1.15 \times 10^{-9}\end{aligned}$$

Case 5. *FG*, in series, must be added to case 4.

$$\begin{aligned}R &= 1.15 \times 10^{-9} + \frac{1.5 \times 10^{-8} \times 4900}{117{,}650}\\ &= 1.15 \times 10^{-9} + 6.25 \times 10^{-10} = 1.775 \times 10^{-9}\end{aligned}$$

Case 6. Resistance of *BCDEFG* is in parallel with *HKG*.

$$\begin{aligned}\frac{1}{\sqrt{R}} &= \frac{1}{\sqrt{1.775 \times 10^{-9}}} + 49\sqrt{\frac{49}{1.5 \times 10^{-8} \times 2.338 \times 10^{4}}}\\ &= 2.38 \times 10^{4} + 1.83 \times 10^{4} = 4.21 \times 10^{4}\\ R &= 5.64 \times 10^{-10}\end{aligned}$$

Case 7. To the total resistance in case 6 must be added, in series, the resistance of *GL*. Beyond *GL* the resistance is negligible.

$$\begin{aligned}R &= 5.64 \times 10^{-10} + \frac{1.5 \times 10^{-8} \times 10{,}530}{74{,}088}\\ &= 5.64 \times 10^{-10} + 2.13 \times 10^{-9}\\ R \text{ for the mine} &= 2.69 \times 10^{-9}\end{aligned}$$

Case 8. The pressure difference to maintain a flow of 60,000 cu ft per min is

$$\begin{aligned}i &= \frac{Rq^2}{5.2} \quad \text{in. of water gage}\\ &= \frac{2.69 \times 10^{-9} \times 60{,}000^2}{5.2} = 1.86 \text{ in. of water gage}\end{aligned}$$

Case 9. By inserting values for q, the mine characteristic curve may be found and plotted.

Case 10. In this case the natural ventilation pressure is considered. As a minimum flow of 60,000 cu ft has been decided upon, it is only necessary to consider the natural ventilation which opposes the fan. This occurs during the summer. The pressure is as follows:

Downcast air column:

a. The surface component is 200 ft with density of 0.0620 lb per cu ft.

b. The mine component is 650 ft with density of 0.0682 lb per cu ft.

Upcast air column:

The surface component is 850 ft with density of 0.0678 lb per cu ft.

$$\begin{aligned}\text{Natural ventilation pressure} &= 200 \times 0.0620 + 650 \times 0.0682 - 850 \times 0.0678\\ &= 0.90 \text{ lb per sq in.}\\ &= 0.90 \div 5.2 = 0.17 \text{ in. of water gage}\end{aligned}$$

These data are obtained from Fig. 183. The upcast column is

$$4250 - 3400 = 850 \text{ ft.}$$

The downcast column is 4050 − 3400 = 650 ft in the mine. To this is added 850 − 650 = 200 ft of atmospheric air giving a total of 850 ft for the downcast column.

The average air densities must be carefully determined.

Total Static Pressure on Fan. To provide pressure for the airways assumed to have negligible resistance, for uncertain resistances in other airways, and for variation in the natural ventilation pressure, a factor of safety of 20 per cent should be included. This factor is applied to the pressure resulting from the mine characteristic at the desired minimum volume.

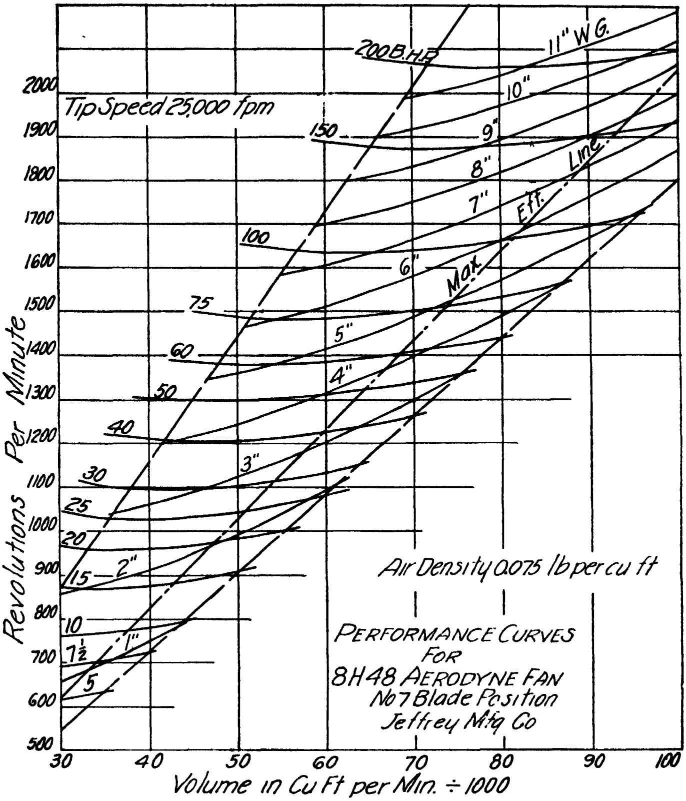

FIG. 184. Characteristic curve for Aerodyne fan. (*The Jeffrey Manufacturing Company.*)

$$20\% \times 1.86 \text{ in.} = 0.37 \text{ in.}$$

$$\text{Total static pressure} = 1.86 + 0.37 + 0.17 = 2.40 \text{ in. of water gage}$$

To complete the preceding problem, characteristic fan curves must be obtained from the fan manufacturers. One is selected that will give the desired minimum volume at the required pressure, with maximum efficiency. This is indicated where the fan pressure-volume (P-v) curve crosses the mine characteristic curve. At this intersection the required minimum pressure and volume must be given by the fan. Future requirements of the fan should be anticipated, when possible. Thus, a fan whose number of blades, or blade pitch can be varied, may save a completely new installation in future years.

In Fig. 184 is given the data for an Aerodyne 8H 48 No. 7 blade position. These curves are for an air density of 0.075 lb per cu ft. The density of the mine air is 0.0682 lb per cu ft. This is equivalent to 0.075/0.0682 × 2.40 = 2.64 in. of water gage when using the curves. The curves show that 60,000 cu ft per min may be obtained with a water gage of 2.64 in. at close to maximum efficiency. The speed of the fan is about 1160 rpm. Slightly more than 33 hp are required.

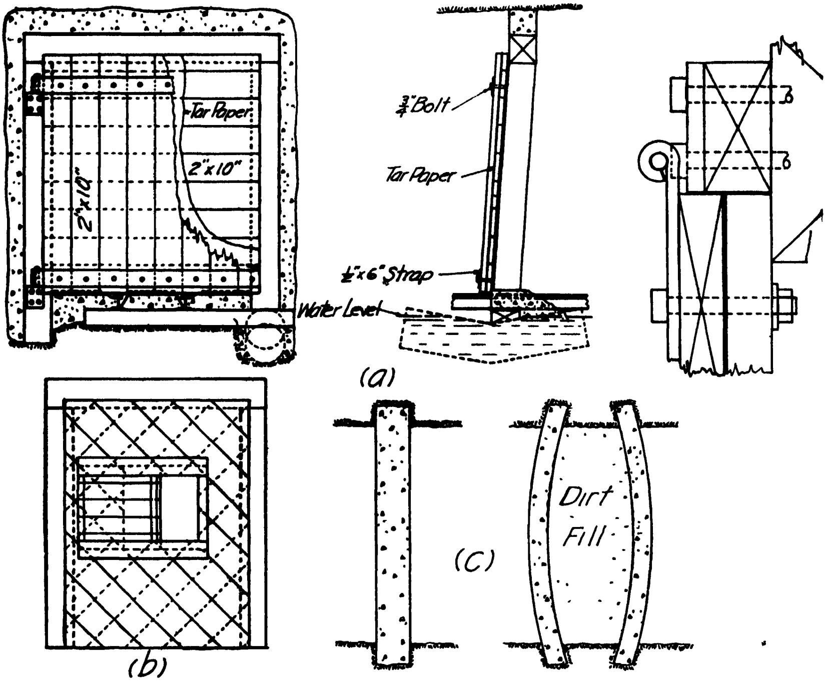

FIG. 185. Ventilation doors and bulkheads.

The theoretical or air horsepower is found from

$$\text{Hp} = \frac{qP}{33{,}000} = \frac{60{,}000 \times 2.64 \times 5.2}{33{,}000} = 25$$

$$\text{Efficiency} = \frac{25}{33} \times 100 = 76\%$$

Ventilation Doors. Mine-ventilation doors may be made of wood or metal, or wood with a light sheet-metal cover. Completely airtight doors are practically impossible to obtain, especially if they are in a haulage drift. Every precaution should be taken to see that the doors fit their frames and the floor of the drift closely. One of the primary reasons for excessive ventilation costs is the leakage of air to parts of the mine where it is not needed. The per cent of air actually reaching the desired destination is very low.[1] Figure 185*a* shows a common type of door. If the volume of air must be changed from time to time, a regulator may be placed in the door. A regulator is shown in Fig. 185*b*. This figure also shows another less desirable way to construct the door. Doors are hung so that they open against the air current. They usually lean slightly so that they are certain to close. It is a good plan to provide a small manway door in haulage drifts.

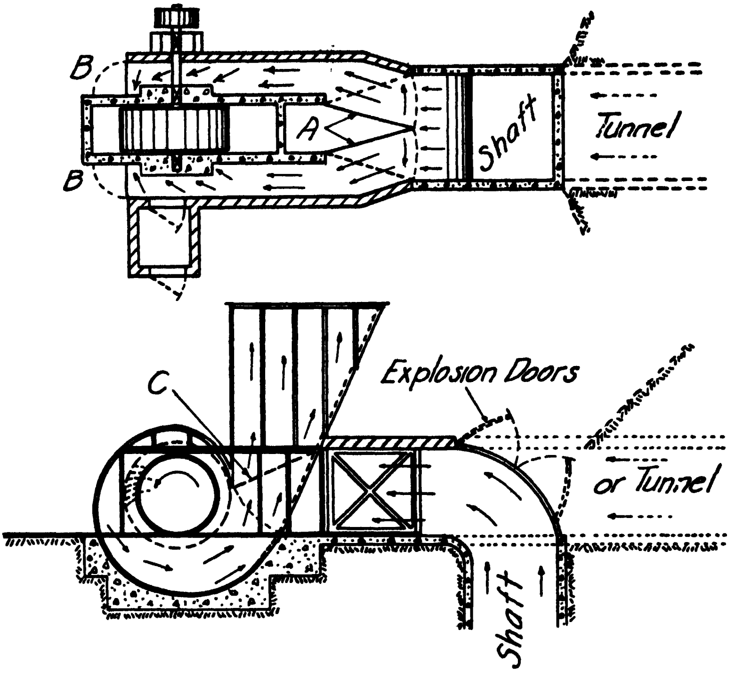

FIG. 186. Connection of centrifugal fan to mine. (*Montgomery, "Theory and Practice of Mine Ventilation."*)

Doors are usually opened automatically for the passage of trains. This may be accomplished by the use of compressed-air arrangements; by the use of elevated rails,[2] which the weight of the train depresses causing a linkage arrangement to open the doors; and by other methods.

Bulkheads (Stoppings). These may be permanent or temporary. They are used when a section of the mine is to be blocked off. Extreme care is necessary to get them even approximately airtight. Poor bulkheads are responsible for a large amount of fugitive air loss. Figure 185c shows two recommended types of stoppings.[1]

---

[1] MONTGOMERY, *op. cit.*, p. 22.

[2] American Mine Door Company, Canton, Ohio.

Connection of Fan to Mine. Figure 186 shows the connection of a centrifugal fan to a mine shaft or tunnel. State and Federal laws should be consulted for the minimum distance the fan must be from the mine opening. In Fig. 187 is shown the installation of an Aerodyne fan at the portal of the ventilating drift at the Sunshine Mine.[2] Figure 188 shows the underground installation at the Bunker Hill and Sullivan Mine.

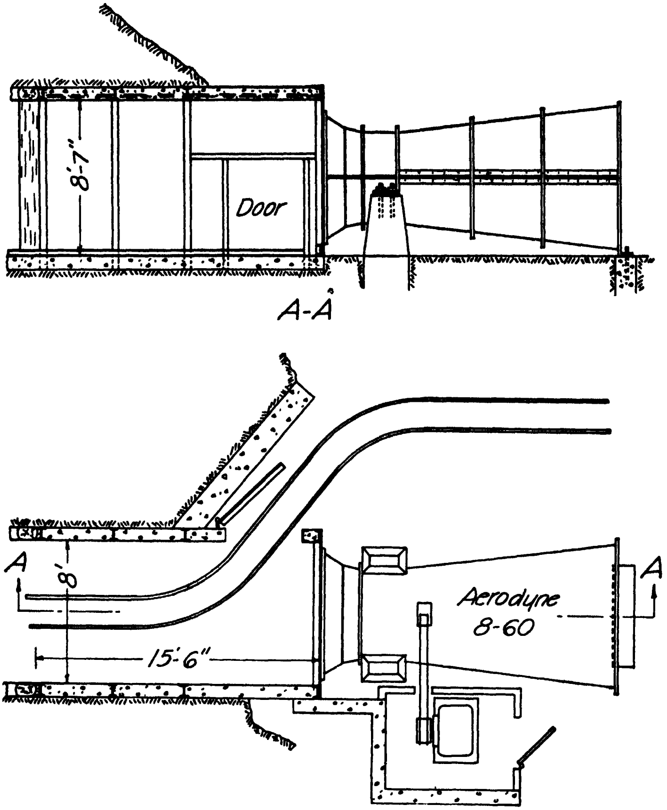

FIG. 187. Ventilation fan at the Sunshine Mine.

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[1] MONTGOMERY, *op. cit.*, p. 109.

[2] By permission of The Sunshine Mining Company.

This fan is located about 5000 ft from the portal of the main haulage tunnel.[1]

The fan manufacturers' recommendations must be carefully followed when connecting the fan to the mine. It is important to have the sides of the intake to the fan and the discharge flare at just the right angle in order to reduce the losses from velocity head to a minimum.

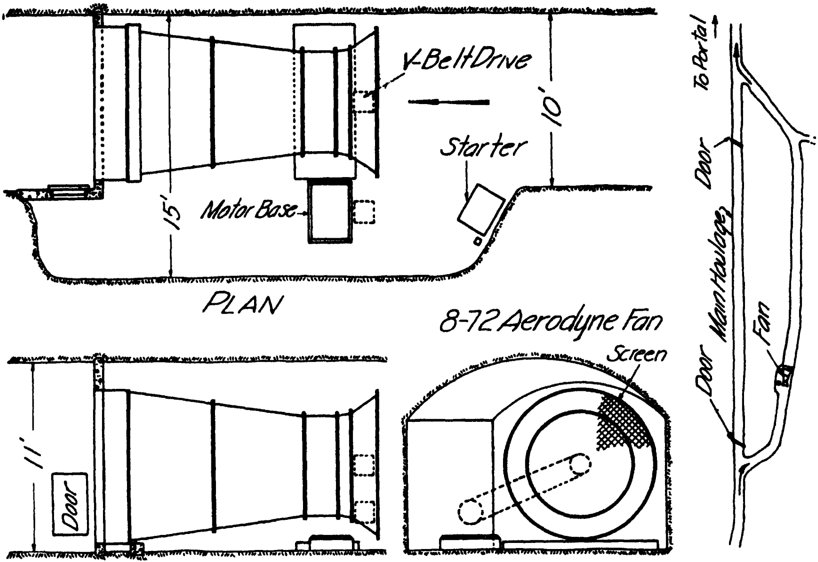

FIG. 188. Underground fan at the Bunker Hill and Sullivan Mine.

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[1] By permission of J. B. Haffner, general manager, Bunker Hill and Sullivan Mine.

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